Addendum for the paper "*Exploitation, skills, and inequality*". Not for publication.

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Abstract

Sections 1, 2, and 4 present additional information on the simulations in the paper. Sections 3 and 5-13 present the results of robustness checks.

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Contents

1 Skill distributions

The figures in this section show distribution of skills s^{ν} in relation to agents' wealth at $t = 0$, W_0^{ν}, for all simulations in the accompanying paper.

Figure 1: Skill vs. Wealth - Model with skilled-labour

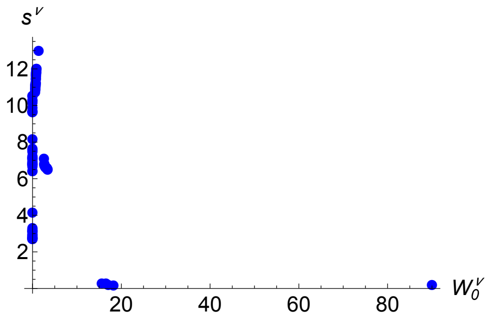


Figure 2: Skill vs. Wealth - Model with wealth taxes

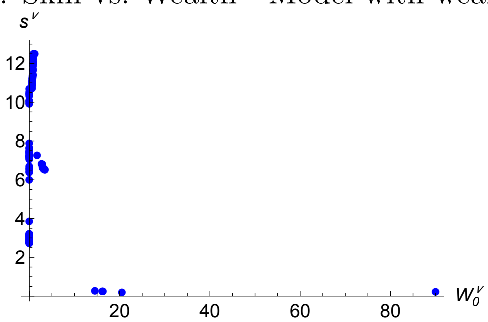


Figure 3: Skill vs. Wealth - Model with wealth equality

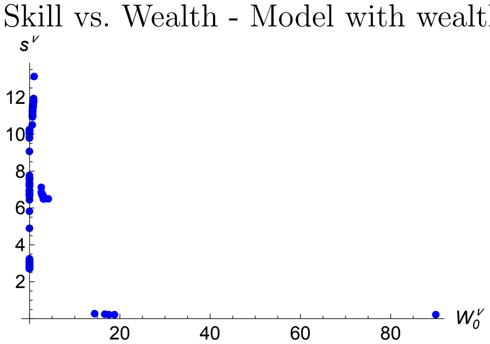


Figure 4: Skill vs. Wealth - Model with socialist allocation

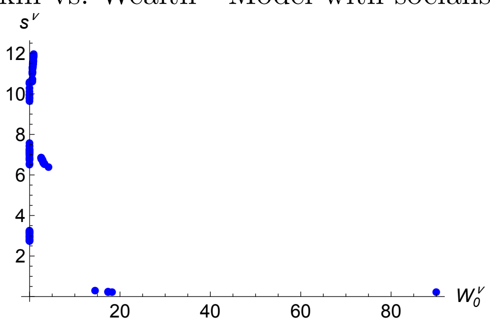
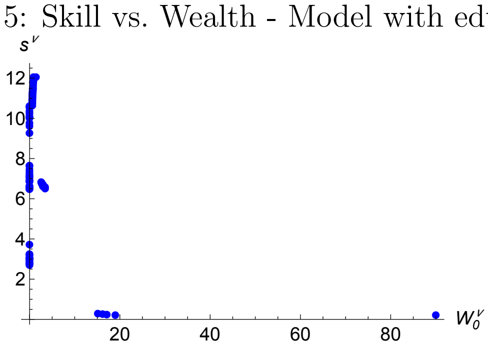


Figure 5: Skill vs. Wealth - Model with education



2 Equilibrium conditions

The figures in this section show that, in all of our simulations, conditions (b)-(d) of Definition 1 in the paper are satisfied, and so we are analysing the equilibrium dynamics of the economies considered.

Figure 6: Equilibrium conditions - Model with skilled-labour

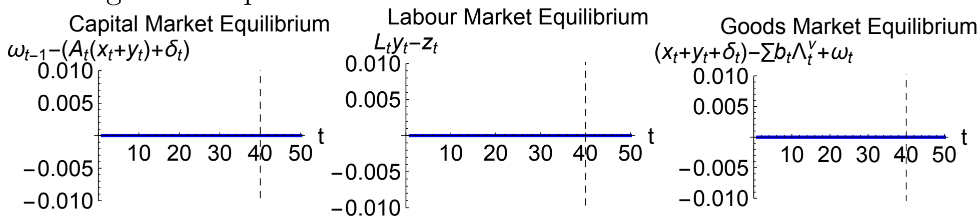


Figure 7: Equilibrium conditions - Model with wealth taxes

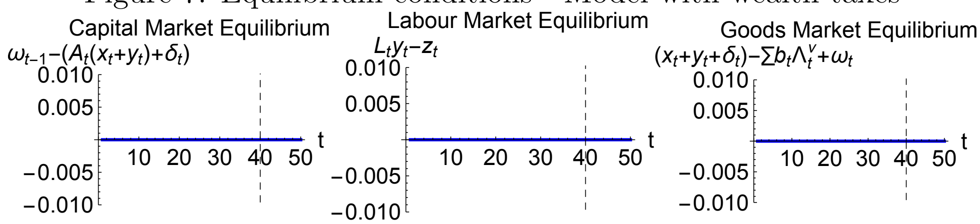


Figure 8: Equilibrium conditions - Model with wealth equality

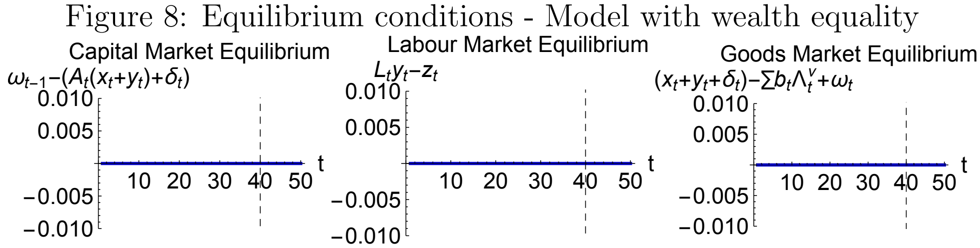


Figure 9: Equilibrium conditions - Model with socialist allocation

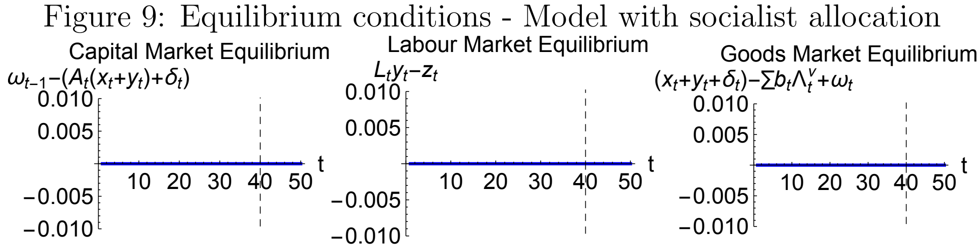
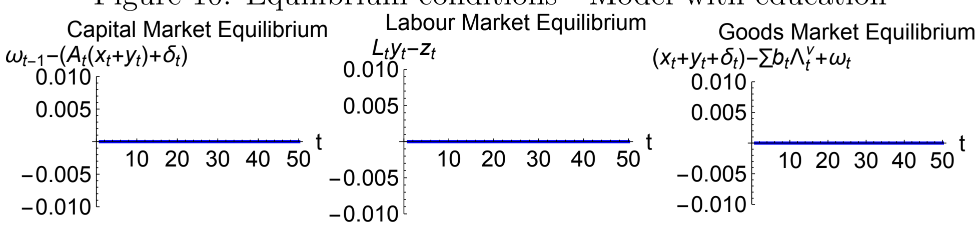


Figure 10: Equilibrium conditions - Model with education



3 Simulation summary results

The figures below report the summary results that are omitted from the main paper, and for the two simulations in section 3 here.

