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Exploitation, skills, and inequality*

Jonathan F. Cogliano,[†] Roberto Veneziani,[‡] and Naoki Yoshihara[§]

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Abstract

This paper uses a computational framework to analyse the equilibrium dynamics of exploitation and inequality in accumulation economies with heterogeneous labour. A novel index is presented which measures the intensity of exploitation at the individual level and the dynamics of the distribution of exploitation intensity is analysed. The effects of technical change and evolving social norms on exploitation and inequalities are also considered and an interesting phenomenon of exploitation cycles is identified. Various taxation schemes are analysed which may reduce exploitation or inequalities in income and wealth. It is shown that relatively small taxation rates may have significant cumulative effects on wealth and income inequalities. Further, taxation schemes that eliminate exploitation also reduce disparities in income and wealth but in the presence of heterogeneous skills, do not necessarily eliminate them. The inegalitarian effects of different abilities need to be tackled with a progressive education policy that compensates for unfavourable circumstances.

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1 Introduction

The common view—at least in economics, both in the mainstream but also for most heterodox scholars—is that the concept of exploitation cannot be defined coherently because of the logical flaws in the labour theory of value. Moreover, the notion of exploitation is considered to be metaphysical and obscure especially outside of economies with the simplest assumptions on preferences, technology and behaviour.

In particular, heterogeneous skills are usually deemed to pose insurmountable problems for the concept of exploitation. At the most general level, Marxian exploitation identifies a discrepancy between the labour 'given' by agents, in some relevant sense, and the labour 'received' by them, in some relevant sense. In simple economies with homogeneous labour, the agents' exploitation status is measured focusing on labour *time*. If individuals possess different skills, however, how should the amounts of labour given and received by them be measured? In units of labour time, or rather in terms of *effective*—or *skill-adjusted*—labour?

According to Roemer [22, 23], exploitation should be measured in units of effective labour but the concept of exploitation thus defined is not normatively meaningful, and the elimination of capitalist exploitation does not necessarily lead to a just society. In fact, an exploitation-free allocation requires income to be allocated in proportion to labour contributed and, in the presence of heterogeneous skills, this implies an unequal income distribution—a phenomenon that Roemer has dubbed 'socialist' exploitation. Actually, using a simple model of the U.S. economy, Roemer [23] has shown that, rather surprisingly, the elimination of exploitation would lead to *higher* income inequality than was actually experienced in the United States. This is an unpalatable conclusion for socialists and egalitarians, especially if skills are inherited and not acquired.[1]

In this paper, we analyse the concept of exploitation in economies with heterogeneous agents and skills. First, we provide a notion of exploitation that is logically coherent, well-defined, and firmly anchored to empirical data. Indeed, we show that exploitation can be defined both at the aggregate and at the individual level by means of an *exploitation index* which measures an agent's effective labour per unit of income received. For each individual, this index is a clearly defined magnitude that can be measured based on available empirical data, and its distribution can be analysed with the standard tools of the theory of inequality measurement.

Second, contrary to Roemer [22, 23], we show that the notion of exploitation is normatively relevant, and the analysis of the distribution of the exploitation index yields distinct insights on the injustice of capitalism and on the effects of redistributive policies. On the one hand, we argue that Roemer's [23] negative conclusions on

[1] Another problem arises in economies with heterogeneous labour *inputs* in production: it is unclear how labour performed by agents with different skills can be made uniform. This is the so-called problem of the reduction of complex labour to simple labour and we do not address it in this paper. For a discussion, see Yoshihara and Veneziani [34].

the inequalities persisting in the socialist allocation critically depend on his specific modelling framework, including his assumptions on preferences, technology and—crucially—the distribution of skills. In his analysis, Roemer assumes that the US labour market is perfectly competitive and high salaries reflect high skills. This is both theoretically and empirically doubtful. On the other hand, as Roemer ([23], p.24) himself notes, even granting that "the socialist allocation, given the distribution of skills in the United States today, would bring with it a relatively high degree of income inequality, ... [one may object that] under socialism, that distribution of skills would change". Yet, his models are inherently static, one period economies and cannot address the issue of evolution of the distribution of skills, income, wealth, and exploitation.

In this paper, we analyse a dynamic generalisation of Roemer's [22] accumulating economy with heterogeneous maximising agents. We assume that initial aggregate capital mimics the empirical wealth distribution for the U.S. and calibrate the distribution of skills in relation to wealth, such that the initial distribution of income is close to the empirical distribution of income for the U.S. Given the complexity of the model, we analyse the dynamics of the economy computationally, which allows us to derive definite conclusions on the distributive variables. The simulations confirm that indeed exploitation, income inequality and wealth inequality provide rather different normative insights and socialists and egalitarians may face trade-offs when implementing various policies.

Nonetheless, with a more realistic distribution of skills, Roemer's [23] negative conclusions are significantly qualified. Whether exploitation disappears due to over-accumulation leading to the disappearance of profits, or by means of wealth taxation, income and wealth inequalities in the socialist allocation are nowhere close to the values in Roemer [23]. The *static* trade-offs are much less severe than suggested by Roemer [23]. Furthermore, if a fraction of the revenues from wealth taxation are devoted to education and the growth of skills, it can be shown that, *dynamically*, the trade-off becomes less severe over time and can be led to vanish in the long run. Socialists and egalitarians may not face a major conundrum after all.

To be sure, our basic economy with constant technology and consumption is rather stylised and displays a rather simple—and rather unrealistic—dynamics whereby accumulation eventually drives exploitation and profits to zero. We therefore extend the model to incorporate endogenous technical change and varying consumption norms. The main conclusions our analysis continue to hold, even though capital using and labour saving technical progress tends to make exploitation persistent in the laissez faire regime: the concept of exploitation is well-defined and it yields normatively relevant insights. Indeed, the economy displays an interesting cyclical pattern, including cycles in exploitation intensity, and our analysis suggests that the exploitation index may shed some light on the cyclical, and crisis-prone, nature of capitalism.

Another contribution of the paper is methodological. Our analysis shows that computational methods can yield relevant insights in Marxian economics, and in social economics more generally. Computational techniques can be extremely useful as a device to generate thought experiments and to address some issues that cannot be easily tackled analytically. Given the complexity of our models, for example, they allow us to derive clear conclusions on our definition of exploitation, on the distribution of the exploitation index and on the dynamics of inequalities and exploitation. Pioneering work applying computational methods to Marxian theory includes Wright [38, 39, 40], Cogliano [5], and Cogliano and Jiang [6], though they focus on price and value theory and the circuit of capital rather than exploitation and class. More related to our work is a recent article by Cogliano et al. [7], which focuses on the mechanisms guaranteeing the persistence of exploitation in competitive economies with homogeneous labour. Our analysis here is significantly more general as it includes heterogeneous skills, alternative taxation schemes, and the implications of endogenous technical change and consumption norms.

2 The framework

In this section, and in the next, we set out the basic framework and the main definitions focusing on the economy with stationary population, technology, preferences, consumption norms, and labour endowments—the *basic economy*. This is for analytical clarity, as the basic economy provides a theoretical benchmark and starting point for our analysis. However, the framework, concepts, and definitions can be easily extended and the results derived continue to hold in more general economies (as confirmed also by the simulations).[2]

Consider a dynamic extension of Roemer's [22] accumulating economy with a labour market and only one good produced and consumed.[3] In every period $t = 1, 2, \ldots$, there is a set $\mathcal{N} = \{1, ..., N\}$ of agents in the economy where ν denotes a generic agent. At the beginning of each t, every agent can produce by activating a Leontief production technique (A, L), where A is the amount of the produced input necessary to produce one unit of output and L is the amount of effective (or skill-adjusted) labour necessary to produce one unit of output. We assume that the economy can produce a surplus $(0 < A < 1)$ and labour is indispensable $(L > 0)$.

In every t, agents are characterised by their endowment of labour time $\zeta^\nu > 0$, a skill factor $s^\nu > 0$, and capital endowment $\omega^\nu_{t-1} \geqq 0$. Agents are endowed with the same amount of labour time which is normalised to one: $\zeta^\nu = 1$ for all $\nu \in \mathcal{N}$. The skill factor s^ν of any agent modifies their labour endowment so that the endowment of *effective labour* of any agent ν is $l^\nu \equiv s^\nu \zeta^\nu = s^\nu$. The distribution of agents' effective labour and wealth endowments at the beginning of t are given by $\Pi = (l^\nu)_{\nu \in \mathcal{N}}$ and $\Omega_{t-1} = \left(\omega^\nu_{t-1}\right)_{\nu \in \mathcal{N}}$, respectively. An agent $\nu \in \mathcal{N}$ endowed with $\left(l^\nu, \omega^\nu_{t-1}\right)$ can engage

[2] We explore economies with technical change and variable consumption norms in section 9 below.

[3] Given our focus on the dynamics of exploitation, the one-good assumption yields no loss of generality. The model can be extended to include n commodities, albeit at the cost of a significant increase in technicalities and computational intensity.

in three types of production activity: she can sell a quantity z_t^ν of her labour power; she can hire others to operate a technique (A, L) at the level y_t^ν; or she can work on her own to operate (A, L) at the level x_t^ν. Total effective labour performed by agent ν at t comprises both self-employed labour and labour sold on the market, and is denoted by $\Lambda_t^\nu \equiv Lx_t^\nu + z_t^\nu$.

Following Roemer [21, 22], we assume that production takes time and current choices are constrained by past events. To be precise, wages are paid ex post and $w_t \geqq 0$ denotes the nominal wage rate at the end of t, but every agent must be able to lay out in advance the operating costs for the activities she chooses to operate using her wealth W_{t-1}^ν. Letting $p_t \geqq 0$ denote the price of the produced commodity at the end of t and beginning of $t+1$, the market value of agent ν's endowment—her wealth—is $W_{t-1}^\nu \equiv p_{t-1}\omega_{t-1}^\nu$. The wealth that is not used for production activities can be invested to purchase goods to sell at the end of the period, δ_t^ν.

Our main behavioural assumption postulates that agents wish to maximise their wealth,[4] subject to consuming a strictly positive amount b of the consumption good per unit of effective labour performed, where b identifies a socially-determined basic consumption standard incorporating social norms, culture, and so on.

Formally, in every t, given prices (p_{t-1}, p_t, w_t), every agent $\nu \in \mathcal{N}$ chooses $\xi_t^\nu \equiv (x_t^\nu; y_t^\nu; z_t^\nu; \delta_t^\nu)$ to maximise her wealth subject to purchasing b per unit of effective labour performed (1) and to the constraints set by her capital (2) and effective labour capacity (3). Formally, every ν solves the following programme MP_t^ν:

$$\max_{\xi_t^\nu \in \mathbb{R}_+^4} W_t^\nu = p_t\omega_t^\nu$$

subject to

$$\begin{aligned} p_t x_t^\nu + [p_t - w_t L]\, y_t^\nu + w_t z_t^\nu + p_t \delta_t^\nu &= p_t b \Lambda_t^\nu + p_t \omega_t^\nu & (1) \\ p_{t-1} A x_t^\nu + p_{t-1} A y_t^\nu + p_{t-1}\delta_t^\nu &= p_{t-1}\omega_{t-1}^\nu, & (2) \\ L x_t^\nu + z_t^\nu &\leqq l^\nu \equiv s^\nu. & (3) \end{aligned}$$

Let $\mathcal{A}^\nu(p_{t-1}, p_t, w_t)$ be the set of actions ξ_t^ν that solve MP_t^ν at prices (p_{t-1}, p_t, w_t). Let $W_t^{*\nu}$ be the value of MP_t^ν—that is the maximum wealth that can be accumulated at t. Let $(p, w) \equiv \{(p_t, w_t)\}_{t=1,...}$ and let $(x^\nu; y^\nu; z^\nu; \delta^\nu) \equiv \xi^\nu = \{\xi_t^\nu\}_{t=1,...}$. A *basic accumulation economy* is defined by agents $\mathcal{N}$, technology (A, L), effective labour endowments Π, and initial capital endowments Ω_0; and is denoted as $E(\mathcal{N}; (A, L); b; \Pi, \Omega_0)$, or, as a shorthand notation, E_0. We suppose that the economy can produce a surplus: $(1 - bL) > A$ or, equivalently, $1 - vb > 0$, where $v = L(1-A)^{-1}$ denotes the embodied labour value.

Let $x_t \equiv \sum_{\nu \in \mathcal{N}} x_t^\nu$, and likewise for y_t, z_t, δ_t, ω_t, c_t, Λ_t, and l. Based on Roemer [22], the equilibrium notion can be defined.

[4] "Accumulate, accumulate! That is Moses and the prophets!" (Marx [16], p. 742).

Definition 1. *A reproducible solution (RS) for* $E(\mathcal{N};(A,L);b;\Pi,\Omega_0)$ *is a vector* (p,w) *and associated actions* $(\xi^\nu)_{\nu\in\mathcal{N}}$*, such that at all* t*:*

(a) $\xi_t^\nu \in \mathcal{A}^\nu(p_{t-1},p_t,w_t)$*, for all* $\nu\in\mathcal{N}$ *(individual optimality);*

(b) $A(x_t+y_t)+\delta_t \leqq \omega_{t-1}$ *(capital market);*

(c) $Ly_t = z_t$ *(labour market);*

(d) $(x_t+y_t)+\delta_t \geqq b\Lambda_t^\nu + \omega_t$ *(goods market).*

At a RS, in every period: (a) all agents optimise; (b) aggregate capital is sufficient for production plans; (c) the labour market clears; (d) aggregate supply is sufficient for consumption and accumulation plans. E_0 can thus be interpreted either as a sequence of generations living for one period or as an infinitely-lived economy analysed in a sequence of temporary equilibria.[5]

For any (p,w), the profit rate at t is $\pi_t = \frac{p_t - p_{t-1}A - w_t L}{p_{t-1}A}$. Given the structure of the economy, we shall focus on equilibria with strictly positive prices, so that the profit rate is well defined at all t.[6] By constraint (2), it immediately follows that at any RS, only (p_t,w_t) matter for individual choices at all t and so we can take the produced commodity as the numéraire, setting $p_t=1$, all t.[7] Let the normalised price vector be denoted as $(\mathbf{1},\widehat{w})$, where $\mathbf{1}=(1,1,...)$ and, at any t, $\widehat{w}_t$ is the real wage rate and $\pi_t = \frac{1-A-\widehat{w}_t L}{A}$. In what follows, with a slight abuse in notation, in the analysis of individual choices at t, we shall simply refer to the price vector $(1,\widehat{w}_t)$.

Given the previous observations and constraints (1)-(2), it follows that at any RS, for all $\nu\in\mathcal{N}$ and all t, the following equation must hold

$$\omega_t^\nu = [1-A-\widehat{w}_t L](x_t^\nu + y_t^\nu) + (\widehat{w}_t - b)(Lx_t^\nu + z_t^\nu) + \omega_{t-1}^\nu. \tag{4}$$

Equation (4) has a number of implications.[8] First, it is immediate to prove from $(1-bL) > A$ that at any RS, if $\omega_{t-1} > 0$, then $\widehat{w}_t \geqq b$ and $\pi_t \geqq 0$, all t. Second, at any t, if the profit rate is strictly positive, then all wealth is used productively,

[5] The concept of RS may seem to impose stringent requirements on individual rationality. For agents trade in the good and labour market at the beginning of each period based on expectations of prices that will form at the end of the period, and in equilibrium these expectations are exactly correct. Yet, the RS is a *temporary* equilibrium notion, and so it imposes much less stringent rationality and consistency requirements than standard *intertemporal* optimisation models. For a thorough defence of the notion of RS, see Cogliano et al [7].

[6] It immediately follows from MP_t^ν that if there is some t' such that $p_{t'}=0$, then at any RS it must be $p_t = 0$ for all $t > t'$.

[7] Differences in beginning-of-period prices, p_{t-1}, and end-of-period prices, p_t, are inconsequential for agents' choices. At the beginning of t, given p_{t-1} and the expected (p_t,w_t), for every unit of wealth stored to be sold at the end of t one foregoes A^{-1} units of output produced at the end of t. Therefore one will invest productively (rather than storing the good) provided $(p_t - w_t L)A^{-1} \geqq p_t$: beginning of period prices do not enter the decision.

[8] The proofs of all of the following claims and of Theorems 1 and 2 below are straightforward extensions of the proofs in Cogliano et al. [7] and are therefore omitted.

and if the wage rate is above the minimum standard b, then the labour constraint (3) binds, for all agents at the solution to MP_t^ν. Formally: at any t, if $\pi_t > 0$, then $A\left(x_t^\nu + y_t^\nu\right) = \omega_{t-1}^\nu$, all $\nu \in \mathcal{N}$; and if $\widehat{w}_t > b$, then $Lx_t^\nu + z_t^\nu = l^\nu$, all $\nu \in \mathcal{N}$. Third, in equilibrium, at any t, the maximum wealth accumulated by any agent is

$$\omega_t^{*\nu} = (1+\pi_t)\,\omega_{t-1}^\nu + (\widehat{w}_t - b)\,l^\nu,$$

and therefore the growth rate of capital for each agent is

$$g_t^\nu = \pi_t + (\widehat{w}_t - b)\,\frac{l^\nu}{\omega_{t-1}^\nu},$$

while the aggregate growth rate of the economy is $g_t = \pi_t + (\widehat{w}_t - b)\,\frac{l}{\omega_{t-1}}$.