Figure 11: Summary results - Model with wealth taxes

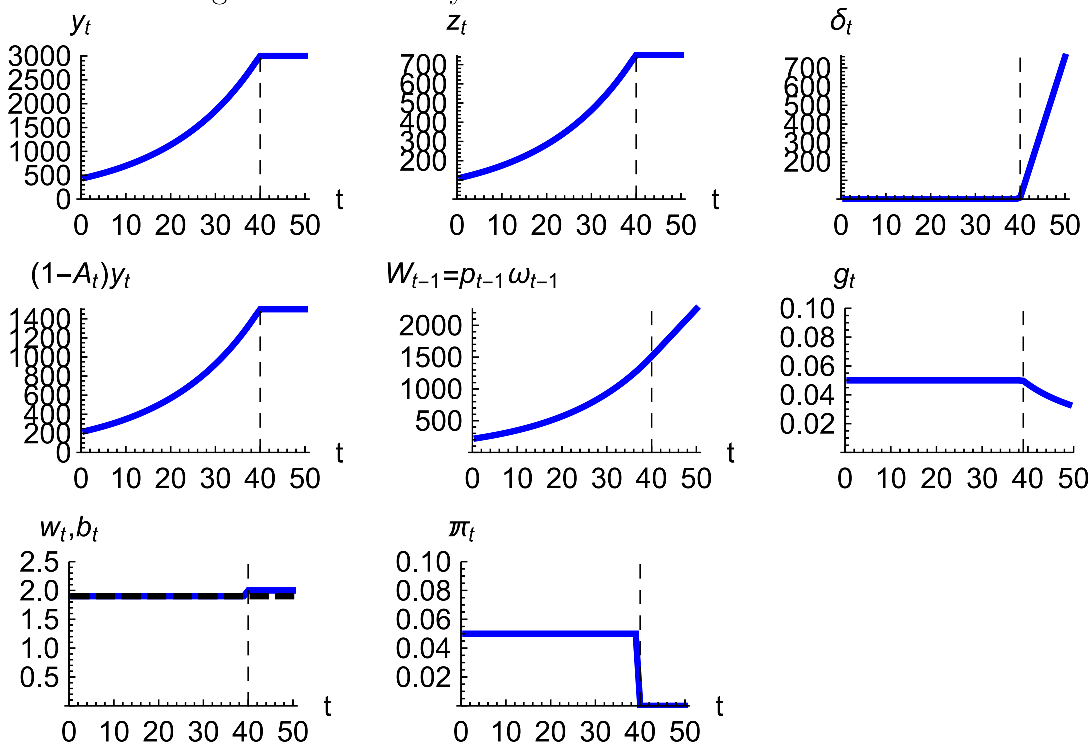


Figure 12: Summary results - Model with wealth taxes and wealth equality

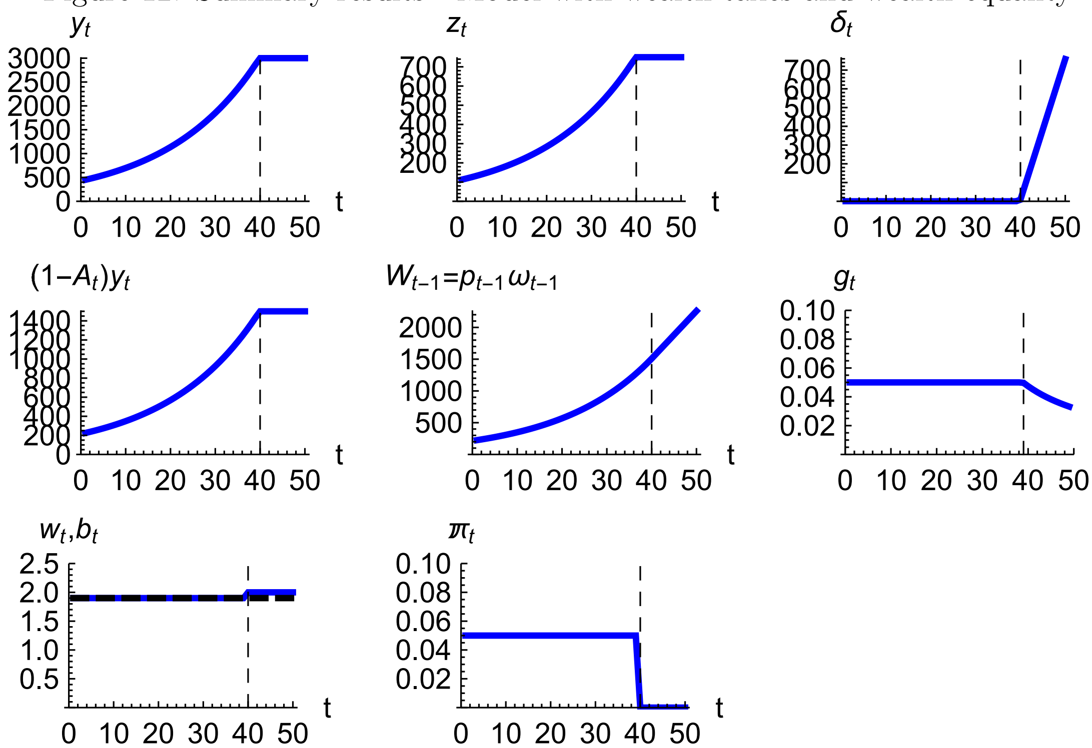
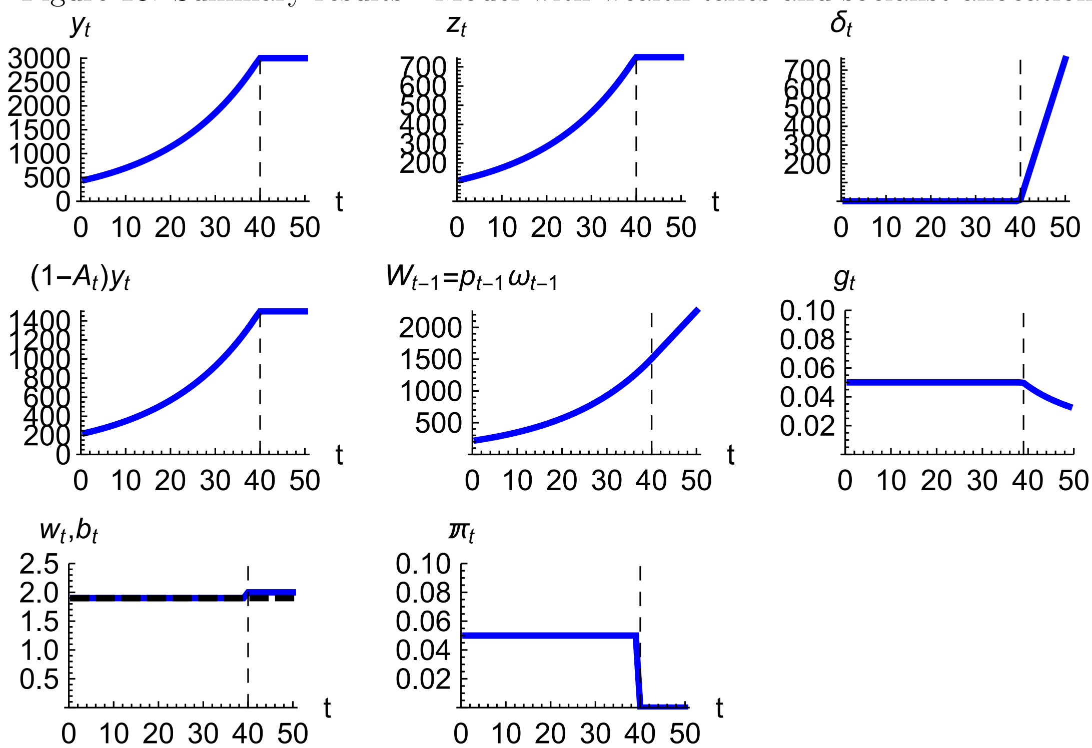


Figure 13: Summary results - Model with wealth taxes and socialist allocation



4 Net income figures

The figures below report the dynamics of the distribution of net income $\pi_t \omega_{t-1}^{\nu} + \widehat{w}_t \Lambda_t^{\nu}$ for all simulations in the paper.