We conclude the analysis of the basic economy by characterising its equilibria.

Theorem 1. *Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be a RS for E_0. At any t:*

(i) If $\pi_t > 0$ and $\widehat{w}_t > b$, then $l = LA^{-1}\omega_{t-1}$;

(ii) If $l > LA^{-1}\omega_{t-1} > 0$ then $\widehat{w}_t = b$;

(iii) If $l < LA^{-1}\omega_{t-1}$ then $\pi_t = 0$.

Theorem 1 defines the theoretical framework for the analysis of the dynamics of the economy. Although it only identifies necessary conditions for the existence of a RS, it does shed some light on how to construct the dynamic general equilibria. Consider part (ii). Suppose $l > LA^{-1}\omega_{t-1}$, some t. If $\widehat{w}_t = b$, then $\pi_t = \pi^{\max} \equiv \frac{1-A-bL}{A}. > 0$ and labour performed does not produce any net income for accumulation, and for all $\nu \in \mathcal{N}$, any $(0; y_t^\nu; z_t^\nu; 0)$ with $Ay_t^\nu = \omega_{t-1}^\nu$ solves MP_t^ν. Therefore since $Ay_t = \omega_{t-1}$ and $l > LA^{-1}\omega_{t-1}$, we can choose a suitable profile $(z_t^\nu)_{\nu\in\mathcal{N}}$ such that $Ly_t = z_t$ and all conditions of Definition 1 are satisfied at t.

Consider part (iii). Suppose $l < LA^{-1}\omega_{t-1}$, some t. If $\pi_t = 0$, then $\widehat{w}_t = \frac{1}{v} > b$ and capital holders are indifferent between using their wealth productively and just carrying it for sale at the end of the period, and for all $\nu \in \mathcal{N}$, any $(0; y_t^\nu; z_t^\nu; \delta_t^\nu)$ with $z_t^\nu = l^\nu$ solves MP_t^ν. Therefore since $z_t = l$ and $l < LA^{-1}\omega_{t-1}$, we can choose a suitable profile $(y_t^\nu)_{\nu\in\mathcal{N}}$ such that $Ly_t = z_t$ and all conditions of Definition 1 are satisfied at t.

3 Exploitation

The concept of exploitation can now be introduced. In what follows, exploitation status is defined in every period t: this is a natural assumption if the model describes a series of one-period economies, otherwise it reflects a focus on *within period* exploitation.[9] Definition 2 identifies exploitation status in terms of the bundles of goods that

[9] For a discussion of *within period* and *whole life* exploitation, see Veneziani [30, 31].

an agent *can* purchase with her income. More precisely, at any RS (p, w) and for all $\nu \in \mathcal{N}$, let c_t^ν satisfy $p_t c_t^\nu = p_t \omega_t^\nu + p_t b \Lambda_t^\nu - p_t \omega_{t-1}^\nu$ for every t. Then, the following definition is an extension of Roemer [22] into economies with heterogenous labour and $b > 0$.

Definition 2. *Agent ν is exploited at t if and only if $\Lambda_t^\nu > v c_t^\nu$; she is an exploiter if and only if $\Lambda_t^\nu < v c_t^\nu$; and she is neither exploited nor an exploiter if and only if $\Lambda_t^\nu = v c_t^\nu$.*

Definition 2 identifies exploitation status focusing on *effective* labour. As argued by Veneziani and Yoshihara [34], this is the natural extension of all of the classic definitions of exploitation, and it is the approach adopted in much of the literature on exploitation in economies with heterogeneous labour (see, e.g., Krause [14]; Duménil *et al* [10]). According to Definition 2, the concept of exploitation measures discrepancies in the amount of labour that agents *contribute* to the economy and the amount that they receive, via their income. The '*contribution view*' incorporates an important normative intuition: an efficient and UEL exploitation-free allocation coincides with the *proportional solution*, a well-known fair allocation rule whereby every agent's income is proportional to her contribution to the economy (Roemer and Silvestre [25]). Proportionality is a strongly justified normative principle, whose philosophical foundations can be traced back to Aristotle, and it can be justified in terms of the Kantian categorical imperative (Roemer [24]). The contribution principle ('To each according to his contribution') is also one of the principles of justice analysed by Marx in the *Critique of the Gotha programme* [15] (for a discussion see Cohen [8]).

Theorem 2 characterises the exploitation status of every agent, based on their wealth per unit of labour performed $\frac{\omega_{t-1}^\nu}{\Lambda_t^\nu}$:

Theorem 2. *Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be a RS for E_0. At any t, if $\pi_t > 0$:*
(i) agent ν is an exploiter $\Leftrightarrow \frac{\omega_{t-1}^\nu}{\Lambda_t^\nu} > \frac{1}{\pi_t} \frac{[1-\widehat{w}_t v]}{v}$;
(ii) agent ν is exploited $\Leftrightarrow \frac{\omega_{t-1}^\nu}{\Lambda_t^\nu} < \frac{1}{\pi_t} \frac{[1-\widehat{w}_t v]}{v}$;
(iii) agent ν is neither exploited nor an exploiter $\Leftrightarrow \frac{\omega_{t-1}^\nu}{\Lambda_t^\nu} = \frac{1}{\pi_t} \frac{[1-\widehat{w}_t v]}{v}$.

Theorem 2 generalises analogous results by Roemer [22] as it allows for unemployed labour. If $\Lambda_t^\nu = l^\nu$, all $\nu \in \mathcal{N}$, then by Theorem 2 exploitation status is determined by the ratio of capital and labour *endowments* as in Roemer [22]. If the economy is characterised by unemployed labour, however, $\Lambda_t^\nu < l^\nu$ for at least some $\nu \in \mathcal{N}$ and exploitation status is determined by the ratio of the capital endowment *and labour performed*, $\frac{\omega_{t-1}^\nu}{\Lambda_t^\nu}$.

Theorem 2 holds if $\pi_t > 0$. If $\pi_t = 0$ then $\widehat{w}_t = (1/v) > b$ and $\Lambda_t^\nu = v c_t^\nu$ for all $\nu \in \mathcal{N}$ and no exploitation exists in the economy according to Definition 2.

This correspondence between profits and exploitation is a standard result in Marxian theory (for a discussion, see Veneziani and Yoshihara [32]).

Theorem 2 provides important normative insights on the structural injustices characterising capitalist economies. Yet, an exclusive focus on the sets of exploiters and exploited agents yields a rather partial, coarse picture of the structure of exploitative relations: two economies with similar numbers of agents belonging to each set may still be very different. Based on Definition 2, it is possible to extend the normative reach of the concept of exploitation and provide a finer and more comprehensive picture of exploitative relations. For Definition 2 allows us to move beyond a purely aggregate analysis and explore the exploitation status of every agent. This immediately raises the issue of the measurement of the *intensity of exploitation*, both at the individual and at the aggregate level. It is certainly desirable to have a notion of exploitation that allows us to make statements such as "agent A is less exploited than agent B", or "Economy C is more exploitative than economy D", or "Economy E is becoming increasingly exploitative over time".

Based on Definition 2, we examine an index that measures exploitation intensity for each individual:

$$\varepsilon_t^\nu = \frac{\Lambda_t^\nu}{vc_t^\nu},$$

An analysis of exploitation status at the individual level raise a number of novel, interesting issues in exploitation theory both at the theoretical and at the empirical level, and it highlights some important formal and conceptual similarities between exploitation theory and the standard theory of inequality measurement. According to the exploitation intensity index ε_t^ν, an agent ν is exploited if and only if $\varepsilon_t^\nu > 1$, whereas they are an exploiter if and only if $\varepsilon_t^\nu < 1$. Yet, the index provides a much finer and nuanced description of exploitation. For each individual, the exploitation index is a well defined magnitude based on available empirical data and the distribution of the exploitation indices can be analysed with the standard tools of the theory of inequality measurement. Just like for income inequalities, one can analyse differences in exploitation intensity across countries, or the evolution of exploitation intensity within a given country over a certain period of time by focusing on the distribution of ε_t^ν. What is the appropriate way of capturing the key characteristics of $(\varepsilon_t^\nu)_{\nu \in \mathcal{N}}$? In this paper, we focus on the Gini coefficient of $(\varepsilon_t^\nu)_{\nu \in \mathcal{N}}$, denoted as γ_t^ε, but alternative measures can be used. We return to this issue in the concluding section.

4 The benchmark simulation routine

This section presents the benchmark routine used for all simulations in the paper, unless otherwise stated.[10] All simulations run for $T = 50$ periods. In each period

[10] All simulations are done using *Mathematica* version 11 and the code is available from the authors upon request.

t, the subsistence level b_t serves as a lower limit for the wage $\widehat{w}_t$, with $\widehat{w}_t = b_t$ for any t in which the economy is capital constrained, $\widehat{w}_t = 1/v_t$ for any t in which the economy is labour constrained, and $b_t \leqq \widehat{w}_t \leqq 1/v_t$ for any t in which the economy is on the knife-edge.

Lemma 3 of Cogliano et al. [7] proves that if $(x_t^\nu; y_t^\nu; z_t^\nu; \delta_t^\nu)$ solves MP_t^ν, then there is another vector $(0; y_t'^\nu; z_t'^\nu; \delta_t^\nu)$ which solves MP_t^ν. In the simulations, this allows us to select one of the many potential solutions of MP_t^ν by setting $x_t^\nu = 0$ for all $\nu \in \mathcal{N}$. Specifically, at any t, we set $\xi_t^\nu = \left(0; A_t^{-1}\omega_{t-1}^\nu; \frac{L_t A_t^{-1}\omega_{t-1}}{l} l^\nu; 0\right)$, $\xi_t^\nu = \left(0; \frac{l}{L_t A_t^{-1}\omega_{t-1}} A_t^{-1}\omega_{t-1}^\nu; l^\nu; \left(1 - \frac{l}{L_t A_t^{-1}\omega_{t-1}}\right)\omega_{t-1}^\nu\right)$, or $\xi_t^\nu = \left(0; A_t^{-1}\omega_{t-1}^\nu; l^\nu; 0\right)$, for all ν, depending on whether the economy is capital constrained, labour constrained, or on the knife-edge. This specification of agents' optimal choices guarantees that Definition 1 is always satisfied across all simulations.[11]

The simulations begin with data on $(\mathcal{N}; (A_0, L_0); b_0; \Pi, \Omega_0)$. Unless otherwise stated, all simulations are initialised with the following standard parameters:

Standard parameters: $N = 100$, $A_0 = 0.5$, $L_0 = 0.25$, and $b_0 = 1.9$.

The distribution of initial aggregate capital Ω_0 mimics the empirical wealth distribution for the U.S. (Allegretto [3]) and is similar to the method used in Cogliano et al. [7]. At $t = 0$, ω_0 is distributed such that there are five groups of agents. The first group comprises 50% of the total population and agents in this group are assigned $\omega_0^\nu = 0$. The top 1% of agents are assigned 40% of ω_0, the next 4% are assigned 30% of ω_0, the next 15% are assigned 20%, and the remaining 10% of ω_0 is distributed to the remaining 30% of N.[12]

The skill factors s^ν are generated such that the initial distribution of income $\left((1+\pi_t)\omega_{t-1}^\nu + \widehat{w}_t\Lambda_t^\nu\right)_{\nu\in\mathcal{N}}$ is close to the empirical distribution of income for the U.S. Using the same sorting of agents as in the determination of Ω_0, at $t = 0$ an initial aggregate skill endowment $s = 750$ is distributed across agents so that the first quintile of agents is assigned 8.12% of s, the second quintile is assigned roughly 18.93% of s, the third quintile is assigned 28.56% of s, and the fourth quintile is assigned roughly 30.79% of s. These s^ν are increasing over the first 80% of agents. The next fifteen percent of agents are assigned roughly 13.42% of s, the next four percent of agents are assigned 0.15% of s, and the top one percent of agents are assigned whatever remains of s, which is inevitably the smallest share of all agents. The skill factors over the

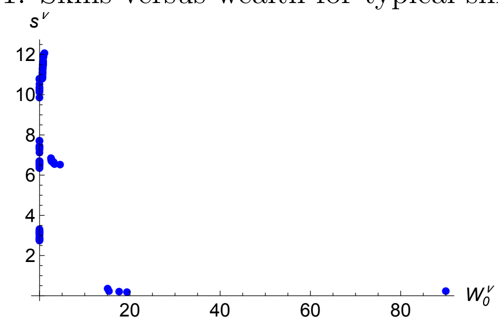
[11] This choice has no implications for the analysis of exploitation, because the agents' exploitation status does *not* depend on the specific solution to MP_t^ν considered. Observe that in the specification of ξ_t^ν we are implicitly assuming that all agents activate the same production technique. This is trivially true in the basic economy, but it also holds in the economy with technical change in section 9 as a corollary of profit maximisation.

[12] There can be some variation in the initial distribution of wealth across models due to different starting points in relation to the knife-edge and randomness built into the initial distribution procedure. However, the differences are sufficiently small that simulation results are unaffected and comparable across models.

final 20% of agents are decreasing in magnitude. Within each group of agents there is a degree of randomness in the assigned skill factors so that agents have different s^ν and s^ν are increasing within each group.

This assignment of skill factors results in the highest skills existing at the top of the fourth quintile of agents, thus these agents have the highest labour income $\widehat{w}_t \Lambda_t^\nu$. After the fourth quintile of agents s^ν are decreasing in wealth as capital income $(1+\pi_t)\omega_{t-1}^\nu$ begins to make up a larger portion of agents' income, with the top one percent of agents deriving nearly all of their income from owning capital. This method of determining $(s^\nu)_{\nu\in\mathcal{N}}$ is applied to all simulations that follow. Figure 1 shows a sample distribution of skills in relation to wealth.

Figure 1: Skills versus wealth for typical simulation



With the allocations of s^ν described above the initial distribution of shares of aggregate income $\left(\frac{(1+\pi_1)\omega_0^\nu+\widehat{w}_1\Lambda_1^\nu}{\sum_\nu (1+\pi_1)\omega_0^\nu+\widehat{w}_1\Lambda_1^\nu}\right)_{\nu\in\mathcal{N}}$ by quintile, top 5% of agents, and top 1% is close to the empirical distribution of income for the U.S. For a typical run of the simulations, at $t=1$, the bottom quintile earns around 3.5-4% of aggregate income, the second quintile earns 8.5-9%, the third quintile 14-15%, the fourth quintile 18-20%, and the fifth quintile 52-55%. The top 5% of agents receives roughly 37% of aggregate income and the top 1% receives 20-22%. There is some variation in the distribution across these groups in different simulations due to the small degree of randomness in determining the distribution of skills and Ω_0, but the small variation does not induce any qualitative differences in the simulations. This initial distribution of income is close to figures reports by the U.S. Bureau of the Census [28] and different measures of the income share of the top 1% reported by Mishel et al. [17] (Table 2AA) for recent years. The Gini coefficient of the initial distribution of income is slightly higher than, yet close to, that reported by the Census Bureau [29] and Guzman [13].

Given the standard parameters and the choice of Ω_0, the economy is initially capital constrained since $l > L_0 A_0^{-1}\omega_0$ and begins far from the knife-edge condition $l = L_0 A_0^{-1}\omega_0$. Starting far from the knife-edge allows for the examination of the evolution of exploitation dynamics.

5 The basic economy with skilled-labour

The simulation of the basic model begins with the *standard parameters* and $A_t = A_0$, $L_t = L_0$, and $b_t = b_0$ all t. The simulation runs by first checking whether the economy is capital constrained, labour constrained, or on the knife-edge and determines $\widehat{w}_t$ accordingly. With $\widehat{w}_t$ determined, π_t is then known and agents solve MP_t^ν. Agents' endowments update according to equation (4) and the simulation repeats as necessary.

Figure 2 reports the summary results for the basic model. The simulation shows steady growth of activity levels (y_t, z_t) and net output $(1 - A)y_t$ until the simulation becomes labour constrained - denoted by the vertical dashed line in the diagrams.[13] The growth rate of aggregate endowments and the profit rate are also steady as long as the simulation is capital constrained.