Figure 14: Distribution of net income - Model with skilled-labour

(a) Distribution of net income over t (b) Gini coefficient of net income

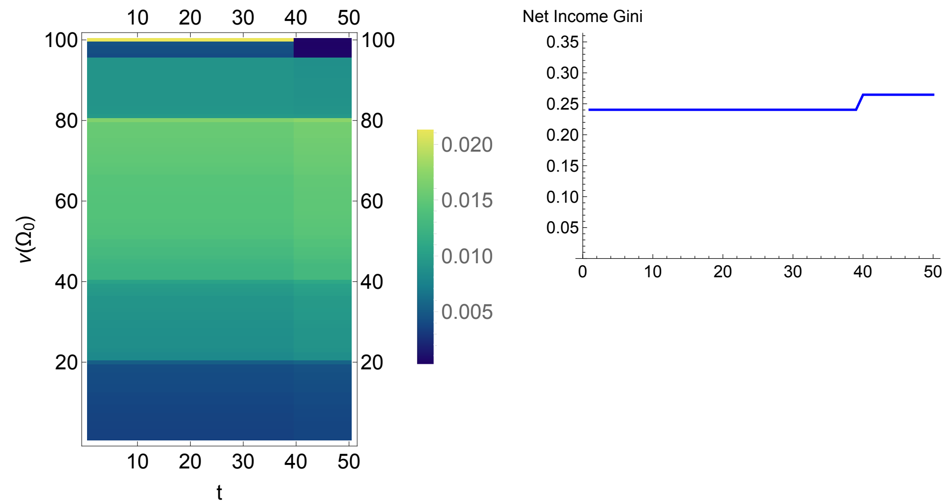


Figure 15: Distribution of net income - Model with wealth taxes

(a) Distribution of net income shares (b) Gini coefficient of net income

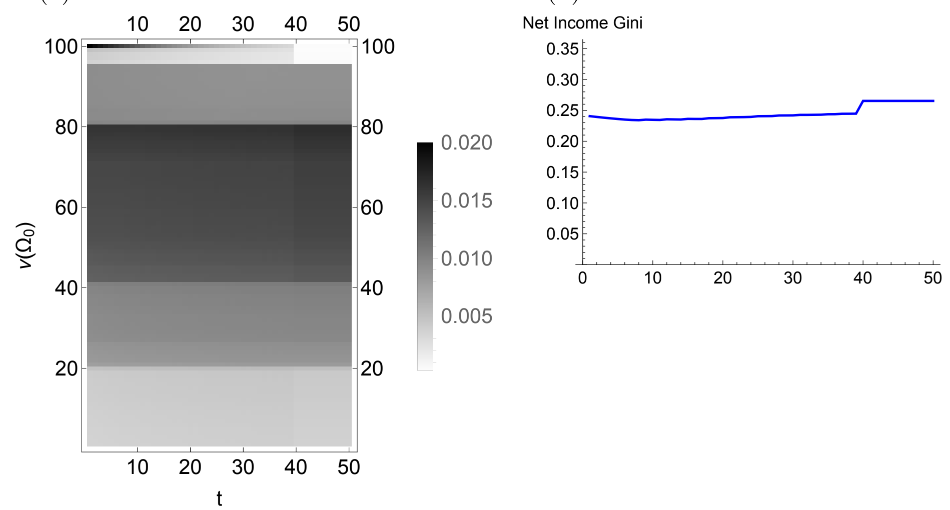


Figure 16: Distribution of net income - Model with wealth taxes and wealth equality

(a) Distribution of net income shares (b) Gini coefficient of net income

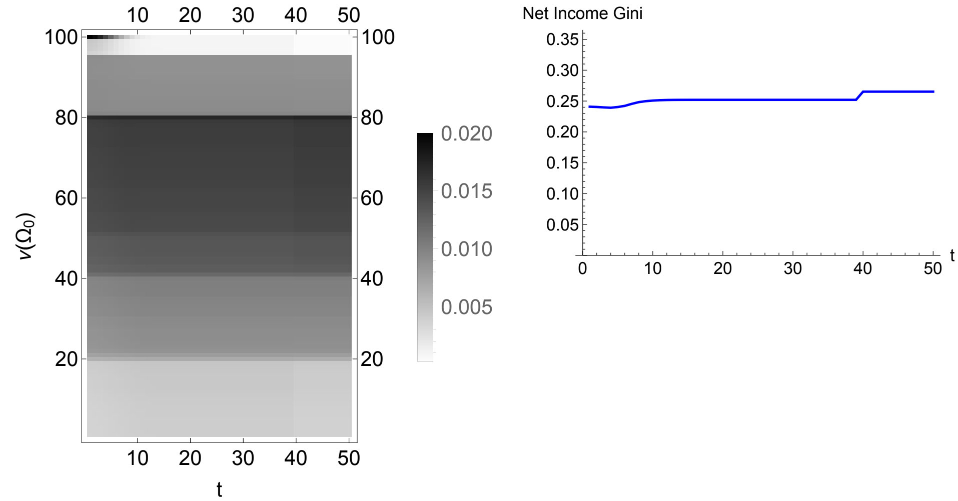


Figure 17: Distribution of net income - Model with wealth taxes and socialist allocation

(a) Distribution of net income shares (b) Gini coefficient of net income

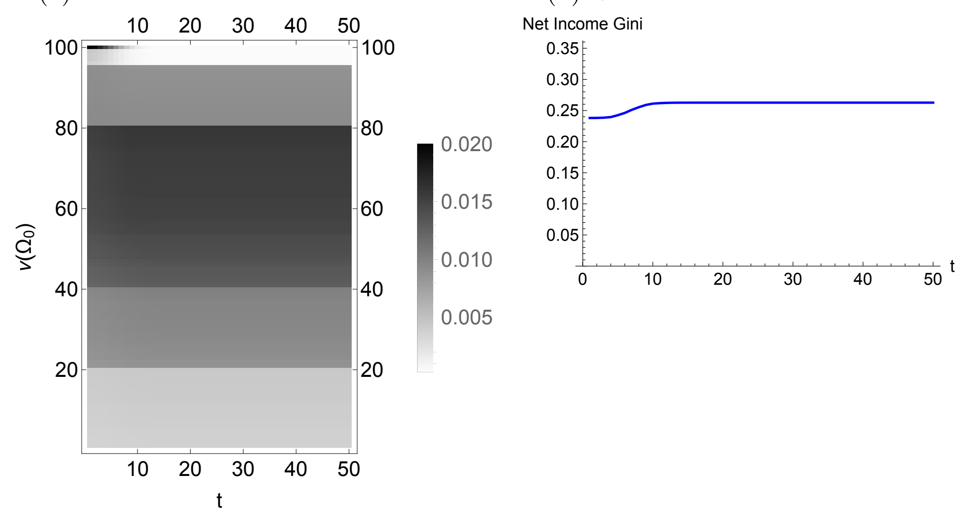
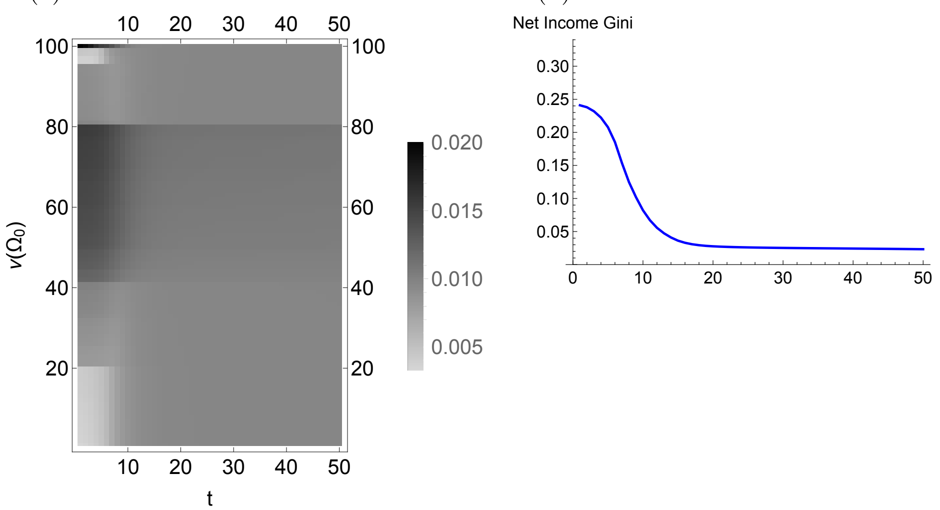


Figure 18: Distribution of net income - Model with education

(a) Distribution of net income shares (b) Gini coefficient of net income



5 An alternative exploitation intensity index

Presuming a close relationship between agents' skills and their labour income, as our simulations and measure of exploitation ε_t^ν do, may not fully correspond to empirical studies of the causes behind earnings differentials. For example, work by Bowles, Gintis, and Osborne [2] finds that education—widely interpreted as a measure of skill(s)—does not fully explain inequality in earnings, which is further confirmed in an intergenerational context by Bowles and Gintis [1]. Important factors like socialisation, family background, and social networks are also important explanatory factors in earnings differentials. An alternative interpretation of skill factors and their determination of income could be to view s^ν as reflecting factors like family background and network connections, i.e. the "skills" agents have are more akin to knowing the "right" people, going to the "right" school(s), and behaving in the "right" way(s). To this end, below we examine the dynamics of the *time-adjusted exploitation intensity index* e_t^ν.

For the model in the paper, figure 19(a) shows that the least-skilled agents at the top and bottom of the vertical axis experience higher degrees of time-adjusted exploitation intensity. This is due to the low skill levels of these agents relative to their labour endowment $\zeta^\nu = 1$. Because their skills are low, they receive little by way of labour income yet put in the same amount of time as relatively high-skilled agents in the "middle class". This pattern is even more apparent in the labour constrained portion of the simulation as the agents who begin the simulation with large amounts of wealth experience the most intense time-adjusted exploitation due to their extremely low skills.

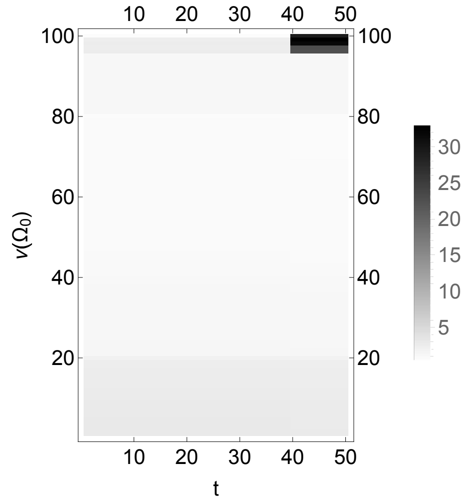
Figures 20(a) and 20(b) show the post- and pre-tax distribution of e_t^ν for the model with Piketty-type wealth taxes. Figure 20(c) shows the pre- and post-tax γ_t^e. The wealth taxes do little to impact the distribution of e_t^ν since redistributing wealth does nothing to Λ_t^ν/s^ν, s^ν, and the average skill level. Interestingly, γ_t^e rises as the simulation becomes labour constrained, this can be thought of as a "natural" level of γ_t^e given the distribution of s^ν.

Figures 21(a) and 21(b) show the post- and pre-tax distribution of e_t^ν for the model under Rule 1. Figure 21(a) shows that redistributive taxes increase e_t^ν for agents with low skills and high wealth, as is also seen in the increase in the Gini coefficient of e_t^ν, γ_t^e, in figure 21(c).

Figures 23(a) and 23(b) respectively show the post- and pre-tax distributions of e_t^ν for the model under Rule 2. Figures 23(a) and 23(b) show that the socialist allocation does little to alleviate exploitation in terms of labour time, increasing e_t^ν for agents who begin the simulation with the highest wealth and low skills. This is also seen in the increase in the Gini coefficient of $(e_t^\nu)_{\nu\in\mathcal{N}}$, γ_t^e, in figure 23(c). The dynamics of exploitation intensity in terms of ε_t^ν under the socialist allocation in section 8 of the

Figure 19: Time-adjusted exploitation - Model with skilled-labour

(a) e_t^{ν}

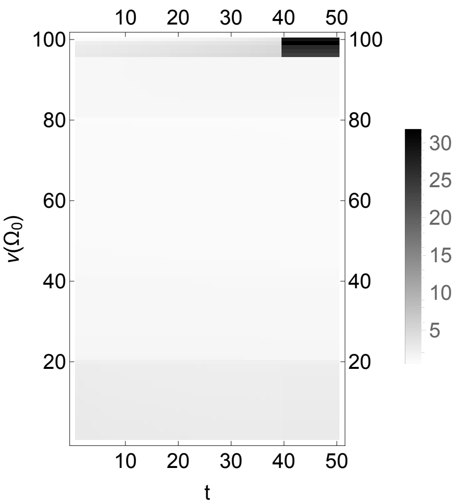


paper was as one would expect, but the distribution of e_t^{ν} reveals a range of experience of exploitation intensity relative to agents with average skills. Agents at the top of the skills hierarchy—the "middle class"—benefit most from their labour time relative to the average-skilled agent, whereas the opposite is true for agents with low skill levels.

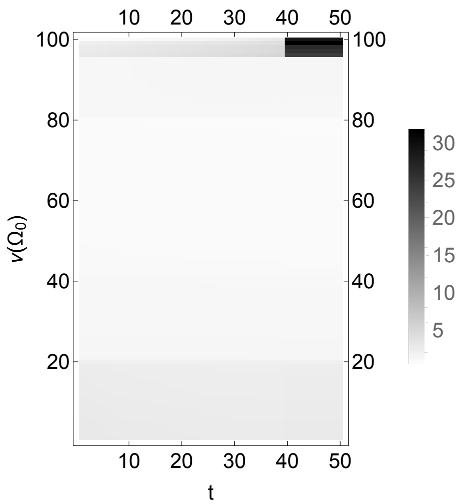
Figure 23 shows dynamics of e_t^{ν} for the model with education in section 10.1 of the paper. As expected, the compression of the skill distribution induces greater equality time-adjusted exploitation, albeit not perfect equality.

Figure 20: Time-adjusted exploitation - Model with wealth taxes

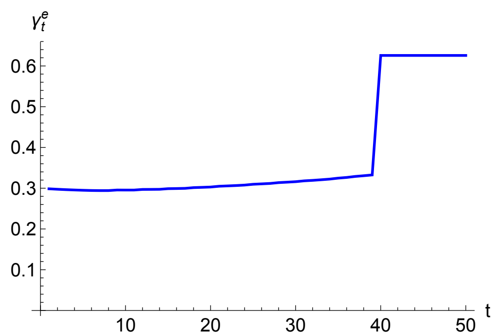
(a) Post-tax e_t^ν



(b) Pre-tax e_t^ν



(c) Gini coefficient of e_t^ν



(d) Distribution of e_t^ν for select t

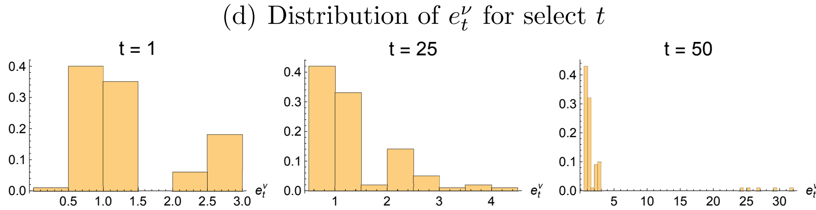
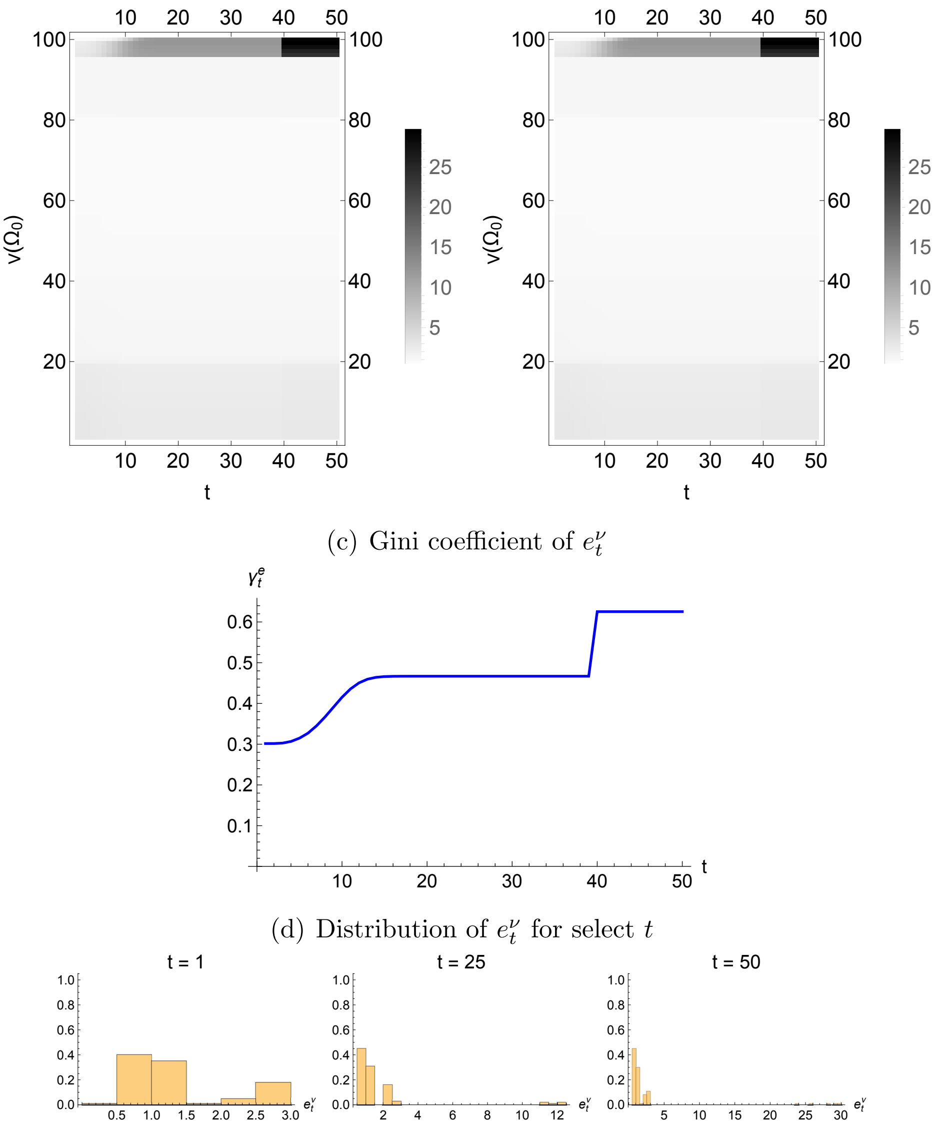