Figure 2: Summary results - Basic model with skilled-labour

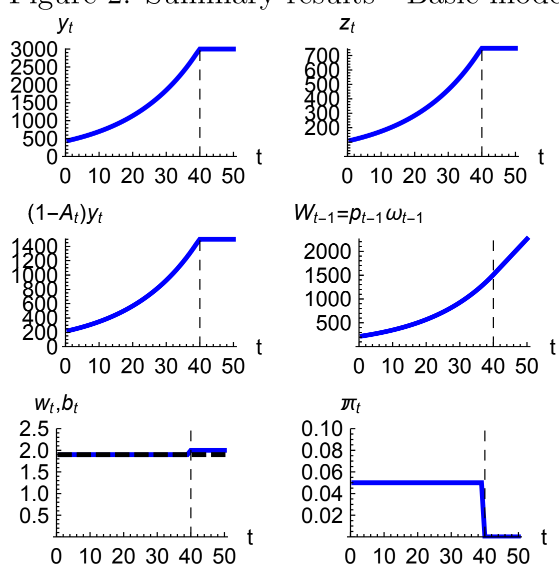


Figure 3(a) shows that the structure of exploitation is relatively stable as long as the simulation is capital constrained, but as soon as the simulation is labour constrained exploitation disappears. Figure 3(b) displays the distribution of the exploitation intensity index ε_t^ν over t. The vertical axis in figure 3(b) shows the agents numbered 1 to 100 arranged by their initial wealth so that the 100th agent is the wealthiest agent, this ordering is abbreviated as $\nu(\Omega_0)$. Figure 3(b) shows a clear pattern of exploitation up until the point at which the economy becomes labour constrained. Agents who are exploited experience $\varepsilon_t^\nu > 1$ consistently and agents who are exploiters experience $\varepsilon_t^\nu < 1$ for all t, until the economy becomes capital constrained. The presence of heterogeneous skills does not significantly alter the structure of exploitative relations: the wealthiest agents exploit and the poorest ones are exploited.

[13]Given our construction of the agents' optimal choices, $x_t = 0$ at all t and therefore the results for this variable are not shown.

The Gini coefficient of exploitation intensity γ_t^{ε}—not shown—is steady at 0.05744 while the simulation is capital constrained and zero thereafter.

Figure 3: Exploitation - Basic model with skilled-labour

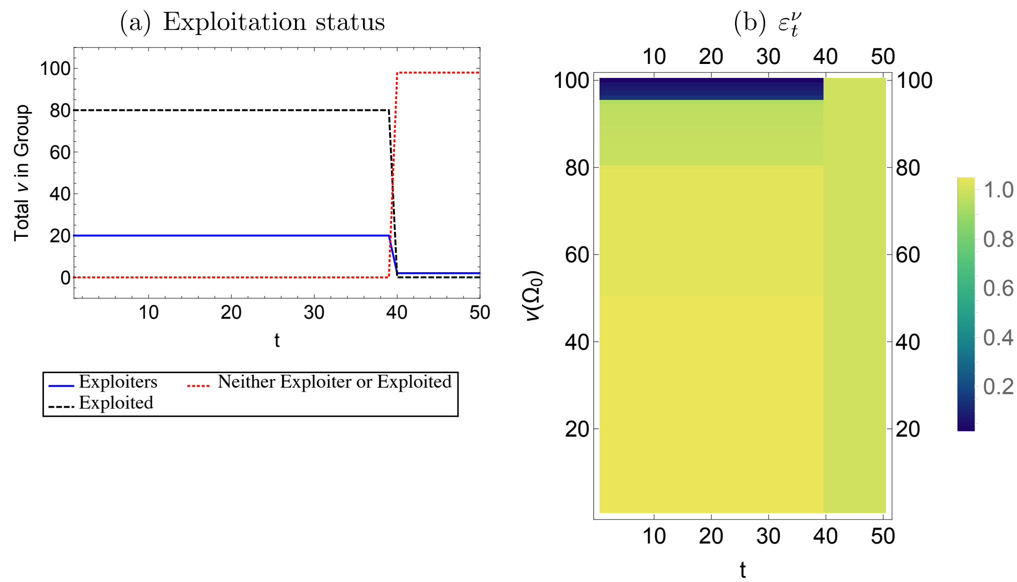


Figure 4(a) shows that the Gini coefficient of wealth, denoted as γ_t^W, is stable until the economy is labour constrained, at which point wealth inequality begins to steadily decline as all agents accumulate. Figure 4(b) shows the distribution of wealth for select t.

Figure 4: Distribution of wealth - Basic model with skilled-labour

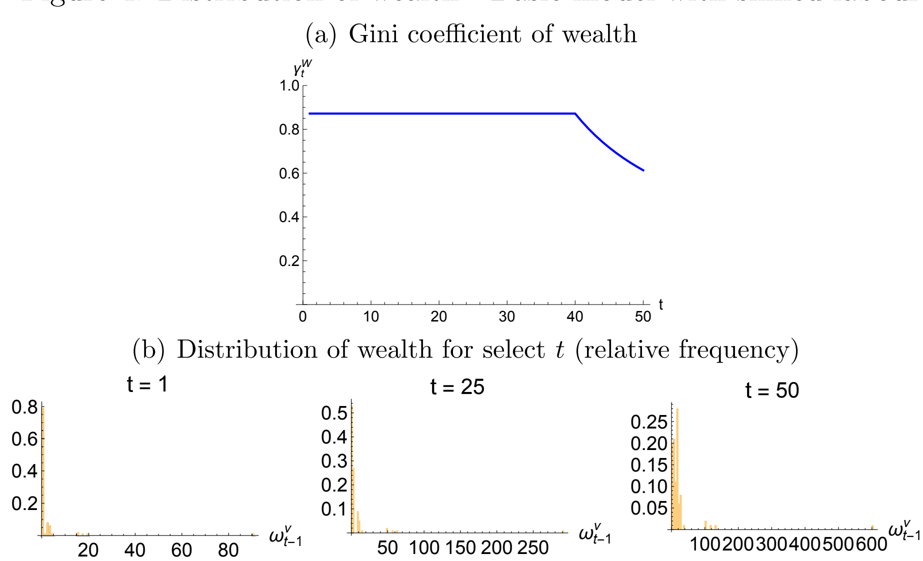
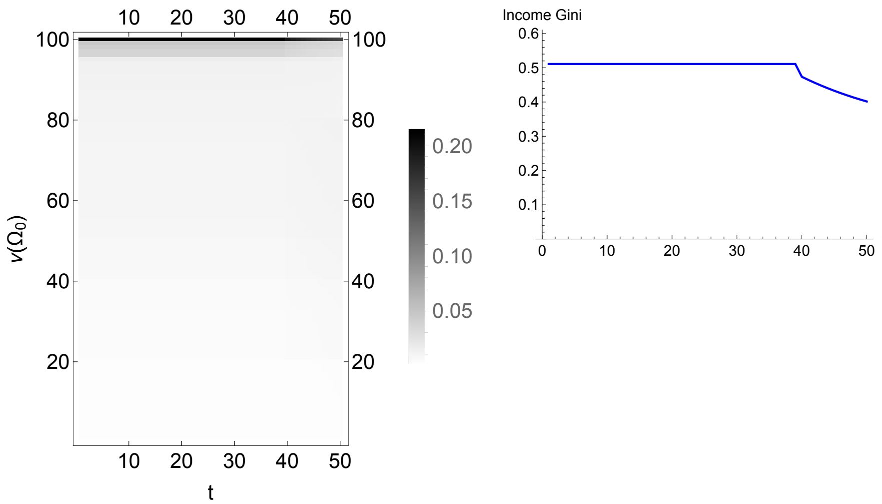


Figure 5 shows the distribution of shares of income $\frac{(1+\pi_t)\omega_{t-1}^{\nu}+\widehat{w}_t\Lambda_t^{\nu}}{\sum_{\nu}(1+\pi_t)\omega_{t-1}^{\nu}+\widehat{w}_t\Lambda_t^{\nu}}$ over the course of the simulation. There is noticeable income inequality, which remains somewhat stable until the simulation becomes labour constrained, at which point income inequality starts declining.

Figure 5: Distribution of income - Basic model with skilled-labour

(a) Distribution of income shares over t (b) Gini coefficient of income



In summary, two general conclusions can be drawn from our results. First, the presence of heterogenous skills in the population poses no insurmountable conceptual problem for exploitation theory: the notion of exploitation remains theoretically robust, conceptually well defined, and grounded on empirically measurable magnitudes. Second, compared with the basic model first explored in Cogliano et al. [7], the incorporation of skills provides a more complex picture from a normative perspective. For the simulations clearly show that exploitation, income inequality and wealth inequality provide rather different normative insights. Before it becomes labour constrained, the economy displays significant and steady levels of exploitation, income inequality and wealth inequality. Yet, while exploitation disappears as soon as the simulation is labour constrained, income and wealth inequality decrease over time but do not go to zero. As originally noted by Roemer [23], this implies that socialists and egalitarians may face significant trade-offs when implementing various policies.

Some questions immediately arise from our results. First, accumulation eventually drives exploitation and profits to zero: after settling on an accumulation path that smoothly leads to overaccumulation, the economy becomes labour constrained. Arguably, this is unrealistic, and exploitation is a persistent feature of capitalist economies. What are the mechanisms that guarantee the persistence of (capital scarcity and) exploitation in capitalist economies? And, relatedly, what is the relation between the persistence of exploitation and the cyclical and crisis-prone dynamics of capitalist economies? Second, until the economy becomes labour constrained, it displays relatively high levels of exploitation and inequality. What kind of policies can be implemented to alleviate this? And are there any policies that can tackle at the same time exploitation and inequalities? We explore the latter question first in the next sections where we extend the basic model to include three different types of redistributive wealth taxes.

6 Basic economy with wealth taxes

In this section we analyse the effect on exploitation and inequality of a wealth taxation scheme similar to that proposed by Piketty [20]. We assume that wealth taxes are paid at the end of t once agents have solved MP_t^ν and determined their wealth $p_t\omega_t^\nu$ for the next time period $t+1$. The tax scheme works to redistribute wealth from relatively wealthy agents to agents with less wealth. Given that all agents consume at subsistence b according to how much labour they perform, any transfer that a relatively poor agent receives adds to their wealth. Because the transfers of wealth via taxation do not affect consumption or agents' decisions of how much labour to perform, the addition of wealth taxes does not affect how agents choose ξ_t^ν to solve MP_t^ν since each agent is still maximising their revenue subject to the usual constraints. Thus wealth taxes can be easily incorporated into the benchmark simulations without altering the optimisation programme, equilibrium conditions, or definitions of exploitation and classes presented earlier in the paper.[14] The details of the tax structure are as follows.

Let the wealth tax that any agent ν pays at the end of any t be denoted by τ_t^ν, with the distribution of tax rates across all ν denoted by $(\tau_t^\nu)_{\nu\in\mathcal{N}}$. All $\nu \in \mathcal{N}$ are assigned a tax rate $\tau_t^\nu \in \{0, 0.005, 0.02, 0.05\}$ according to where they fall in the distribution of wealth at the end of t, $(\omega_t^\nu)_{\nu\in\mathcal{N}}$.[15] Agents with wealth at or below the median face $\tau_t^\nu = 0$, agents with wealth between the median and the 75th percentile face $\tau_t^\nu = 0.005$, agents with wealth at or above the 75th percentile up to and including the 99th percentile face $\tau_t^\nu = 0.02$, and agents at the top one percent of the wealth distribution face $\tau_t^\nu = 0.05$. At the end of every t, taxes are collected from agents for whom $\tau_t^\nu > 0$ and the wealth collected through taxes is evenly redistributed to the $\nu \in \mathcal{N}$ for whom $\tau_t^\nu = 0$. Taxes are redistributed evenly across ν with $\tau_t^\nu = 0$.

The simulation is initialised as in section 4 and $A_t = A_0$, $L_t = L_0$, and $b_t = b_0$ all t. The summary results are qualitatively the same as the basic model and not

[14]There is an additional reason to focus on a scheme of wealth taxation that has no effect on the rate of accumulation. Exploitation can be eliminated either by pushing profits to zero or by redistributing wealth so as to make labour performed proportional to income. By assuming taxation to have the particular structure in this paper, we are able to separate the dynamics of exploitation arising from changes in profitability (and capital scarcity), and the dynamics of exploitation determined by changes in wealth distribution.

[15]The top tax rate on wealth is adopted from the minimum suggested by Piketty [20].

pictured here.[16]

Figure 6(a) reports the post-tax dynamics of exploitation over the course of the simulation. As in the basic model, there is a consistent structure of exploitation as long as the simulation remains capital constrained, but as the simulation evolves and taxes redistribute wealth, the number of exploited agents decreases as the number of exploiters rises. Figure 6(b) shows the post-tax distribution of ε_t^ν over the simulation. Two features of the distribution of ε_t^ν are worth emphasising. First, as in the basic economy, exploitation does not disappear until the simulation is labour constrained. Unlike in the basic economy, in which $(\varepsilon_t^\nu)_{\nu \in \mathcal{N}}$ is constant over time, however, the distribution of exploitation intensity varies over the course of simulation as a result of the wealth taxes.

Second, as expected, agents at the top of the wealth distribution do not experience exploitation due to their high wealth holdings (and relatively low skills), but as the simulation progresses social relations become less exploitative and they come closer to the exploitation threshold as their wealth is taxed away. Perhaps more surprisingly, agents at the bottom of the wealth distribution do not experience exploitation after $t > 1$ either, due to the receipt of wealth transfers and their relatively low skills. The behaviour of ε_t^ν at the extremes of the distribution creates an interesting situation where agents with mid-range values of s^ν and little to no wealth experience exploitation most intensely. Some agents within this group possess enough wealth to pay taxes while others possess no wealth and receive transfers, however, this group effectively constitutes a "middle class" that, due to their skill levels, perform the most effective labour relative to their potential means of consumption.

Figure 6(c) shows post-tax values of γ_t^ε, which decreases only slightly over the course of the simulation until the economy becomes labour constrained. The slight "saw-tooth" pattern in γ_t^ε is the result of agents shifting between different wealth tax rates as their endowments are redistributed, thereby altering vc_t^ν.

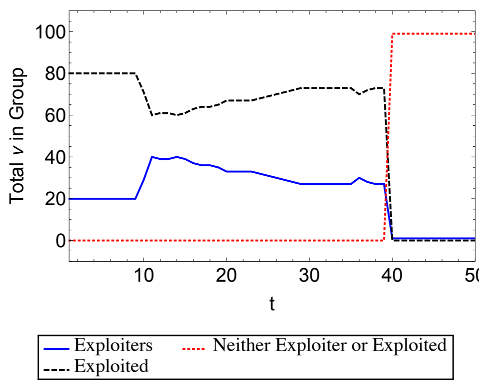
Figure 7(a) shows the Gini coefficient of wealth, γ_t^W, while Figure 7(b) shows the distribution of wealth for select t. Interestingly, while wealth taxation has a relatively small impact on exploitative relations, γ_t^W steadily, and rapidly, declines over t, clearly showing how effective even small tax rates on wealth can be in reducing wealth inequality.

Figure 8 shows the dynamics of the post-tax distribution of income $(1 + \pi_t)\omega_{t-1}^\nu + \widehat{w}_t \Lambda_t^\nu$. Figure 11(a) shows the post-tax distribution of income shares across agents for all t: as all agents start accumulating thanks to wealth redistribution, the distribution of income shares tends to shrink over t, and inequality tends to decrease as non-labour income tends to become more equal. The residual income inequality is due to skill differentials, but is much smaller than at the beginning of the simulation. Once the simulation is labour constrained, capital income is zero for all agents, and income

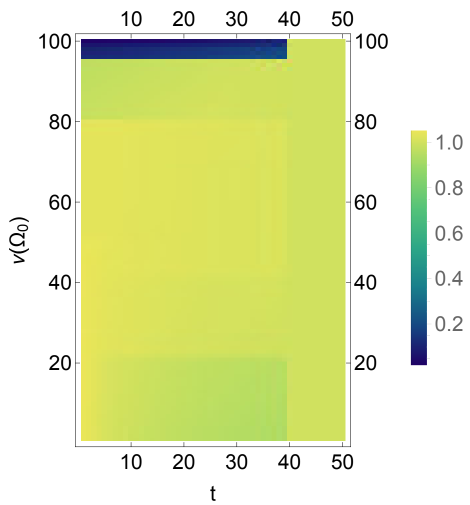
[16] Because the behaviour of the aggregate variables is qualitatively the same across the two models, wealth taxes are macroeconomically neutral in that they do not alter the aggregate performance of the economy. This allows us to focus exclusively on the distributional effects of taxation.

Figure 6: Exploitation - Basic model with wealth taxes

(a) Post-tax Exploitation status



(b) Post-tax ε_t^ν



(c) Gini coefficient of ε_t^ν

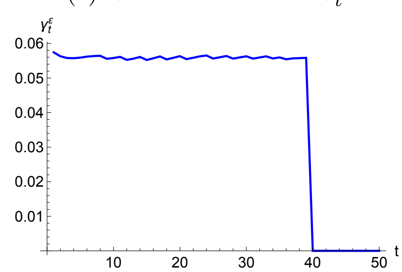
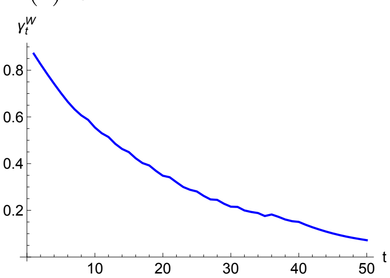
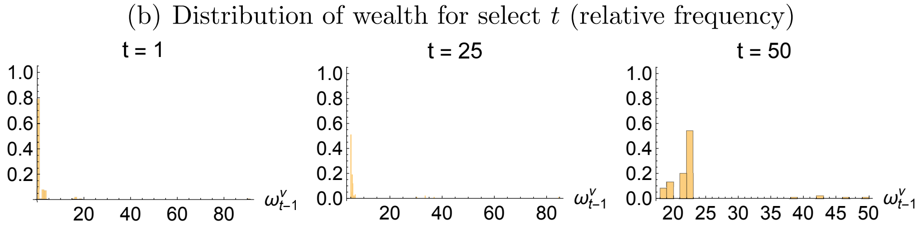


Figure 7: Distribution of wealth - Basic model with wealth taxes

(a) Gini coefficient of wealth



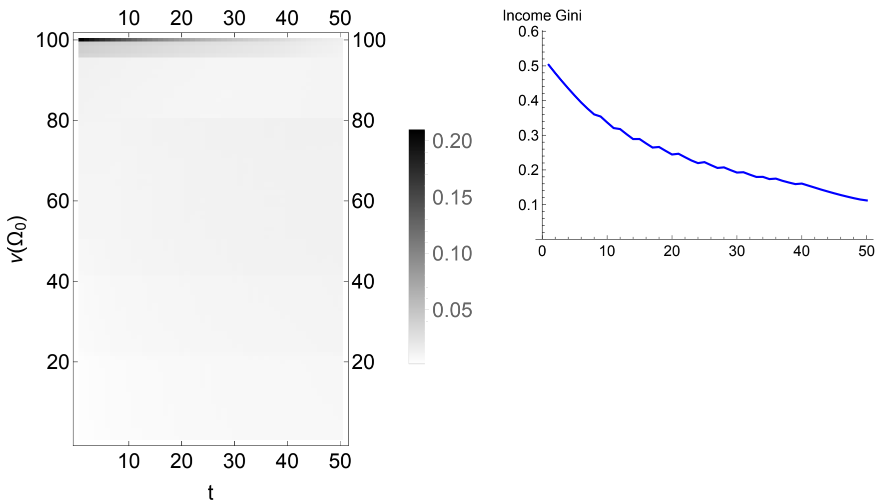
(b) Distribution of wealth for select t (relative frequency)



inequality depends only on $(s^\nu)_{\nu\in\mathcal{N}}$. Figure 8(b) shows the dynamics of the post-tax Gini coefficient of income. As expected, the redistribution of wealth bolsters the non-labour income of agents who begin the simulation with little to no wealth, thereby rendering income more equal over time.

Figure 8: Distribution of income - Basic model with wealth taxes

(a) Post-tax distribution of income shares (b) Gini coefficient of income



In summary, the model shows the effectiveness of rather modest wealth taxes of the type suggested by Piketty [20]. Given the very low taxation levels chosen, in every period, wealth taxation has apparently negligible effects: in every period, the pre-tax and post-tax distributions of income, wealth and exploitation are extremely similar. Yet, wealth taxes have significant cumulative effects over time yielding major reductions in wealth and income inequality in a relatively short period of time.

Nonetheless, two features of the taxation scheme analysed should be noted. First, although Piketty-type taxes have significant effects on inequalities, they do not eliminate wealth inequality completely, except in the very long run.[17] Second, a generic tax on wealth does not alter the fundamentally exploitative structure of a capitalist economy, and exploitation disappears only towards the end of the simulation when accumulation drives profits to zero. In the next two sections, we explore two alternative tax schemes to address these issues: a more robust wealth taxation scheme to eliminate wealth inequalities in a finite number of periods and a tax scheme specifically meant to elilminate exploitation.

[17]Moreover, income inequality does not disappear completely, even in the long run, due to earning inequalities deriving from skill differentials.

7 Basic model with wealth taxes to equalise wealth

In this section we extend the basic model to incorporate wealth taxes that are meant to quickly eliminate wealth inequality. At the end of any period t, after agents solve MP_t^ν, let $\omega_t'^\nu$ be the endowment of any $\nu \in \mathcal{N}$ according to equation (4). The taxation scheme is formalised in Rule 1:

Rule 1 (**Wealth Equality**). *If, at the end of t, $\pi_t > 0$ and the Gini coefficient of $(\omega_t')_{\nu \in \mathcal{N}}$ is positive, $\gamma_t^{\omega'} > 0$, then $\omega_t'^\nu$ is taxed at rate τ_t^ν according to where agents fall in the wealth distribution:*

$$\begin{aligned} \tau_t^\nu &= \beta_t \left(1 - \frac{\mu_t [\omega_t']}{\omega_t'^\nu}\right) && \textit{if and only if } \omega_t'^\nu > \mu_t [\omega_t'] , \\ \tau_t^\nu &= 0 && \textit{if and only if } \omega_t'^\nu < \mu_t [\omega_t'] , \end{aligned}$$

where $\beta_t = \min [0.05t, 1]$ and $\mu_t [\omega_t']$ denotes average endowment after MP_t^ν is solved.

At all t, let N_t^0 denote the number of agents with wealth below the average who pay no taxes. Agents' wealth at period $t+1$, ω_t^ν, is determined as follows:

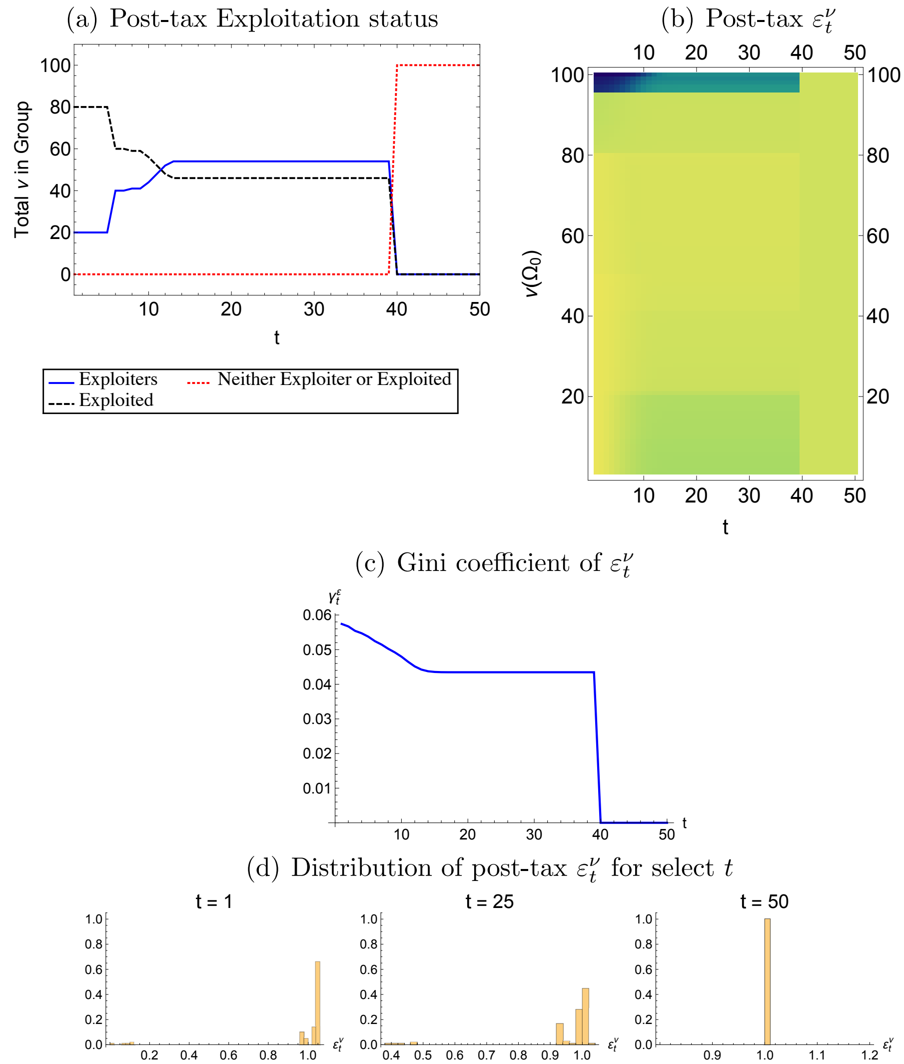
$$\begin{aligned} \omega_t^\nu &= (1 - \tau_t^\nu)\omega_t'^\nu && \textit{if and only if } \tau_t^\nu > 0, \\ \omega_t^\nu &= \omega_t'^\nu + \frac{\sum_{\nu \in \mathcal{N}} \tau_t^\nu \omega_t'^\nu}{N_t^0} && \textit{if and only if } \tau_t^\nu = 0. \end{aligned}$$

According to Rule 1, after solving MP_t^ν, agents with endowments $\omega_t'^\nu$ above the average pay a tax rate such that their wealth for $t+1$ is brought closer to the average endowment by a distance determined by β_t. Agents with wealth $\omega_t'^\nu$ below the average pay no taxes and receive an equal share of the total tax revenue, $\sum_{\nu \in \mathcal{N}} \tau_t^\nu \omega_t'^\nu$. Agents with $\omega_t'^\nu = \mu_t [\omega_t']$ pay no taxes and receive no transfers. Rule 1 runs as long as both $\gamma_t^{\omega'}$ *and* the profit rate are positive, so that wealth is not taxed when the economy is labour constrained (i.e. for $t > 40$).

The simulation is initialised as in section 4 and $A_t = A_0$, $L_t = L_0$, and $b_t = b_0$ all t. The simulation occurs in the following sequence: (1) check whether the economy is capital constrained, labour constrained, or on the knife-edge and set $\widehat{w}_t$ and π_t accordingly; (2) solve MP_t^ν; (3) check that $\gamma_t^{\omega'} > 0$ and $\pi_t > 0$, and if appropriate use Rule 1 to redistributed wealth; (4) repeat as necessary.

The summary results of the simulation are qualitatively the same as in the basic model and are therefore omitted. Figure 9 shows the post-tax dynamics of exploitation. As expected, the redistribution of wealth quickly reduces the number of agents who are exploited (figure 9(a)) and the distribution of ε_t^ν is more compressed than in the basic model (figure 9(b)), yet a robust middle class of skilled agents exists which remains exploited—albeit at a low level—even when wealth is equalised, until the economy becomes labour constrained and profits vanish. As figure 9(c) shows γ_t^ε declines and quickly reaches a stable level just above 0.04. Rule 1 reduces overall

Figure 9: Exploitation - Basic model with wealth taxes and wealth equality



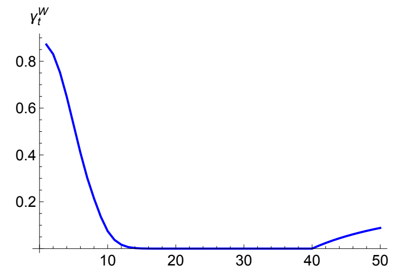
inequality in exploitation intensity, also shown in figure 9(d), but does not eliminate it entirely.

Figure 10 shows the Gini coefficient of wealth, γ_t^W, and the distribution of ω_{t-1}^ν for select t. As expected, γ_t^W sharply decreases over time and falls to zero in twenty time periods.

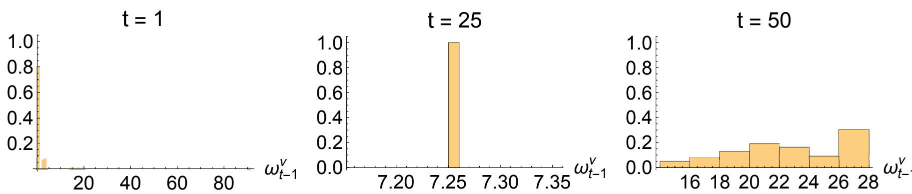
Figure 11 shows the post-tax distribution of income over the simulation. By sharply reducing inequalities in capital income, rule 1 has a strong egalitarian effect on the distribution of income to a point where shares of aggregate post-tax income range from 0.00517 to 0.0136 when wealth equality is achieved. The inequality in income is not negligible—some agents earn twice as much income as others due to skill differentials—but it is a dramatic improvement over the laissez faire income

Figure 10: Distribution of wealth - Basic model with wealth taxes and wealth equality

(a) Gini coefficient of wealth



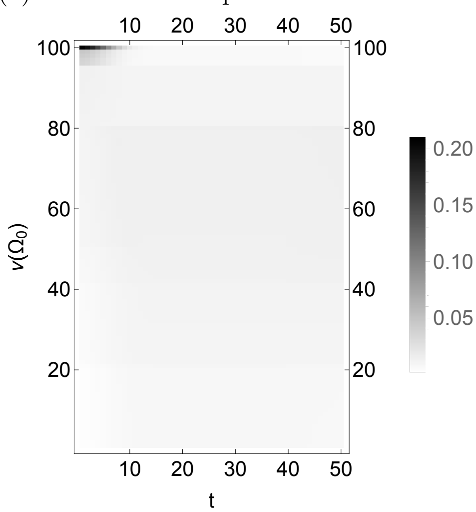
(b) Distribution of wealth for select t (relative frequency)



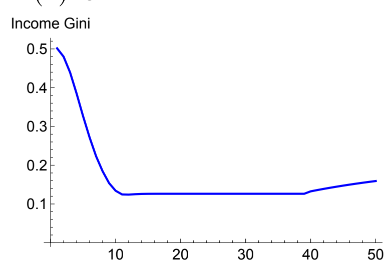
distribution in the basic economy.

Figure 11: Distribution of income - Basic model with wealth taxes and wealth equality

(a) Distribution of post-tax income over t



(b) Gini coefficient of income



As noted, rule 1 does not eliminate exploitation. The redistribution of wealth affects agents with the lowest skill levels at the very bottom and top of the income distribution, while agents with the highest skills are largely unaffected. It is not clear what is necessarily desirable from a societal point of view. While some may find it desirable to achieve wealth equality, this equal right to returns from wealth creates "an unequal right for unequal labour" (Marx [15], p. 24) and simple-minded wealth egalitarianism may create inequalities along the lines of skill and ability.

8 Basic model with a socialist allocation

In this section, we extend the basic model to include wealth taxes aimed at achieving a socialist allocation. Roemer [23] defines a *socialist allocation* as one in which agents receive a share of total output proportional to their effective labour performed.[18] From Theorem 2, this means that at any period t such that $\pi_t > 0$, a socialist allocation can be achieved only if, for any $\nu \in \mathcal{N}$,

$$\omega_{t-1}^{\nu} = \Lambda_t^{\nu} \frac{1}{\pi_t} \left(\frac{1 - \widehat{w}_t v_t}{v_t} \right). \tag{5}$$

In other words, a socialist wealth taxation scheme must bring endowments to a level proportional to the effective labour performed by each agent. At first sight, equation (5) seems to suggest that, at the end of every period t (and the beginning of period $t+1$), the calculation of the relevant tax rates would require anticipating the equilibrium labour supply of all agents (Λ_{t+1}^{ν}), as well as equilibrium distribution ($\widehat{w}_{t+1}$,π_{t+1}) and technology (v_{t+1}). As it turns out, this is unnecessary in our model and at the end of any period t, a well-defined socialist taxation scheme can be defined based on past observed variables.

For all $\nu \in \mathcal{N}$, at any t, let Ψ_t^{ν} be defined as follows:

$$\Psi_t^{\nu} = \Lambda_t^{\nu} \frac{\omega_t}{L_t y_t}.$$

This expression for Ψ_t^{ν} denotes the wealth of each agent *at the beginning of* $t+1$ that is consistent with a socialist allocation at $t+1$. To see this, note that in equation (5) $\Lambda_t^{\nu} \frac{1}{\pi_t} \left(\frac{1-\widehat{w}_t v_t}{v_t} \right) = \Lambda_t^{\nu} \frac{A_t}{L_t}$. Then, observe that in any period t such that $\pi_t > 0$, at a RS, $\Lambda_t^{\nu} \frac{\omega_t}{L_t y_t} = \Lambda_t^{\nu} \frac{A_{t+1} y_{t+1}}{L_t y_t}$. Therefore $\Psi_t^{\nu} = \Lambda_{t+1}^{\nu} \frac{1}{\pi_{t+1}} \left(\frac{1-\widehat{w}_{t+1} v_{t+1}}{v_{t+1}} \right) = \Lambda_{t+1}^{\nu} \frac{A_{t+1}}{L_{t+1}}$ if and only if $\frac{\Lambda_t^{\nu}}{L_t y_t} = \frac{\Lambda_{t+1}^{\nu}}{L_{t+1} y_{t+1}}$, and the latter equality holds in equilibrium as long as $\pi_t > 0$, and the economy is capital constrained.