Figure 21: Time-adjusted exploitation - Model with wealth taxes and wealth equality



(a) Post-tax e_t^ν

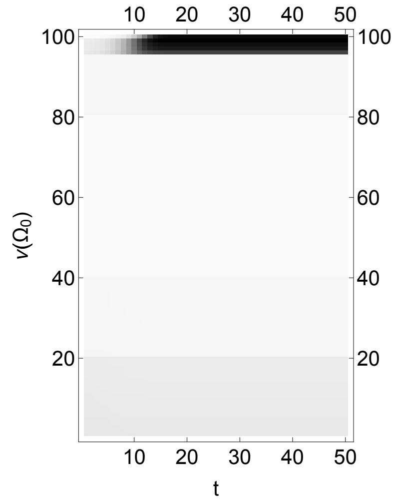
(b) Pre-tax e_t^ν

(c) Gini coefficient of e_t^ν

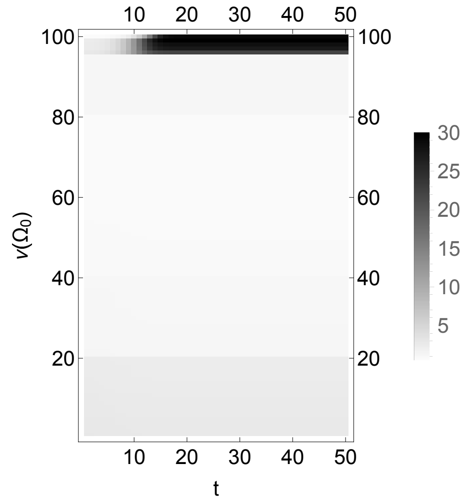
(d) Distribution of e_t^ν for select t

Figure 22: Time-adjusted exploitation - Model with wealth taxes and socialist allocation

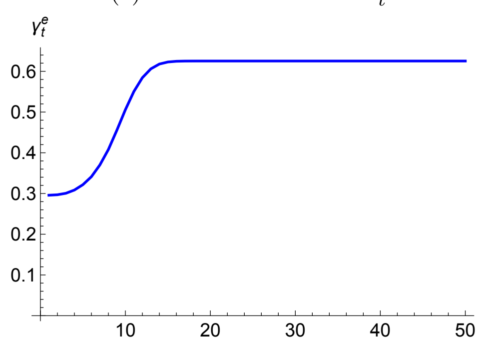
(a) Post-tax e_t^ν



(b) Pre-tax e_t^ν



(c) Gini coefficient of e_t^ν



(d) Distribution of e_t^ν for select t

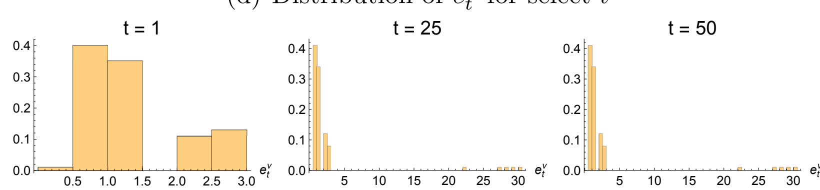
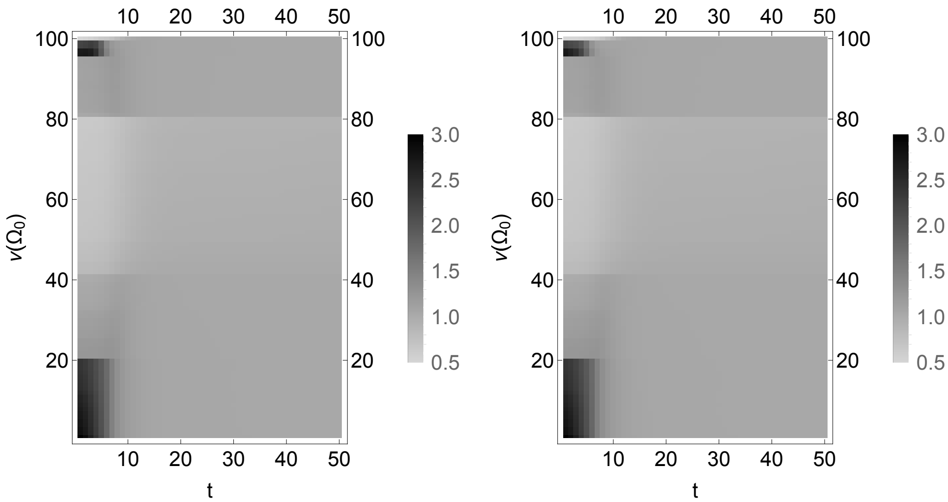
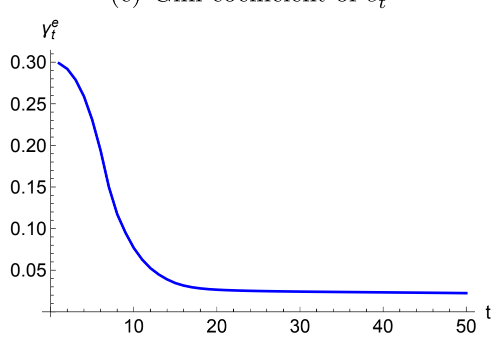


Figure 23: Time-adjusted exploitation - Model with education

(a) Post-tax e_t^ν (b) Pre-tax e_t^ν



(c) Gini coefficient of e_t^ν



6 Classes in the simulations

Following Roemer [3], *classes* can be defined based on the way in which agents relate to the means of production. Let (a_1, a_2, a_3) be a vector where $a_i \in \{+, 0\}$, $i = 1, 2, 3$, where "+" means a positive entry. In every t, agent ν is said to be a member of class (a_1, a_2, a_3), if there is $\xi_t^\nu = (x_t^\nu; y_t^\nu; z_t^\nu; \delta_t^\nu) \in \mathcal{A}^\nu(1, \widehat{w}_t)$ such that $(x_t^\nu; y_t^\nu; z_t^\nu)$ has the form (a_1, a_2, a_3). The notation $(+, +, 0)$ implies, for instance, that an agent works in her own 'shop' and hires others to work for her; $(+, 0, +)$ implies that an agent works both in her own 'shop' and for others; and so on. Although there are eight conceivable classes, only the following four are theoretically relevant.

$$
\begin{aligned}
C_t^1 &= \{\nu \in \mathcal{N} \mid \mathcal{A}^\nu(1, \widehat{w}_t) \text{ has a solution of the form } (+, +, 0) \setminus (+, 0, 0)\}, \\
C_t^2 &= \{\nu \in \mathcal{N} \mid \mathcal{A}^\nu(1, \widehat{w}_t) \text{ has a solution of the form } (+, 0, 0)\}, \\
C_t^3 &= \{\nu \in \mathcal{N} \mid \mathcal{A}^\nu(1, \widehat{w}_t) \text{ has a solution of the form } (+, 0, +) \setminus (+, 0, 0)\}, \\
C_t^4 &= \{\nu \in \mathcal{N} \mid \mathcal{A}^\nu(1, \widehat{w}_t) \text{ has a solution of the form } (0, 0, +)\}.
\end{aligned}
$$

The notation $(a_1, a_2, a_3) \setminus (a_1', a_2', a_3')$ means that agent ν is a member of class (a_1, a_2, a_3) but not of class (a_1', a_2', a_3').

Theorem 3 proves that $C_t^1 - C_t^4$ represent a partition of the set of agents.

Theorem 1 *Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be a RS for E_0. At any t, if $\pi_t > 0$:*

$$
\begin{aligned}
&\textit{(i)}\ \nu \in (+, +, 0)\setminus(+, 0, 0) \Leftrightarrow Ly_t^\nu > z_t^\nu \textit{ for all } \xi_t^\nu \in \mathcal{A}^\nu(1, \widehat{w}_t); \\
&\textit{(ii)}\ \nu \in (+, 0, 0) \Leftrightarrow Ly_t^\nu = z_t^\nu \textit{ for some } \xi_t^\nu \in \mathcal{A}^\nu(1, \widehat{w}_t); \\
&\textit{(iii)}\ \nu \in (+, 0, +)\setminus(+, 0, 0) \Leftrightarrow Ly_t^\nu < z_t^\nu \textit{ for all } \xi_t^\nu \in \mathcal{A}^\nu(1, \widehat{w}_t); \\
&\textit{(iv)}\ \nu \in (0, 0, +) \Leftrightarrow W_{t-1}^\nu = 0.
\end{aligned}
$$

Theorem 1 characterises the class structure of the accumulating economy, based on the way in which agents relate to the means of production. An immediate implication of Theorem 1 is that the class status of each agent is related to her productive endowments. More precisely, at any RS for E_0 and any period t with $\pi_t > 0$: $\nu \in C_t^1$ if and only if $LA^{-1}\omega_{t-1}^\nu > l^\nu$ and $\nu \in C_t^4$ if and only if $W_{t-1}^\nu = 0$. Furthermore, if $\widehat{w}_t > b$, then $\nu \in C_t^2$ if and only if $LA^{-1}\omega_{t-1}^\nu = l^\nu$ and $\nu \in C_t^3$ if and only if $LA^{-1}\omega_{t-1}^\nu < l^\nu$; whereas if $\widehat{w}_t = b$, then $\nu \in C_t^2$ if and only if $LA^{-1}\omega_{t-1}^\nu \leqq l^\nu$ and $C_t^3 = \varnothing$.