Recall that, at the end of any t, for any $\nu \in \mathcal{N}$, $\omega_t^{\prime\nu}$ is ν's endowment according to equation (4). We consider the following tax scheme:

Rule 2 (**Socialist Allocation**). *Consider any period t such that $\pi_t > 0$. For any $\nu \in \mathcal{N}$, if, at the end of t, $\omega_t^{\prime\nu} \neq \Psi_t^{\nu}$, then $\omega_t^{\prime\nu}$ is taxed at rate τ_t^{ν} according to where*

[18] Whereas in communism the allocation of economic goods will be independent of productive contributions. "From each according to his ability, to each according to his needs!" (Marx [15], p. 24).

agents fall in the distribution of $(\omega_t'^\nu)_{\nu \in \mathcal{N}_t}$*:*

$$\begin{aligned} \tau_t^\nu &= \beta_t \left(1 - \frac{\Psi_t^\nu}{\omega_t'^\nu}\right) && \textit{if and only if} \quad \omega_t'^\nu > \Psi_t^\nu, \\ \tau_t^\nu &= 0 && \textit{if and only if} \quad \omega_t'^\nu < \Psi_t^\nu, \end{aligned}$$

where $\beta_t = \min[0.05t, 1]$*.*

Let $\mathcal{N}_t^0 \subset \mathcal{N}$ *be the subset of agents who pay no taxes at* t*. The wealth any agent has available at* $t+1$ *is:*

$$\begin{aligned} \omega_t^\nu &= (1 - \tau_t^\nu)\omega_t'^\nu && \textit{if and only if} \quad \tau_t^\nu > 0, \\ \omega_t^\nu &= \omega_t'^\nu + \frac{l^\nu}{\sum_{\nu \in \mathcal{N}_t^0} l^\nu} \sum_{\nu \in \mathcal{N}} \tau_t^\nu \omega_t'^\nu && \textit{if and only if} \quad \tau_t^\nu = 0. \end{aligned}$$

According to Rule 2, agents with wealth greater than the level consistent with a socialist allocation are taxed, while those whose wealth is below the level consistent with a socialist allocation receive a portion of total tax revenues, $\sum_{\nu \in \mathcal{N}} \tau_t^\nu \omega_t'^\nu$, consistent with their share of effective labour in the subset of relatively poor agents $\mathcal{N}_t^0$. Thus, wealth is redistributed until each agent holds wealth in proportion to their effective labour performed, as specified in equation (5).

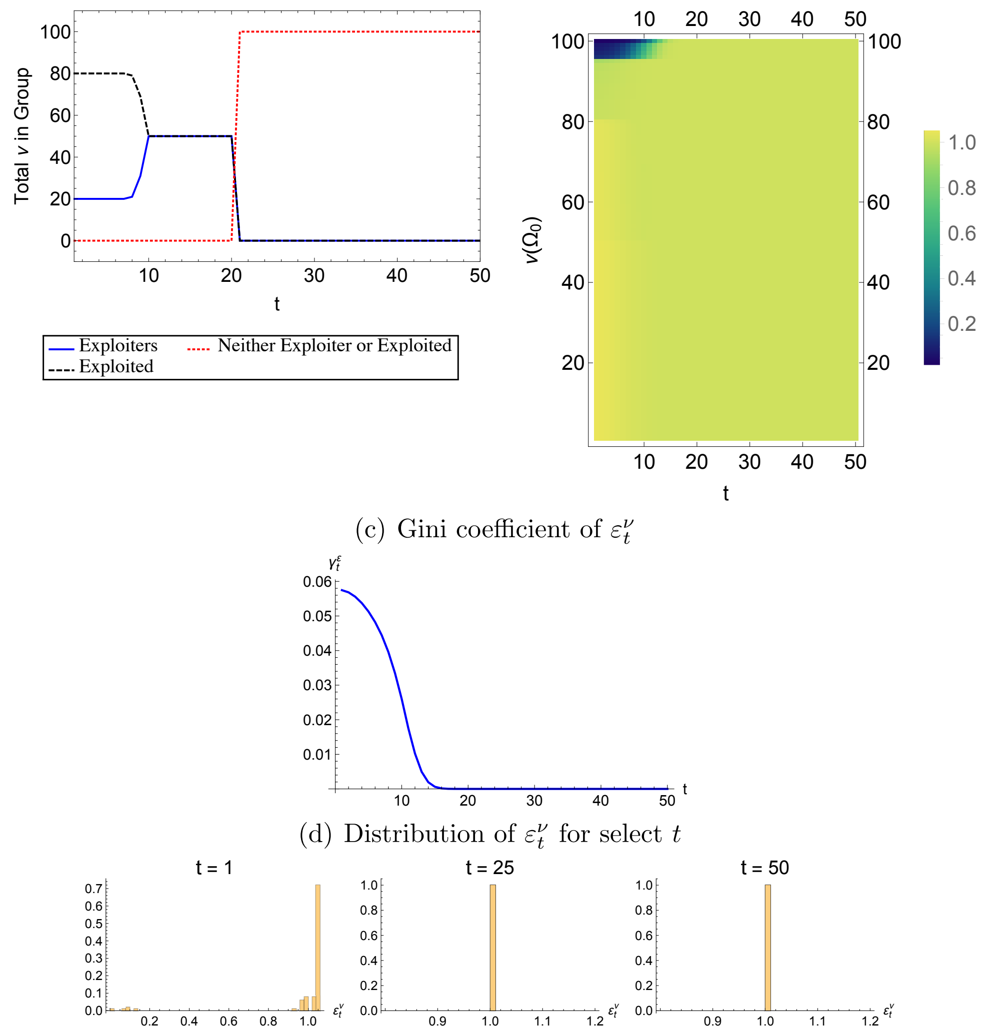
Rule 2 is run for the basic model. The simulation is initialised as in section 4 and $A_t = A_0$, $L_t = L_0$, and $b_t = b_0$ all t. The simulation occurs in the following sequence: (1) check whether the economy is capital constrained, labour constrained, or on the knife-edge and set $\widehat{w}_t$ and π_t accordingly; (2) solve MP_t^ν; (3) check whether $\omega_t'^\nu \neq \Psi_t^\nu$ for any $\nu \in \mathcal{N}$ and $\pi_t > 0$, and if appropriate apply Rule 2; (4) repeat as necessary.

The aggregate results of the model are qualitatively the same as earlier versions of the basic model—continuing the trend of taxes being macroeconomically neutral—and omitted for space concerns. Figure 12 shows the the dynamics of exploitation. Figure 12(a) shows post-tax exploitation status. As expected, the redistribution of wealth quickly ends exploitation while moving all agents to the middle classes, as seen in figure 12(b), where post-tax $\varepsilon_t^\nu = 1$ for all ν by $t = 20$. This is confirmed by γ_t^ε shown in figure 12(c).

Figure 13 shows the Gini coefficient of wealth, γ_t^W, and the distribution of ω_{t-1}^ν for select t: γ_t^W reaches its minimum value of 0.2578 at $t = 11$, and while this is much less wealth inequality than the start of the simulation, it is not insignificant. Thus, the socialist allocation is not consistent with views calling for wealth equality. In fact, as Rule 1 shows, wealth equality leads to inequalities of other kinds.

Figures 14 shows the distribution of income over the simulation. Income inequality is dramatically reduced over the course of the simulation, yet not eliminated entirely due to the heterogeneity in skills. Once the socialist allocation is achieved at $t = 21$, agents' income will be proportional to their effective labour, thus agents with the highest skills receive the highest incomes—agents get out what they put into the economy. While there is noticeable income inequality during the phases of the

Figure 12: Exploitation - Basic model with wealth taxes and socialist allocation



(a) Post-tax Exploitation status

(b) Post-tax ε_t^{ν}

(c) Gini coefficient of ε_t^{ν}

(d) Distribution of ε_t^{ν} for select t

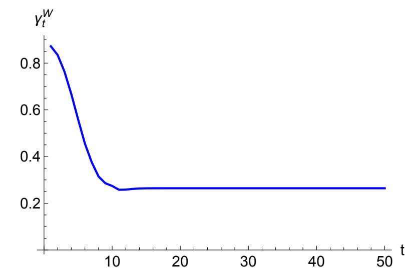
simulation where the socialist allocation has been achieved, it is not nearly as unequal as the initial distribution of income. During the socialist phase of the simulation post-tax shares of income range from 0.0003214 to 0.015986.

9 Endogenising consumption and technical change

The basic economy provides an important benchmark for the analysis of exploitation and inequalities in advanced economies. Its rather simple dynamics eventually leading to overaccumulation and profits falling to zero allows us to clearly distinguish

Figure 13: Distribution of wealth - Basic model with wealth taxes and socialist allocation

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

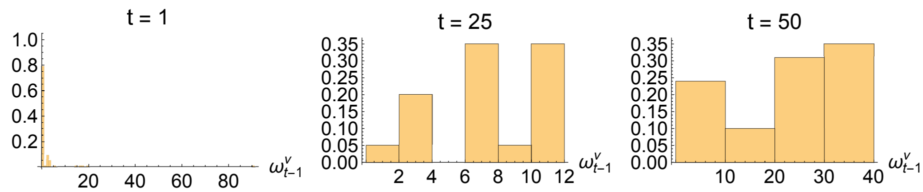
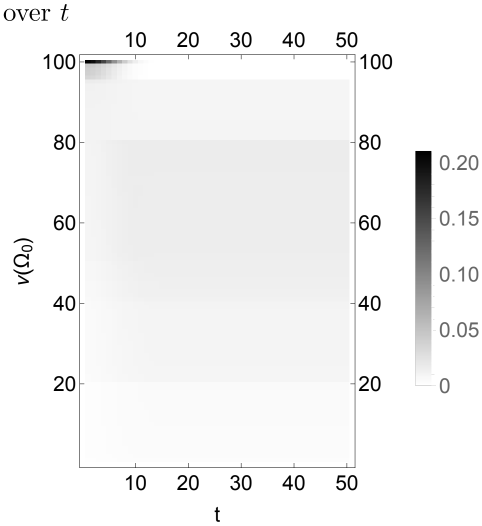
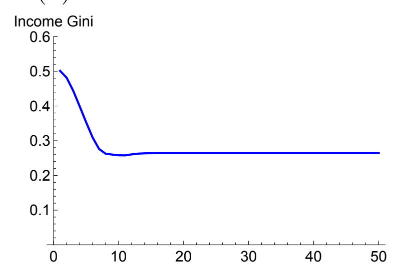


Figure 14: Distribution of income - Basic model with wealth taxes and socialist allocation

(a) Post-tax Distribution of income shares over t



(b) Gini coefficient of income



between different mechanisms for the elimination of exploitation—namely, egalitarian redistributive policies or the extinction of the profit motive. Nonetheless, in portraying an ecomomy which smoothly runs into the labour constraint, the model misses two important features of capitalist economies. First, as already noted, exploitation is a persistent feature of capitalist economies, and it is important to investigate the mechanisms that guarantee its persistence. Second, and related, the dynamics of capitalist economies are all but smooth and capitalism is characterised by periodic and recurrent crisis. It is therefore worth considering the relation between exploitation, inequalities, and the cyclical and crisis-prone dynamics of capitalist economies. In this section we try to address these questions by extending the basic model to allow both consumption and technology to be determined endogenously and change over time, thus incorporating some key properties of capitalism as a dynamic system.

Concerning consumption, we assume that b_t is the product of social norms, allowing it to vary over time so that it keeps pace with the growth rate of the economy. This is empirically reasonable as the long-run evolution of capitalist economies has indeed been characterised by an increase in (average) consumption opportunities and consumption norms have evolved over time. Theoretically, it reflects some key Marxian insights on the social nature of consumption and the idea that consumption norms depend on the general level of development of the economy.[19]

To be specific, we assume that consumption norms grow at the same rate as aggregate capital—our proxy for the level of development of the economy. This allows the economy to settle on a steady growth path but it is important to emphasise that none of our insights on profits, exploitation and class depends on this specification. For example, all of our key conclusions continue to hold if consumption norms depend on labour productivity, rather than wealth, or indeed, if consumption norms do not change at all. Formally,

$$b_t = b_{t-1} \cdot \left(1 + \phi \frac{\omega_{t-1} - \omega_{t-2}}{\omega_{t-2}} \right), \tag{6}$$

where the parameter ϕ captures the degree to which the degree of development of the economy influences consumption norms.

Concerning technology, it is certainly restrictive to assume (A, L) to remain constant over time, and regardless of changes in prices and distribution. A fundamental feature of capitalism as a dynamic system is its constant tendency to revolutionise production. Further, labour-saving technical progress may play a key role in the dynamics of exploitation by guaranteeing the persistent abundance of labour.

In this section, we assume that at the beginning of each production period t, there is a finite set, $\mathcal{P}_t$, of Leontief production techniques (A_t, L_t) with the properties described in section 2 above which can be activated by all agents. When the profit rate falls beneath a certain threshold, capitalists increase their efforts to innovate and introduce new capital-using labour-saving techniques. Formally,

[19] The co-evolution of accumulation and workers' consumption is sometimes considered to be one of the defining features of historical trajectories of capitalist economies à la Marx (Duménil and Levy [9], p.206).

$$
\begin{aligned}
(A_t, L_t) &= (A_{t-1}, L_{t-1}), \text{ if } \pi^{(\widehat{w}_t;A_{t-1},L_{t-1})} = \frac{1 - A_{t-1} - \widehat{w}_t L_{t-1}}{A_{t-1}} \geqq \pi^*, \\
(A_t, L_t) &= (A', L'), \text{ if } \pi^{(\widehat{w}_t;A_{t-1},L_{t-1})} = \frac{1 - A_{t-1} - \widehat{w}_t L_{t-1}}{A_{t-1}} < \pi^*,
\end{aligned}
$$

where π^* is the capitalists' minimum profitability benchmark, which depends on economic, institutional and even cultural factors. The new technique (A', L') is chosen such that $A' \geqq A_{t-1}$, $L' < L_{t-1}$, and $\pi' = \frac{1-A'-\widehat{w}_t L'}{A'} > \pi^{(\widehat{w}_t;A_{t-1},L_{t-1})}$. Capitalists decide whether to introduce new techniques as soon as they know b_t and the real wage $\widehat{w}_t$—and thus $\pi^{(\widehat{w}_t;A_{t-1},L_{t-1})}$.

This formulation of technical progress is both theoretically appropriate and empirically reasonable. Several recent studies find strong empirical support for capital-using labour-saving technical change as a prevailing pattern in capitalist economies, leading to historical increases in labour productivity (see, for example, Flaschel et al. [11] and Tavani and Zamparelli [27]). Theoretically, our model incorporates key insights from both classical-Marxian and evolutionary analyses of technical change, in that the innovation process is fundamentally *profit-driven*, and innovations are both *discontinuous* and *local*.

Technical progress is *profit-driven* because only profitable changes are adopted. This is a defining feature of the classical-Marxian framework, as Duménil and Levy [9] have argued, but it is also a key assumption in the Schumpeterian literature (see, for example, the classic papers by Nelson et al. [18] and Aghion and Howitt [1]). But the innovation process is linked to the trajectory of the profit rate also because significant declines in profitability spur innovation activities and thus tend to yield changes in production processes. This is consistent with standard Marxian insights, whereby "a declining profit rate will lead at some point to a structural crisis, and 'something' will happen with respect to technical change" (Duménil and Levy [9], p.203). But the Schumpeterian literature also emphasises the strongly countercyclical nature of R&D investments both theoretically (Aghion and Howitt [1]; Wälde [35]) and empirically (Aghion et al. [2]).[20]

The *discontinuous* nature of technical change incorporates a Schumpeterian view of innovation as a jerky process (Nelson et al. [18]; Wälde [35]). Formally, our modelling of technical change can be interpreted either as the reduced form—and limit point—of a more complex stochastic process whereby the likelihood of (profitable) innovations increases with R&D efforts, and the latter increase as profitability decreases. But it can also be seen as incorporating satisficing behaviour conceptually analogous to that formalised by Nelson et al. [18] in their classic evolutionary model of technical change in which "Firms with positive capital in the current state retain the production technique of that state, with probability one, if their currently calcu-

[20]We thank Peter H. Matthews for alerting us to this literature.

lated gross return on capital exceeds 0.16. ... Firms that do not make a gross return of 0.16 undergo a probabilistic technique-change process.” ([18], p.95).

We also follow the classical-Marxian and evolutionary literature in assuming that innovations are *local*: agents do not have a global scan of alternatives and search around existing processes (see, for example, Nelson et al. [18]; Duménil and Levy [9]). Therefore when innovations occur, they yield relatively small changes in technical coefficients.

9.1 The laissez-faire economy

The simulation is initialised as in section 4 except for $b_0 = 1.75$.[21] We set $\phi = 1$ and $\pi^* = 0.01$. When $\pi^{(\widehat{w}_t; A_{t-1}, L_{t-1})} < \pi^*$, the new technique prevailing at $t+1$ is identified by first selecting a profit rate, π', from the set of all previous profit rates $\{\pi_\psi\}_{\psi<t}$, such that $\pi_\psi > \pi^*$ and then randomly choosing an increase in A_t in the range $[0.01, 0.03]$ and setting $L_{t+1} = \frac{1-A_{t+1}-A_{t+1}\pi'}{\widehat{w}_{t+1}}$. To ensure that $A_{t+1} < 1$ a limit is set such that $A_{\max} = 0.991$. In the event that the pseudo-randomly determined π' and A_{t+1} entail a negative L_{t+1}, π' is adjusted downward by 0.02 so that $L_{t+1} > 0$.