A fundamental insight of Marxian exploitation theory is the existence of a tight relation between class positions and exploitation status. This is captured in the Class-Exploitation Correspondence Principle (CECP) (Roemer [3]), according to which in equilibrium agents in the upper classes are exploiters and agents in the lower classes are exploited if they work at all.[1]

The next result proves that the CECP holds in the accumulation economies considered in this paper.

Theorem 2 (CECP) *Let $\left((\mathbf{1}, \widehat{w}), (\xi^{\nu})_{\nu \in \mathcal{N}}\right)$ be a RS for E_0. At any t, such that $\pi_t > 0$, if $\nu \in C_t^1$ then ν is an exploiter and if $\nu \in C_t^3 \cup C_t^4$ with $\Lambda_t^{\nu} > 0$ then ν is exploited. Furthermore, if $\widehat{w}_t > b$ then:*

$$
\begin{aligned}
\nu &\in C_t^1 \Leftrightarrow \nu \text{ is an exploiter;}\\
\nu &\in C_t^2 \Leftrightarrow \nu \text{ is neither exploited nor an exploiter;}\\
\nu &\in C_t^3 \cup C_t^4 \Leftrightarrow \nu \text{ is exploited.}
\end{aligned}
$$

As noted in section 6 of the paper, the introduction of wealth taxation raises an interesting conceptual issue concerning the definition of exploitation, leading to a distinction between pre-tax and post-tax exploitation analysis. In the case of classes, this distinction does not arise: classes are defined based on the agents' optimal position in the labour market, which is in turn determined by based on the optimal solution to MP_t^{ν}, which is independent of wealth taxation.

Figure 24 reports the dynamics of classes for the model. As expected the structure of classes is stable and there is a correspondence between exploitation status and class while the simulation is capital constrained.

Figure 25 reports the dynamics of classes for the model with wealth taxes in section 6 of the paper. Unlike in the benchmark model, the addition of wealth taxes modifies the relation between exploitation and class status: while the CECP formally continues to hold, wealth taxes cause the class structure of society to undergo significant changes. Figure 25(a) shows that as soon as wealth is redistributed through taxation, the agents in C_t^4 move into C_t^2 as they begin accumulating wealth, with some climbing the ladder into C_t^1 by the time the simulation becomes labour constrained.

Figure 26 shows the dynamics of class for the model under Rule 1. Similar to figure 25, as wealth is redistributed, C_t^4 empties into C_t^2 and the number of agents in C_t^1 increases with accumulation until the simulation becomes labour constrained.

Figure 27 shows classes in the model under Rule 2. Interestingly, the redistribution of wealth under Rule 2 leads to every agent winding up in C_t^2 as their wealth falls inline with their effective labour capacity.

[1] The restriction that agents in the lower classes spend some of their time working is theoretically appropriate restriction since the exploitation status of agents who do not engage in any economic activities is unclear.

Figure 24: Class - Model with skilled-labour

(a) Class status (b) CECP

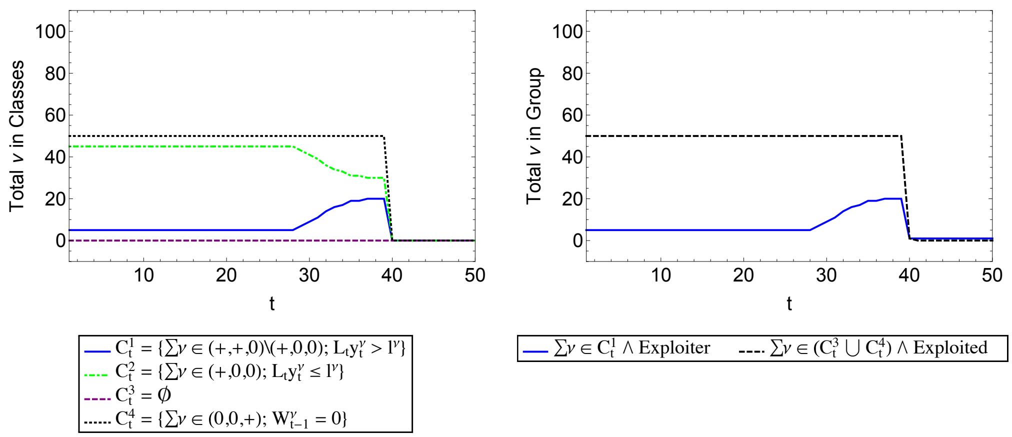


Figure 25: Class - Model with wealth taxes

(a) Class status (b) CECP

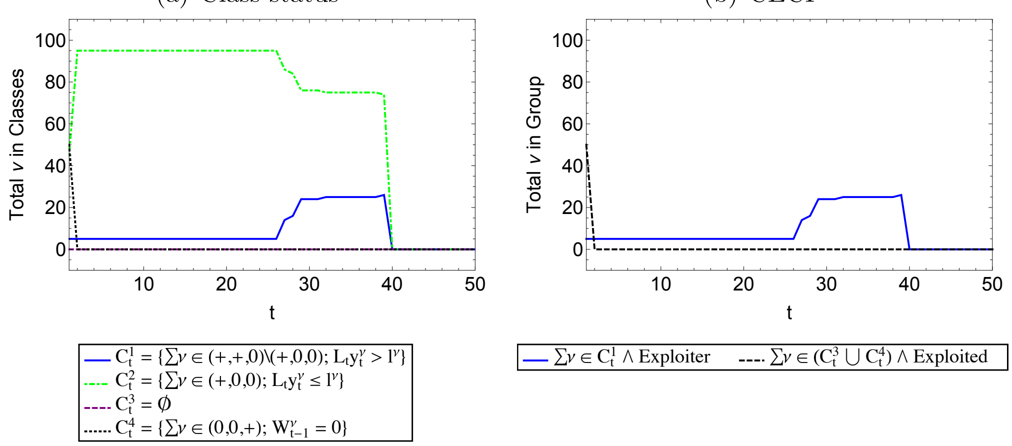


Figure 26: Class - Model with wealth taxes and wealth equality

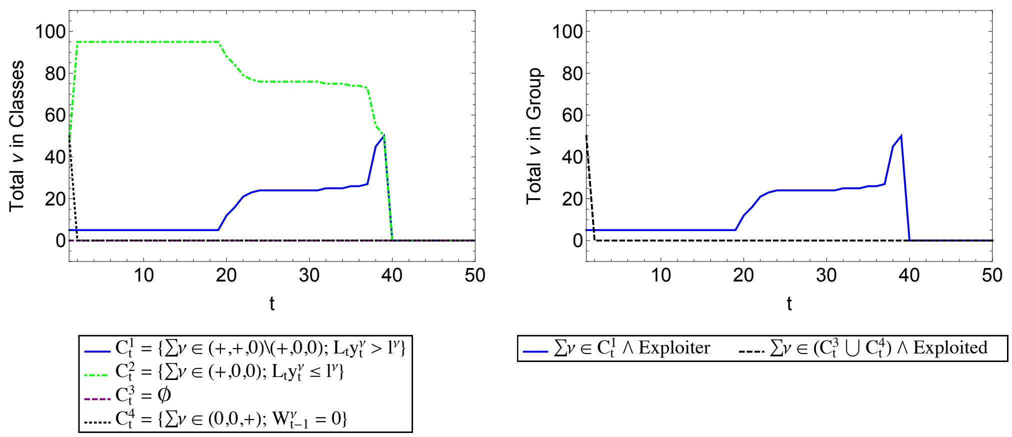
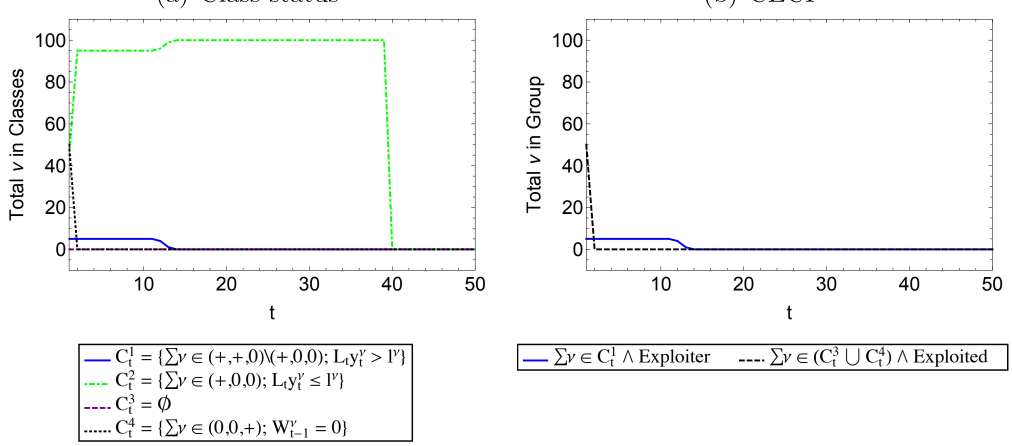


Figure 27: Class - Model with wealth taxes and socialist allocation



7 Pre-tax comparisons

The figures below present some information comparing the pre- and post-tax for relevant variables in the simulations presented in the main paper.