The simulation occurs in the following sequence: (1) check whether the economy is capital constrained, on the knife-edge, or labour constrained and set $\widehat{w}_t$ and π_t accordingly; (2) solve MP_t^ν; (3) check $\pi_{t+1} < \pi^*$ and adjust (A_{t+1}, L_{t+1}) if necessary; (4) repeat until $t = T$.

Figure 15 displays the summary results. The most striking feature of these results is the cycles in z_t, g_t, and π_t, which are a result of the interaction of accumulation, the growth of subsistence, and technical change. As accumulation progresses wealth accumulates at rate g_t, driving the growth of subsistence b_t, and the demand for labour z_t. As z_t and b_t rise, with $\widehat{w}_t = b_t$, the profit rate π_t falls and eventually reaches the threshold π^*, at which point a new technique of production is implemented, restoring the profit rate and reducing z_t. The cycle then repeats itself. It is the interaction of technical change and capitalist behaviour that generates persistent growth cycles. Figure 16 displays A_t, L_t, and embodied labour value for all t. As expected A_t rises in incremental steps as L_t and v_t decrease.

Figure 17(a) shows that there is a stable configuration of exploitation over the course of the simulation. Figure 17(b) displays the distribution of the exploitation intensity index for all agents, showing a very interesting pattern of *exploitation cycles*. As accumulation progresses ε_t^ν decreases for all propertyless agents thanks to the increase in subsistence, but as soon as technical change takes place exploitation intensity increases again for them while decreasing for wealthy agents. Exploitation cycles are also apparent in γ_t^ε (figure 17(c)): exploitation intensity decreases as accumulation progresses, but this tendency is subverted by the arrival of technical change.

[21] This value for b_0 is to ensure a wide enough range of possible profit rates given the mechanism generating technical change.

Figure 15: Summary results - Model with endogenous subsistence and technical change

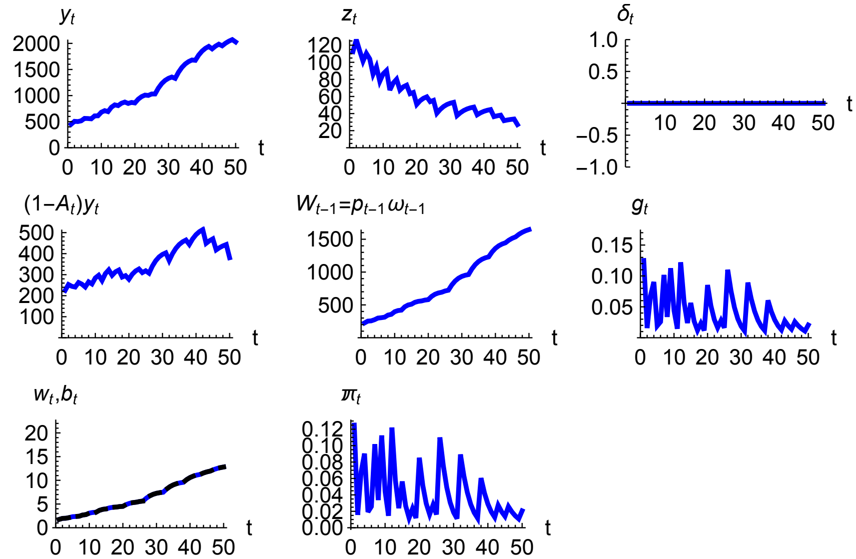


Figure 16: Technology and labour values - Model with endogenous subsistence and technical change

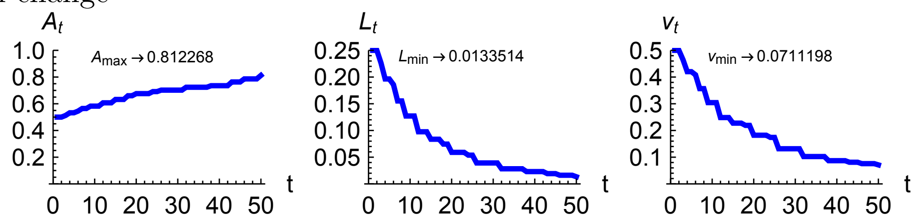


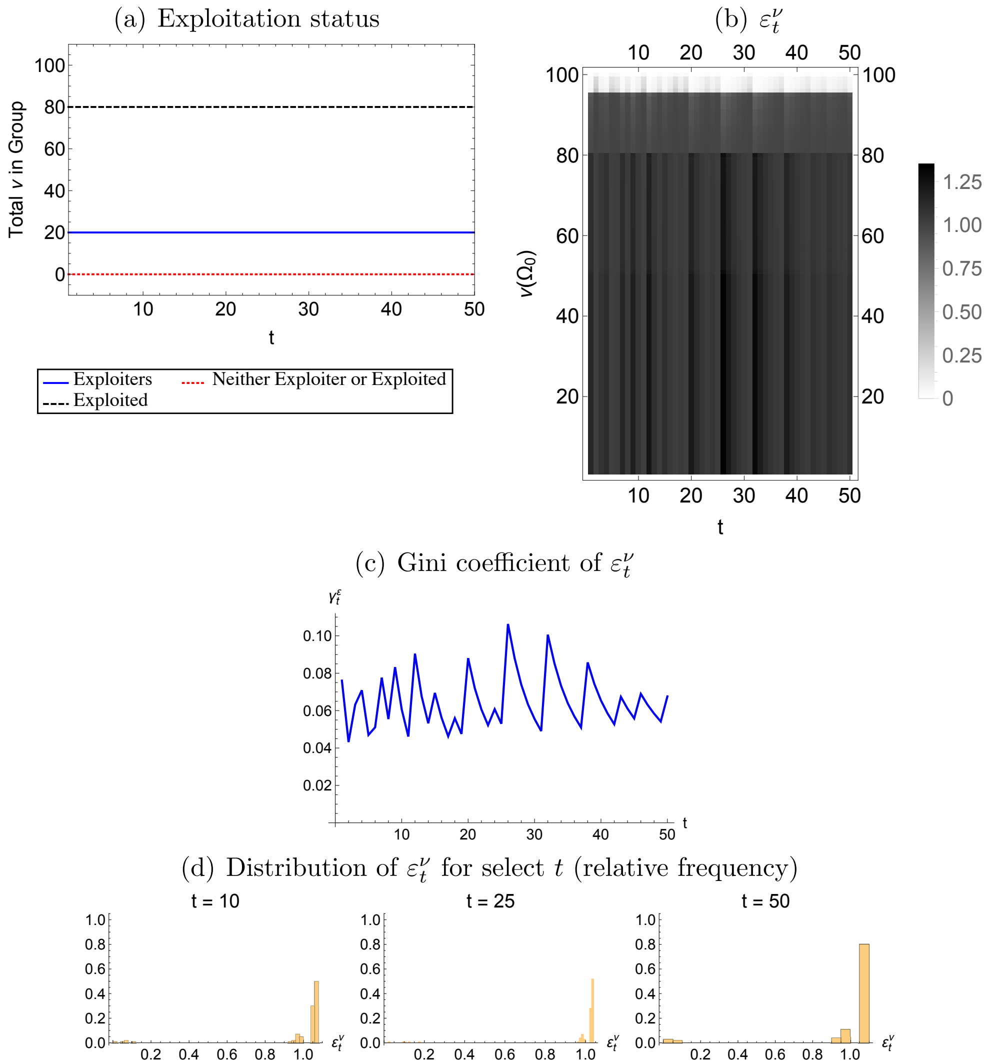
Figure 17(d) shows the distribution of ε_t^ν for select t.[22]

Wealth inequalities are persistent and show no tendency to diminish. Indeed, despite the existence of cycles in exploitation intensity, the accumulation rate, and the profit rate, wealth inequality remains constant over the course of the simulation, with the Gini coefficient of wealth (not depicted) is equal to 0.87204 for the whole simulation.

Figure 18 shows the distribution of income $(1+\pi_t)\omega_{t-1}^\nu + \widehat{w}_t\Lambda_t^\nu$ over the course of the simulation. Figure 18(a) shows the distribution of individual *shares* of total income across $\nu \in \mathcal{N}$ over all t—individual shares of income are calculated as $\frac{(1+\pi_t)\omega_{t-1}^\nu + \widehat{w}_t\Lambda_t^\nu}{\sum_\nu ((1+\pi_t)\omega_{t-1}^\nu + \widehat{w}_t\Lambda_t^\nu)}$. There is a clear cyclical pattern in income shares, with agents at the upper-end of the distribution earning nearly 37% of total income at the peak of the cycles. Figure 18(b) shows the Gini coefficient of income over t, which has an expected cyclical pattern around an upward trend which derives from the increasing

[22] The varying amplitude of the cycles in γ_t^ε, and in the other variables, is due to the stochastic nature of technical progress.

Figure 17: Exploitation - Model with endogenous subsistence and technical change

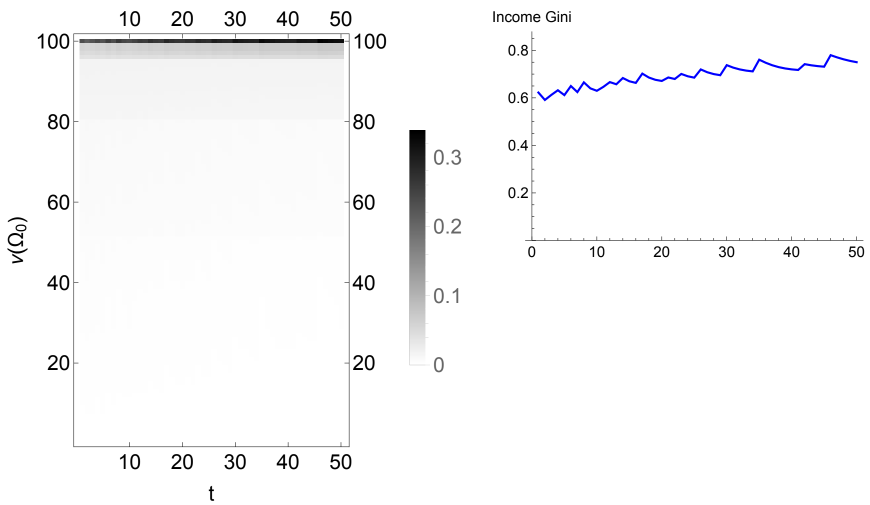


polarisation of wealth.

The results support the claim that capital-using labour-saving technical change can help to explain the persistence of exploitation in accumulation economies (Skillman [26]). The key role of technical change in this context is to make capital persistently scarce relative to labour and to maintain labour unemployment throughout. Hence, the question now is how to eliminate exploitation as the profit motive remains in place due to ongoing technical change. This question returns our focus to taxation and wealth redistribution.

Figure 18: Distribution of income - Model with endogenous subsistence and technical change

(a) Distribution of income shares over t (b) Gini coefficient of income



9.2 Endogenous consumption and technical change with a socialist allocation

In this section, we extend the model with endogenous technical change and consumption to include wealth taxes to achieve a socialist allocation using Rule 2. The simulation is initialised as in section 4 except for $b_0 = 1.75$. We set $\phi = 1$ and $\pi^* = 0.01$.

The simulation occurs in the following sequence: (1) check whether the economy is capital constrained, on the knife-edge, or labour constrained and set $\widehat{w}_t$ and π_t accordingly; (2) solve MP_t^ν; (3) check $\omega_t'^\nu \neq \Psi_t^\nu$ for all $\nu \in \mathcal{N}$, and apply Rule 2 if appropriate; (4) check $\pi_{t+1} < \pi^*$ and adjust (A_{t+1}, L_{t+1}) if necessary; (5) repeat until $t = T$.

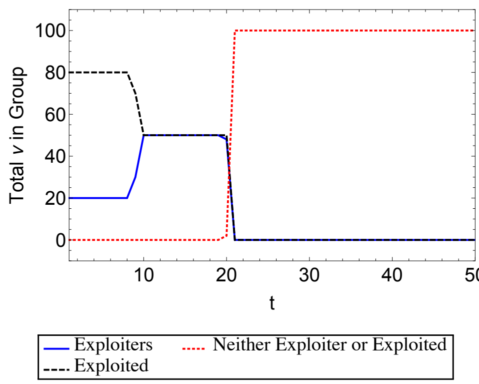
The summary results as well as the evolution of technology are qualitatively identical to those of the model in section 9 and are therefore omitted. This finding is not overly surprising: given our assumptions, wealth taxes have purely redistributive effects and do not significantly affect the behaviour of aggregate variables. Thus, among other things, they do not act as stabilisers for business cycles.

Figure 19 shows the exploitation status of agents throughout the simulation: as expected, Rule 2 quickly ends exploitation, with $\varepsilon_t^\nu = 1$, all $\nu \in \mathcal{N}$, once the socialist allocation is achieved (figures 19(a) and 19(b)). Perhaps more surprisingly, Rule 2 significantly dampens exploitation cycles *even when the economy is transitioning to the socialist allocation* (figures 19(a) and 19(c)).

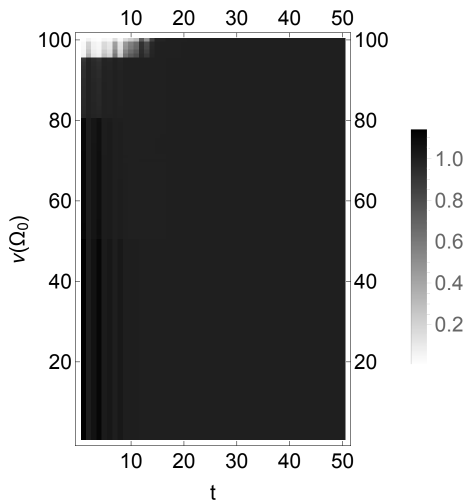
Figure 20 displays the distribution of ω_{t-1}^ν over the simulation. Wealth inequality rapidly decreases but it does not vanish: once the socialist allocation is realised, γ_t^W

Figure 19: Exploitation - Model with endogenous subsistence and technical change with wealth taxes and socialist allocation

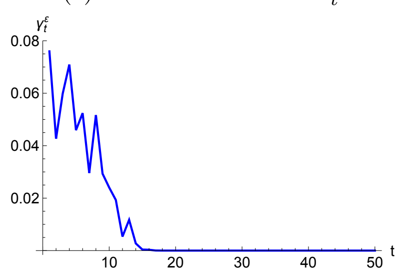
(a) Post-tax Exploitation status



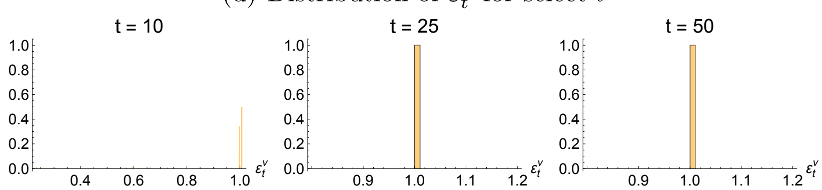
(b) Post-tax ε_t^ν



(c) Gini coefficient of ε_t^ν



(d) Distribution of ε_t^ν for select t



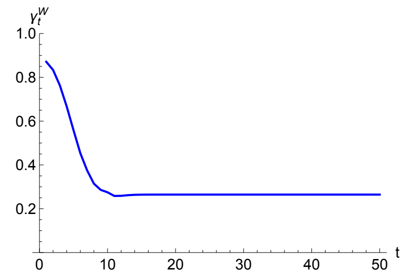
remains at 0.264637 for the rest of the simulation.

Figure 21 shows the distribution of income. Rule 2 quickly reduces income inequality, yet as figure 21(b) shows, the Gini coefficient of income settles at 0.264637 which entails noticeable income inequality.[23] In the socialist allocation, income shares range from 0.000315 to 0.01736.

[23]The Gini coefficient of income is the same as γ_t^W once exploitation is eliminated because the socialist allocation requires wealth holdings to be in proportion to each agents' effective labour, which in turn determines agents' earnings.

Figure 20: Distribution of wealth - Model with endogenous subsistence and technical change with wealth taxes and socialist allocation

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

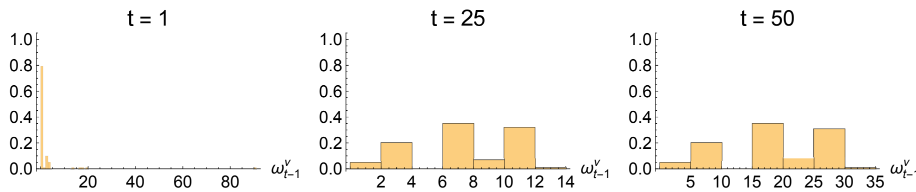
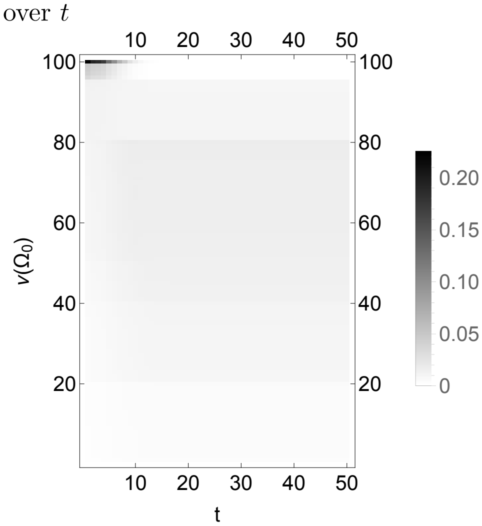
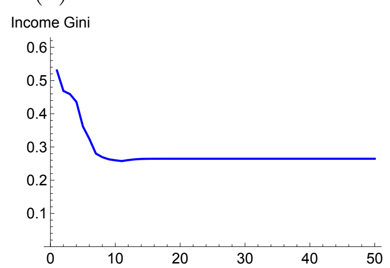


Figure 21: Distribution of income - Model with endogenous subsistence and technical change with wealth taxes and socialist allocation

(a) Post-tax distribution of income shares over t



(b) Gini coefficient of income



10 Socialism, education and skills

The above simulations raise the relevant question of how best to resolve the apparent trade-offs faced by socialists and egalitarians. For those concerned with equality, the inequalities in wealth and income in the models under Rule 2 are likely unacceptable.

Whereas, the inequality in exploitation in the simulations under Rule 1 is undesirable from the socialist perspective. At the heart of these trade-offs is the heterogeneity of labour, and perhaps the best way to satisfy both sets of concerns is through an education system designed to eliminate inequalities in skills. The simulations below consider such a possibility by using wealth taxes according to Rule 2 and diverting a portion of tax revenue to augmenting agents' skills so that the distribution of s^ν is compressed over t.

Let ϵ_t^ν denote an agent's claim to a share of the social fund available for education. At the end of t

$$\epsilon_t^\nu = \frac{1}{1+\exp\left(s_t^\nu\right)} \Bigg/ \sum_\nu \frac{1}{1+\exp\left(s_t^\nu\right)},$$

where "exp" denotes Euler's number. The above equation uses an inverted logistic function so that as agents' skills increase over t their claim on the education fund decreases, thus education will make skills asymptotically approach uniformity over time, yet not ever reaching perfect uniformity.

Agents' skills at $t+1$ are updated as follows:

$$s_{t+1}^\nu = s_t^\nu \left(1 + \epsilon_t^\nu \sigma_t \sum_\nu \tau_t^\nu \omega_t^{\prime\nu}\right),$$

where σ_t denotes the portion of overall tax revenue dedicated to education. This algorithm can be added on to the end of Rule 2 so skill factors are updated after tax revenue is collected. Modifying agents' skills in this way raises the skills of all agents over time, but agents who begin the simulation with low skills are prioritized in the process to introduce much greater equality in effective labour. Thus, agents who begin the simulation with the highest skills will benefit from education, albeit much less so than agents who begin at bottom of the skills distribution. The education algorithm is added to the basic model of section 8 and the endogenous technical change model of section 9.2. Results are reported and discussed below.

10.1 Basic economy with education

The figures below report the results of adding the above described education system to Rule 2 for the basic economy. This simulation uses the standard parameters and sets $\sigma_t = 0.25$ for all t, thus 25% of tax revenue during each t is diverted to augmenting agents' skills. Figure 22 reports the summary results, which show that the continuous augmentation of the aggregate skill endowment s_t prevents the economy from ever becoming labour constrained. There is also a noticeable dip in the accumulation rate g_t at the start of the simulation as a portion of aggregate wealth is used to alter skills, rather than being used for further accumulation, however, this decline is quickly corrected and accumulation proceeds as normal after roughly $t = 15$. Figure 23 shows γ_t^s over the course of the simulation. The effect of education in reducing inequalities in skills is immediately apparent in the decline of γ_t^s and its value of 0.0232483 at $t = 50$.

Figure 22: Summary results - Basic model with education

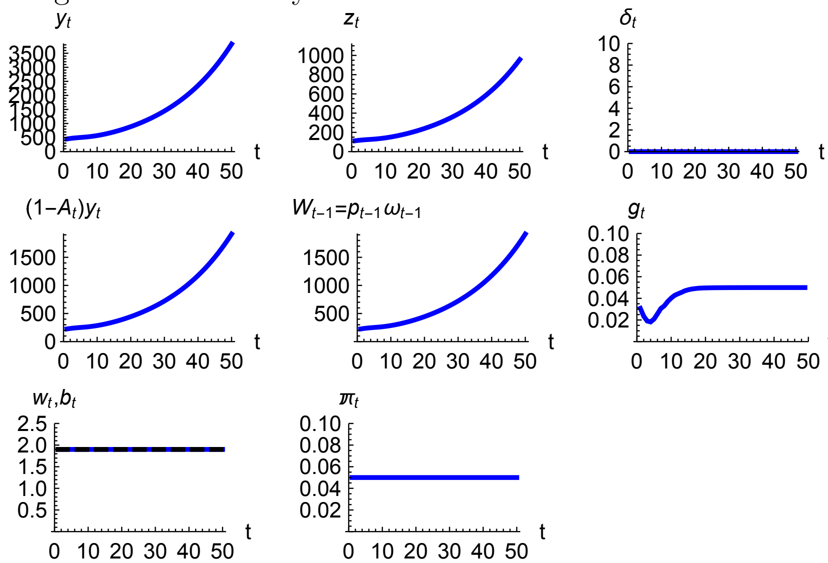


Figure 23: Skill inequality - Basic model with education

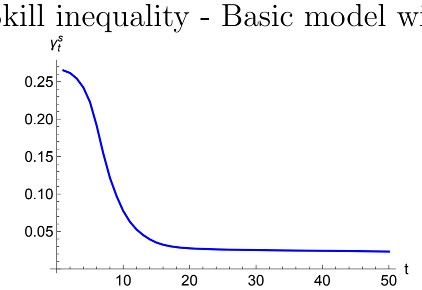
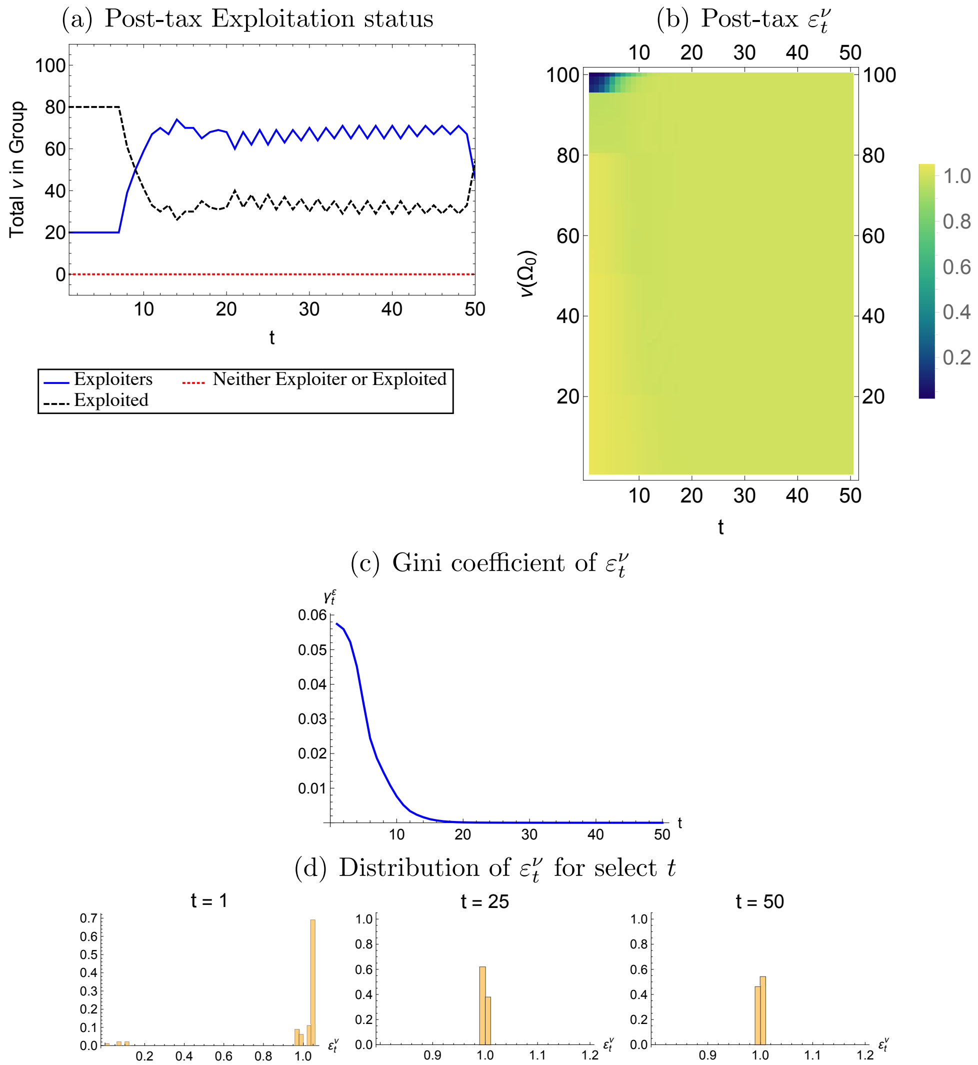


Figure 24 shows the dynamics of post-tax exploitation over the course of the simulation. Figure 24(a) shows that, over time, exploitation is not completely eliminated. However, figures 24(b)-24(d) show that the differences in post-tax exploitation intensity after $t = 20$ are very small. The small differences in ε_t^{ν} during later time periods of the simulation shows that while there is a clear delineation of exploited and exploiter agents, the differences between them are small and driven by the small degree of heterogeneity in skills. These small differences are confirmed by γ_t^{ε} reaching 9.4709×10^{-6} at $t = 50$.

Figure 25 shows the dynamics of the distribution of wealth over the course of the simulation. Figure 25(a) shows that γ_t^W quickly declines and reaches a value of 0.0234334 at $t = 50$ as a result of wealth taxes. This level of γ_t^W is lower than that of the simulation in section 8 due to the compression of the distribution of skills. This result is still consistent with the proportional solution, where wealth is redistributed

Figure 24: Exploitation - Basic model with education



(a) Post-tax Exploitation status (b) Post-tax ε_t^ν

(c) Gini coefficient of ε_t^ν

(d) Distribution of ε_t^ν for select t

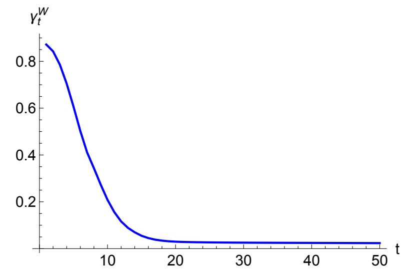
in proportion to effective labour capacity, yet the asymptotic convergence of effective labour induces greater wealth equality than earlier iterations of Rule 2 taxes.

Figure 26 shows the distribution of income for the simulation. The quick compression of the income distribution is apparent in both figures 26(a) and 26(b), where the redistribution of wealth in conjunction with the compression of the skills distribution induces greater income equality with a much lower Gini coefficient of income—0.0233454 at $t = 50$—than that of section 8.

The pre- and post-tax behaviour of exploitation intensity and wealth are similar to that of the simulation in section 8 and omitted for space concerns.

Figure 25: Distribution of wealth - Basic model with education

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

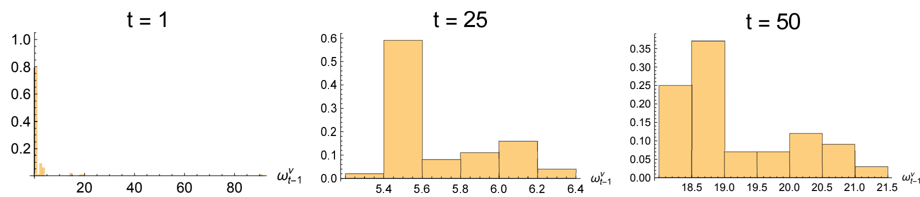
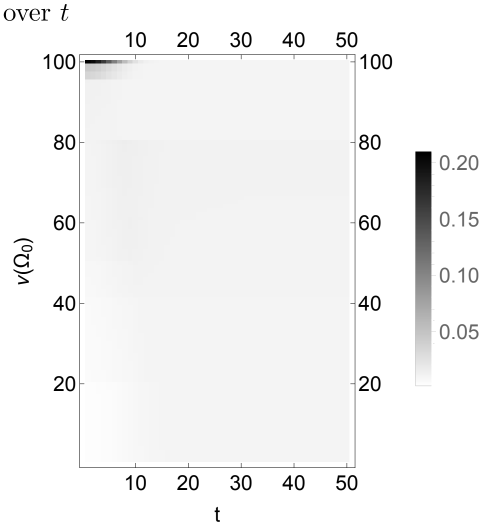
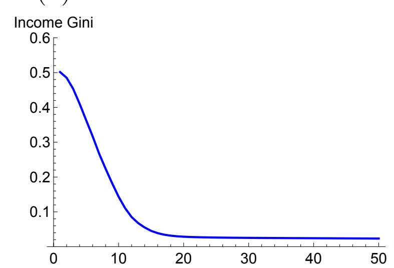


Figure 26: Distribution of income - Basic model with education

(a) Post-tax distribution of income shares over t



(b) Gini coefficient of income



10.2 Endogenous consumption and technical change with education

The simulation below adds education to the model of section 9.2. The standard parameters are used, except for $b_0 = 1.75$, and $\sigma_t = 0.25$ all t. Figure 27 shows the summary results, which exhibit the expected cyclical pattern induced by the interaction of accumulation and technical progress. There is a noticeable, and expected, dip in g_t during the early time periods for the same reasons as in the previous simulation, but the dip in figure 27 coincides with a fall in π_t due to rising b_t that actually causes a negative g_t for one time period. This temporary disaccumulation can be seen as a necessary step in setting the economy on a new trajectory for exploitation, wealth, and income equality. Figure 28 shows the steadily declining γ_t^s, which reaches a value of 0.030191 at $t = 50$.

Figure 27: Summary results - Model with endogenous subsistence and technical change with education

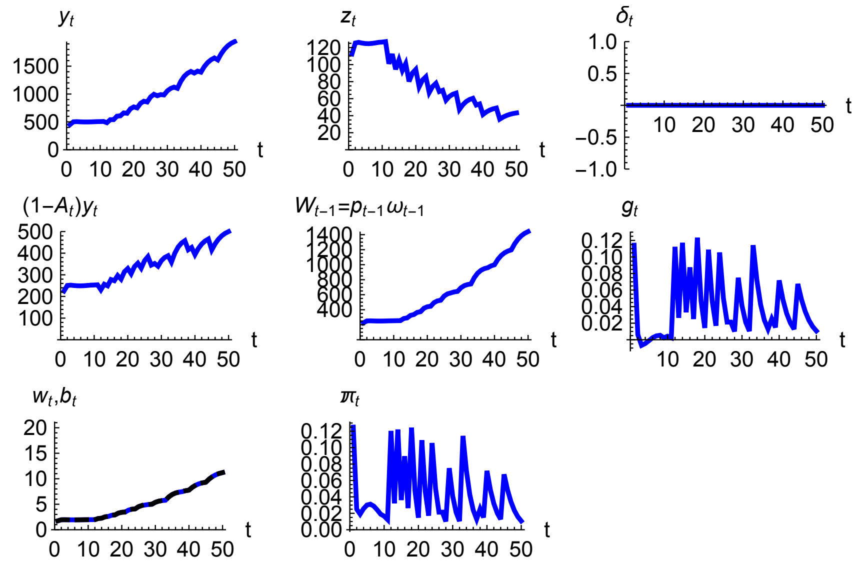


Figure 28: Skill inequality - Model with endogenous subsistence and technical change with education