Figure 28 presents some pre-tax comparisons for exploitation intensity, wealth, and income for the economy with Piketty-type taxes.

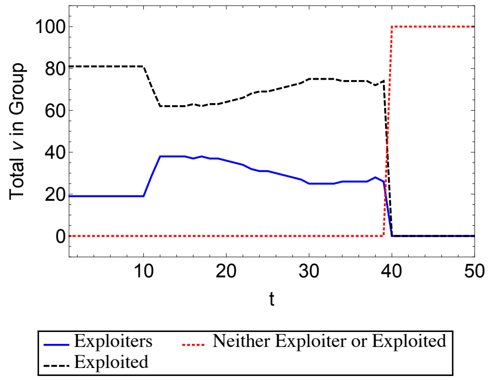
Figure 29 shows some comparative results for pre-tax magnitudes of exploitation intensity and income for the economy with taxes following Rule 1. Figures 29(d) and 29(e) show pre- and post-tax distributions of ε_t^ν for the time period during which overall taxes are the highest, $t = 5$.

Figure 30 shows some comparative figures for pre-tax magnitudes of exploitation and income for the economy with taxes under Rule 2. Figures 30(d) and 30(e) show the pre- and post-tax distributions of ε_5^ν, i.e. the time period during which overall tax collection is highest.

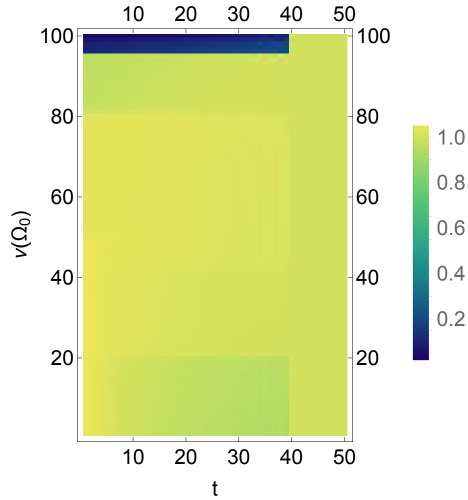
Figure 31 shows pre-tax comparative figures for exploitation, income, and ε_t^ν for $t = 5$, the time period during which taxation is highest, for the economy with taxes according to Rule 2 and education.

Figure 28: Pre-tax comparison - Model with wealth taxes

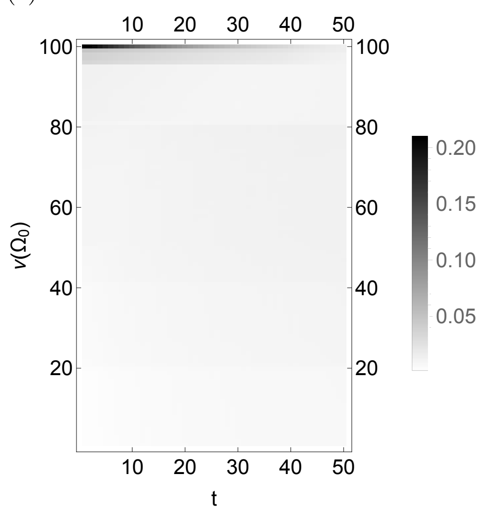
(a) Pre-Tax Exploitation status



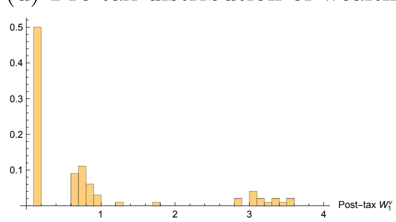
(b) Pre-tax ε_t^{ν}



(c) Pre-tax distribution of income shares



(d) Pre-tax distribution of wealth



(e) Post-tax distribution of wealth

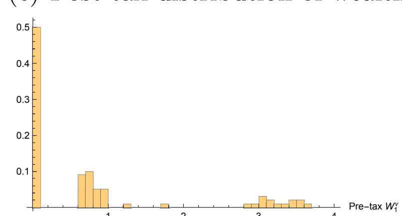
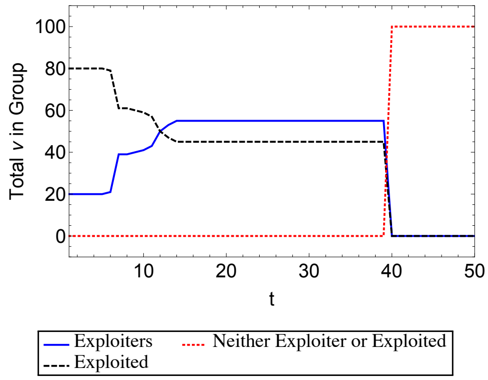
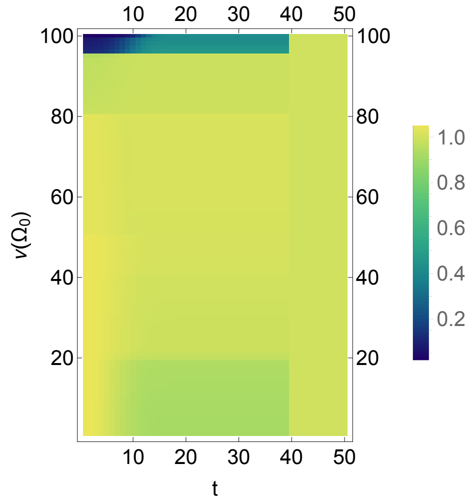


Figure 29: Pre-tax comparison - Model with wealth taxes and wealth equality

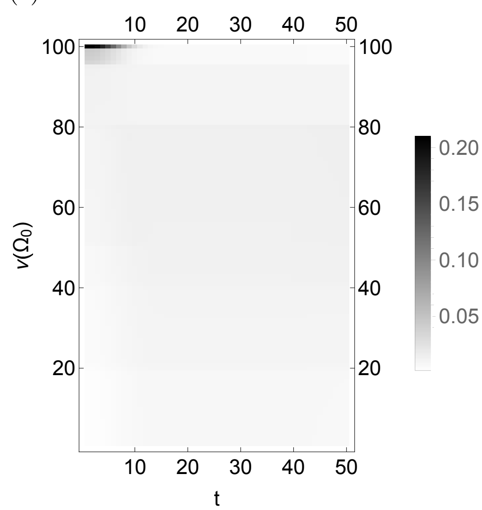
(a) Pre-tax exploitation status



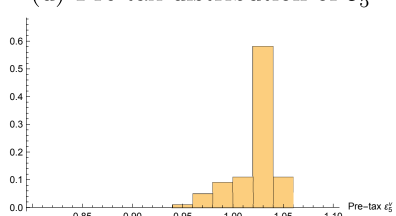
(b) Pre-tax ε_t^ν



(c) Pre-tax distribution of income shares



(d) Pre-tax distribution of ε_5^ν



(e) Post-tax distribution of ε_5^ν

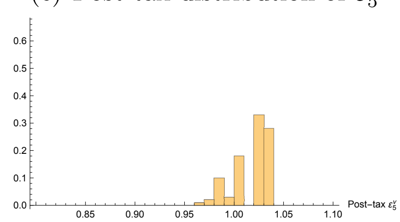
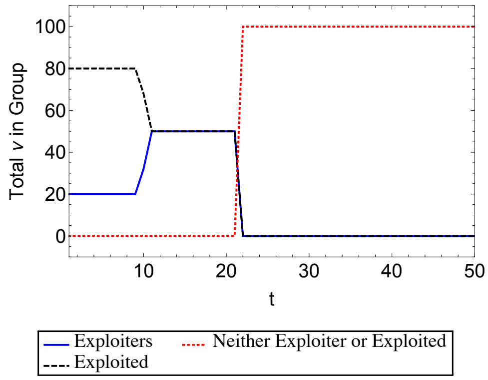
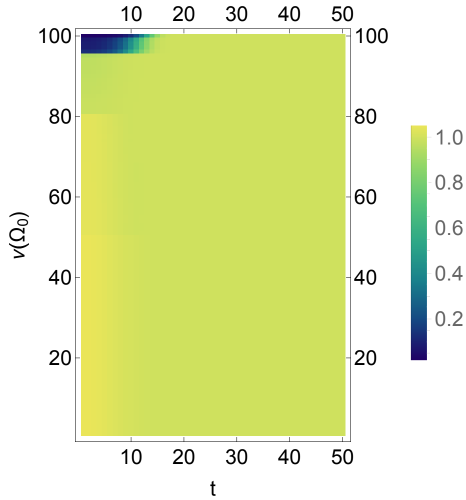