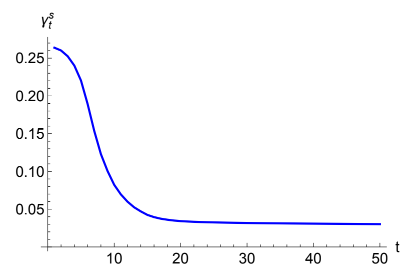
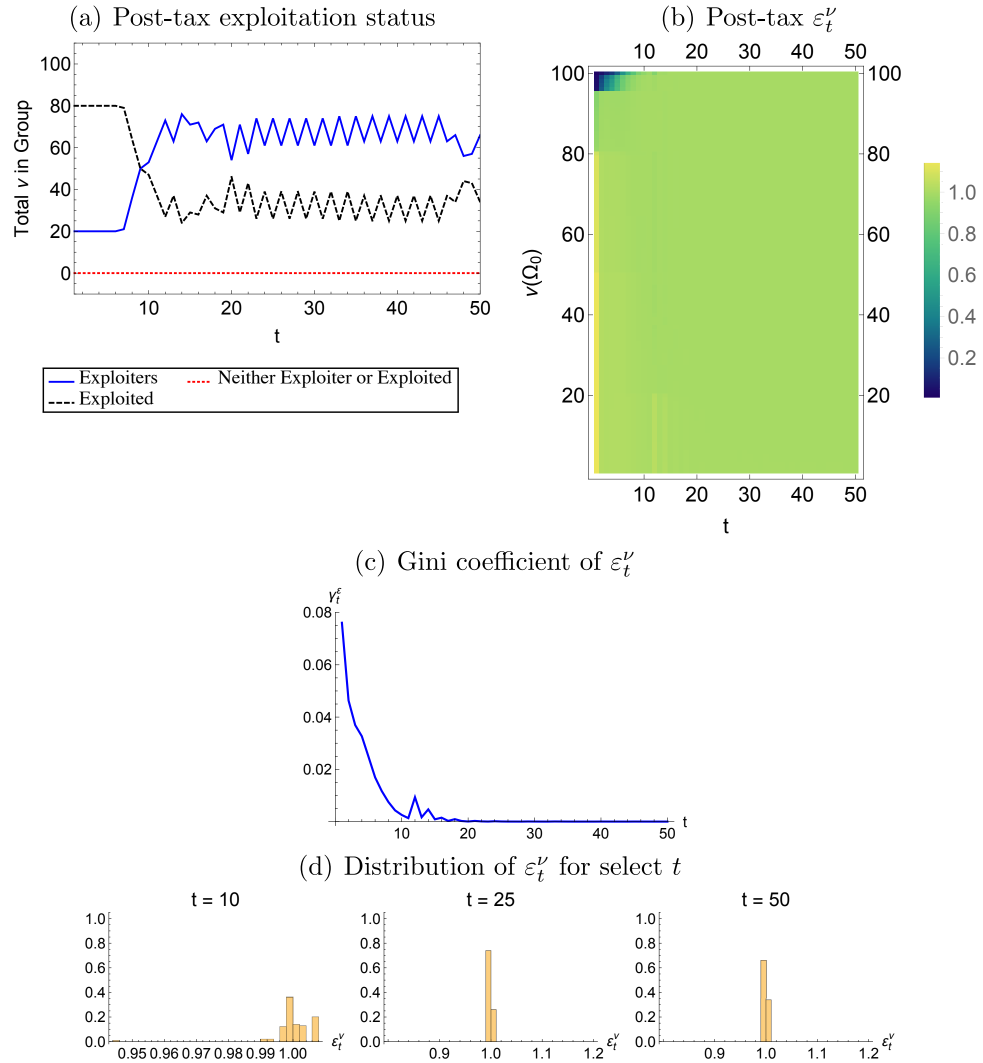


Figure 29 shows the post-tax dynamics of exploitation. Consistent with the previous simulation, there remains a clear distinction between exploiter and exploited agents throughout the simulation. However, similarly, the inequalities in ε_t^ν are small and result from the asymptotic convergence of skills. These small differences are evident in figures 29(b)-29(d) where the distribution of ε_t^ν is highly compressed starting around $t = 15$ with $\gamma_{50}^\varepsilon = 3.09536 \times 10^{-6}$.

Figure 30 shows the dynamics of the distribution of wealth over the simulation. The combined effect of taxes according to Rule 2 and education modifying the com-

Figure 29: Exploitation - Model with endogenous subsistence and technical change with education



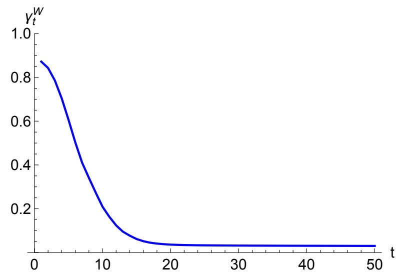
position of the proportional allocation induces greater wealth equality than the simulation in section 9.2. For instance, γ_t^W reaches a value of 0.0302918 at $t = 50$.

Figure 31 displays the dynamics of income inequality over the simulation. As expected, the distribution of income is steadily compressed as wealth inequality falls and skills asymptotically converge. The Gini coefficient of income at $t = 50$ is 0.0302666 and much lower than the Gini coefficient of income for the simulation in section 9.2.

The incorporation of education in both the basic economy and the economy with endogenous consumption and technical change leads to a high degree of equality in exploitation intensity, wealth, and income, although not perfect equality. The degree inequality in these scenarios may be tolerable for both egalitarians and socialists given

Figure 30: Distribution of wealth - Model with endogenous subsistence and technical change with education

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

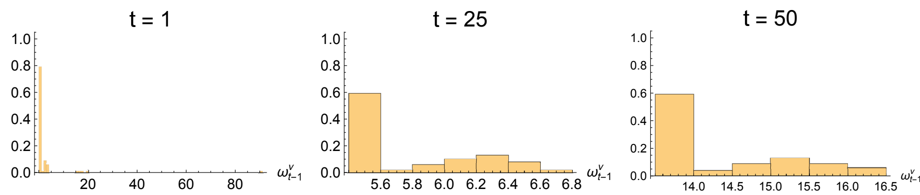
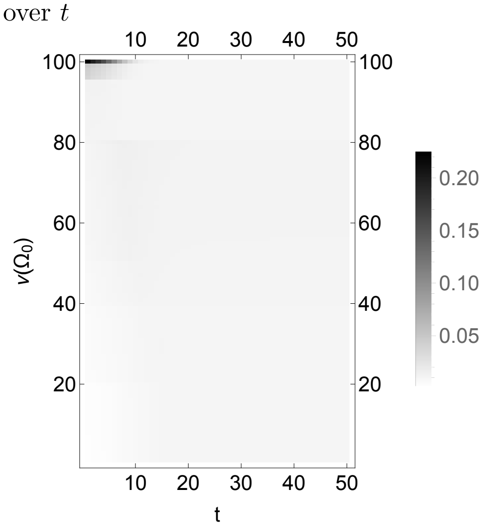
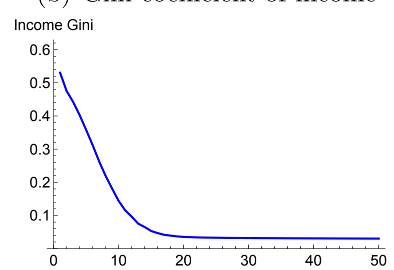


Figure 31: Distribution of income - Model with endogenous subsistence and technical change with education

(a) Post-tax distribution of income shares over t



(b) Gini coefficient of income



that any inequalities are the result of innate heterogeneities in labour. Arguably, it is not unreasonable to think that any education system will not able to fully homogenise certain innate heterogeneities talents and predispositions that people may have. Overall, these results speak to the importance of education as part of what could be considered a socialist project—one aimed at ameliorating social inequities stemming from heterogeneities in skills in addition to eliminating wealth inequalities causing systemic problems of exploitation.

11 Robustness

A bevy of alternative simulations have been run to ensure that our results are robust to changes in simulation parameters and specifications of the models. In this section, we briefly summarise the main findings: a detailed description can be found in the Addendum.

11.1 Skills

First, we have analysed both the basic economy and the model with endogenous consumption and technical progress under the special case of homogeneous labour, or $l^\nu = 1$ for all $\nu \in \mathcal{N}$. The addition of skills does not significantly affect the macro-behaviour of the economy as shown in the summary results and the distribution of wealth. However, as expected, the dynamics of the distributions of the exploitation intensity index and income are significantly different, and there is no trade off between the elimination of exploitation and the reduction of wealth and income inequalities.

Second, it may be objected that our results depend on the specific assumptions concerning the distribution of skills. Although we believe that our assumption is empirically justified, we have run a set of simulations under alternative assumptions on $(s^\nu)_{\nu \in \mathcal{N}}$. To be precise: (i) skills assigned to be increasing in agents' wealth so that the wealthiest agents have the highest effective labour capacity of all $\nu \in \mathcal{N}$; (ii) skills assigned to be decreasing in wealth so that the wealthiest agents have the lowest effective labour capacity; (iii) skills that are normally distributed and ordered according to Ω_0. We have analysed the basic economy and the model with endogenous consumption and technical change under these alternative assumptions on the distribution of skills, including versions with wealth taxes, taxes according to Rule 1, and taxes following Rule 2. These alternative assumptions on skills do not qualitatively alter the simulation results, save for the patterns of exploitation intensity. As expected, the patterns of exploitation intensity in these alternative scenarios are driven by the distribution of skills, with certain commonalities to the simulations reported here. Specifically, agents with the highest skills relative to their wealth experience the most intense exploitation. The addition of wealth taxes, Rule 1 taxes, and Rule 2 taxes also has the same effect on simulations with alternative skill distributions as the simulation reported here.

Third, we have considered an alternative method of incorporating heterogeneous labour was explored whereby agents have the same skills but different endowments of *labour time*, with agents at the lower end of the wealth distribution having more time available to spend laboring relative to wealthier agents. In these simulations agents are divided into quartiles and assigned different values of l^ν depending on which quartile they fall in. This is a simpler way of introducing heterogeneous labour in the model, but it does not qualitatively alter any of the simulation results.

11.2 Taxes

First, in all economies with taxation, we have compared the dynamics of all variables in pre-tax as well as in post-tax terms. Given the structure of the optimisation programme, there is no qualitative difference in the evolution of the two sets of variables. Indeed, the relatively low taxation rates imply that in any given period the pre- and post-tax distributions are extremely similar.

Second, all results are robust to various perturbation of the taxation schemes, including different tax rates or different rules concerning the distribution of tax proceeds.

Third, in section 9.2, we have presented the results of the simulation of the model with endogenous technical change and consumption norms under the socialist taxation Rule 2. However, we have also examined the effect of Piketty-type wealth taxes and Rule 1. The summary results are qualitatively identical to those in figure 15. In both cases wealth inequalities decrease rapidly and tend to vanish. Compared to section 9.2, the main difference concerns the dynamics of exploitation, with the distribution of ε_t^ν displaying a clear cyclical pattern: accumulation reduces exploitation intensity for agents at top of the skill distribution, while ε_t^ν increases for low-skilled agents, until technical change increases ε_t^ν for highly skilled agents, reducing ε_t^ν for low-skilled agents, and the cycle begins to repeat. Interestingly, agents in the middle of the skill distribution—those in the second quintile and between the 80th-95th percentiles of the income distribution—experience little variation in exploitation intensity. The shift in the distribution of wealth toward agents who begin the simulation with low skill levels and zero wealth increases the inequality in exploitation intensity, causing large fluctuations in γ_t^ε after $t = 10$.

As in the basic economy, wealth taxes eventually render wealth inequality a non-factor and any differences in income will be due to differences in labour performed at the prevailing wage.

11.3 Classes

Following Roemer [22], we have also analysed the equilibrium structure and dynamics of classes, and the relation between class and exploitation status, in all economies considered. First, it is possible to generalise Roemer's [22] definition of classes and his celebrated *Class-Exploitation Correspondence Principle* to economies with heterogeneous skills. In both the basic economy, and the economy with endogenous technical change and consumption norms, a remarkably stable class structure emerges in equilibrium until the economy becomes labour constrained and classes disappear. The introduction of wealth taxation does reduce class polarisation but it does not eliminate classes, except in the case of Rule 2.

11.4 Alternative measures of income and exploitation

In all simulations, we analyse the dynamics of the distribution of potential income, $(1+\pi_t)\,\omega_{t-1}^{\nu}+\widehat{w}_t\Lambda_t^{\nu}$, because it captures the total income that agents may devote to consumption of accumulation in every given period—which includes beginning-of-period endowments. The main results and conclusions of the paper remain unchanged if one focuses instead on the *flow* of income deriving from productive endowments and consider $\pi_t\omega_{t-1}^{\nu}+\widehat{w}_t\Lambda_t^{\nu}$.

The exploitation index defined in section 3 measures exploitation intensity according to agents' *effective* labour performed. Alternatively, one may argue that the concept of exploitation is meant to capture some inequalities in the distribution of material well-being and free hours that are—at least *prima facie*—of normative relevance (Fleurbaey [12]). For example, they may be deemed relevant because material well-being and free hours are key determinants of *individual well-being freedom* (Veneziani and Yoshihara [33]). But they are also relevant in approaches that link exploitation and the Marxian notion of alienation in production (Buchanan [4]). From this perspective, the key variable of normative interest is labour *time*.

In constructing an index measuring the intensity of exploitation according to the amount of *time* agents work we immediately encounter a difficulty. While the numerator of such index can be taken to correspond to the unadjusted labour hours that agents spend in production, there is no obvious way of defining the denominator which should measure the amount of labour hours that agents receive in their (notional) bundle c_t. The amount vc_t provides a skill-adjusted (via v) quantity of labour and this needs to be transformed into an amount of labour time factoring out skills. There is no natural, or obvious way to transform vc_t into labour time and any choice is inevitably counterfactual.[24]

We explore an alternative option by dividing vc_t by the average level of skill in the economy, so that the exploitation index e_t^{ν} captures potential inequalities in the labour hours supplied to obtain one unit of potential consumption. Letting $\lambda_t^{\nu}\equiv\Lambda_t^{\nu}/s^{\nu}$ denote the labour *time* agents spend, the exploitation intensity index is

$$e_t^{\nu}=\frac{\lambda_t^{\nu}}{vc_t^{\nu}/\frac{\sum_{\nu}s^{\nu}}{N}}.$$

Observe that unlike for ε_t^{ν} there is no clear threshold to define exploitation status: the consideration of effective labour in this framework renders thresholds for agents'

[24] One may divide vc_t by s^{ν} to measure the time that ν receives given her skills. But note that this is not necessarily equal to the actual amount of unadjusted time used to produce c_t. Moreover, in this case we would trivially be back to ε_t. Interestingly, this suggests an alternative and intriguing interpretation of ε_t^{ν} as measuring the amount of time given and received by ν if she was the only productive agent in the economy, or if all agents were alike.

exploitation status according to e_t^ν less clear, but the variation in $(e_t^\nu)_{\nu\in\mathcal{N}_t}$ yields insight into differential relationships between the labour time agents perform and their available resources for potential consumption.

In all simulations, inequalities in the distribution of $(e_t^\nu)_{\nu\in\mathcal{N}_t}$ increase and eventually stabilise at a pretty high level. Both in the basic model and in the economy with endogenous technical change and consumption norms, the least-skilled agents at the top and bottom of the vertical axis experience higher degrees of time-adjusted exploitation intensity. This is due to the low skill levels of these agents relative to their labour endowment $\zeta^\nu = 1$. Because their skills are low, they receive little by way of labour income yet put in the same amount of time as relatively high-skilled agents in the "middle class". This pattern is even more apparent if the economy becomes labour constrained or when wealth inequalities disappear, because the agents who begin the simulation with large amounts of wealth experience the most intense time-adjusted exploitation due to their extremely low skills. Significant inequalities in e_t^ν remain even at the socialist allocation.

Inequalities in e_t^ν are only resolved in the models with education, or under the special case of homogeneous labour. The compression of the skills distribution caused by education leads to a convergence of effective labour performed by agents and their labour-time expended. Consistent with other results for the models with education, this convergence of effective labour and labour-time—and by extension ε_t^ν and e_t^ν—is asymptotic with any inequalities in e_t^ν being very small at $t = T$.

12 Conclusions

This paper shows that, contrary to the received wisdom, a notion of exploitation exists that is logically coherent, well-defined, and firmly anchored to empirical data. Exploitation can be defined both at the aggregate and at the individual level by means of an *exploitation index* which measures an agent's effective labour per unit of income received. For each individual, this index is a clearly defined magnitude that can be measured based on available empirical data, and its distribution can be analysed with the standard tools of the theory of inequality measurement. Further, the notion of exploitation is normatively relevant, and the analysis of the distribution of the exploitation index yields distinct insights on the injustice of capitalism and on the effects of redistributive policies. It may also shed some light on the cyclical and crisis-prone nature of capitalism. In short, the news of the death of exploitation theory are greatly exaggerated.

In closing this paper, it is worth mentioning some open questions. The analysis in section 9 suggests that capital using labour saving technical progress may play a crucial role in guaranteeing the persistence of exploitation in capitalist economies. This is certainly an important insight but it is worth noting that it derives from a rather specific mechanism to determine distributive variables: technical change plays a crucial role in creating labour unemployment which in turn forces the real wage down to the (socially determined and time-evolving) subsistence level. This immediately raises two issues. First, it appears that labour unemployment is a necessary determinant of the persistence of exploitative relations. Yet it would be important, both normatively and theoretically, to analyse the persistence of exploitation in economies with full employment. Second, with the richest 5% of the population holding around 70% of the wealth and employing a mass of propertyless agents (50% of the population), and the issues of power and class solidarity that this polarised wealth distribution raises, it seems natural to analyse a more complex model for the determination of the key distributive variables. We leave both issues for further research.

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