Figure 30: Pre-tax comparison - Model with wealth taxes and socialist allocation

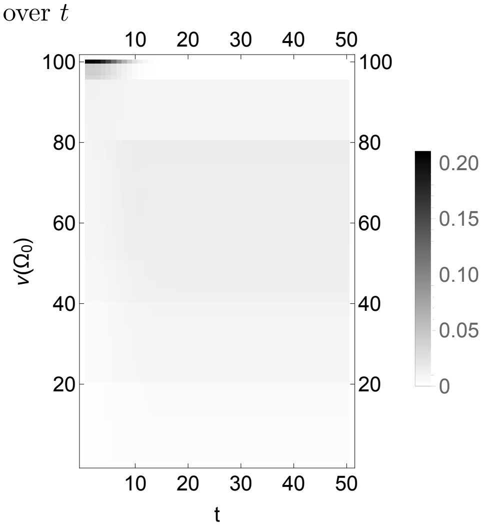
(a) Pre-tax exploitation status



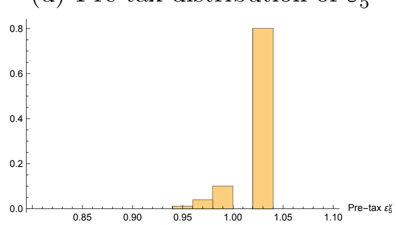
(b) Pre-tax ε_t^ν



(c) Pre-tax distribution of income shares over t



(d) Pre-tax distribution of ε_5^ν



(e) Post-tax distribution of ε_5^ν

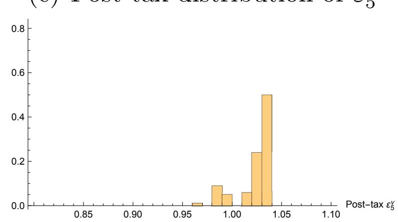
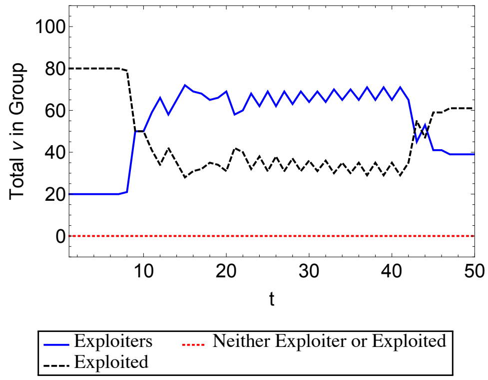
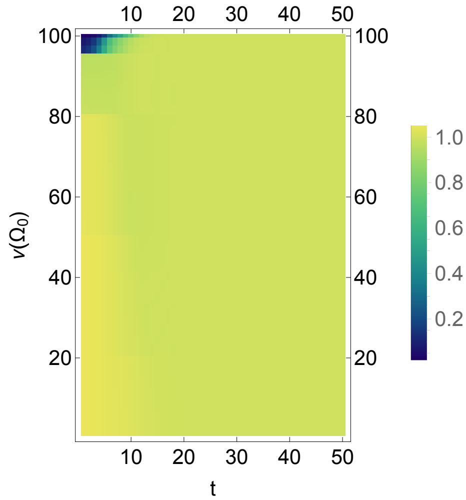


Figure 31: Pre-tax comparison - Model with education

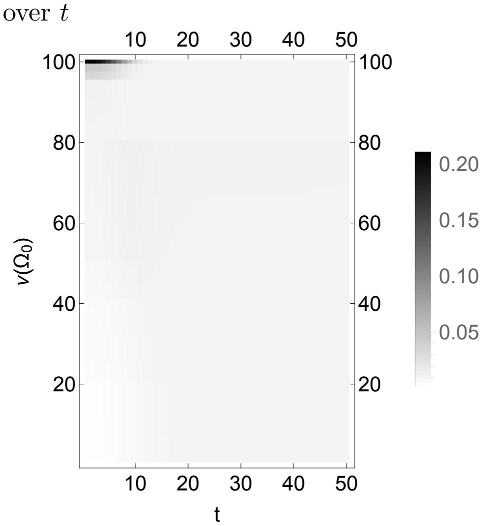
(a) Pre-tax exploitation status



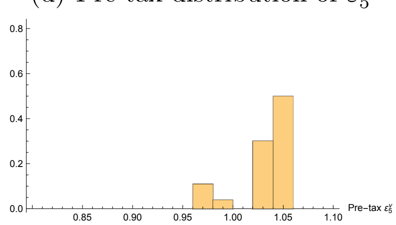
(b) Pre-tax ε_t^ν



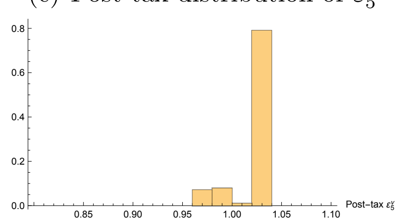
(c) Pre-tax distribution of income shares over t



(d) Pre-tax distribution of ε_5^ν



(e) Post-tax distribution of ε_5^ν



8 Zero subsistence assumption

In this section we analyse the case of Roemer's [3] assumption of $b_t = 0$ for all t for the models in the paper, except for the extension with education. In the simulations that follow, the method for determining the initial distributions of skills and wealth is the same as that of the accompanying paper, and we use the parameters from the paper with the modification that $b = 0$.

Given the assumption of zero subsistence, the exploitation intensity index ε_t^ν is modified slightly to avoid the computationally troublesome division by zero. For all simulations that follow, in the case where an agent has $\omega_{t-1}^\nu = 0$ their ε_t^ν is set arbitrarily high to reflect the high degree of intensity in exploitation experienced by propertyless agents.

8.1 The economy with zero subsistence

This section presents the economy without wealth taxes for the case of $b = 0$. The results of the simulation over T can be found in figures 32-35. As figure 32 shows, setting the subsistence level to zero has no effect on the dynamics of the aggregate behaviour of the model. The only difference between the results immediately below and those of the corresponding simulation in the main paper is the time at which the economy become labour constrained. The dramatically lower subsistence in this case causes the economy to become labour constrained after only a few time periods, due to the faster rate of accumulation.

Figure 33 shows the dynamics of exploitation status and intensity over the course of the simulation. As figure 33(a) shows, the structure of exploitation status is stable for the short period of time in which the economy is capital constrained, and exploitation disappears after the labour constraint is binding for all agents. Figure 33(b) shows the distribution of ε_t^ν across agents over t. Due to the zero subsistence level, a slight adjustment to the calculation of ε_t^ν is made to avoid a zero value of vc_t^ν. For the case in which agents have $\omega_{t-1}^\nu = 0$ and vc_t^ν would go to zero since $b = 0$ for all agents, $\varepsilon_t^\nu = s^\nu \times 10^{10}$. This captures the fact that agents with $\omega_{t-1}^\nu = 0$ experience intense exploitation relative to others, and it preserves the differences in exploitation intensity induced by the distribution of skills. Figure 33(c) takes a slightly different approach to setting ε_t^ν for agents with $\omega_{t-1}^\nu = 0$. In figure 33(c), for agents with $\omega_{t-1}^\nu = 0$, $\varepsilon_t^\nu = 15$. This is done to select an arbitrarily high level for ε_t^ν while not obfuscating the fact that agents who possess small amounts of wealth and high skill levels still experience exploitation.

Figure 34 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality is stable until the economy is labour constrained, at which point wealth inequality begins to fall up until it reaches a stable level consistent with the distribution of skills - clearly shown by the path of γ_t^W. Figure 35 shows the dynamics of the distribution of income over t. As expected, the distribution of shares of income and the Gini coefficient of income are stable until the economy becomes labour constrained, at which point income inequality is dramatically reduced to a state consistent with the distribution of skills.

Figure 32: Summary results - Model with zero subsistence

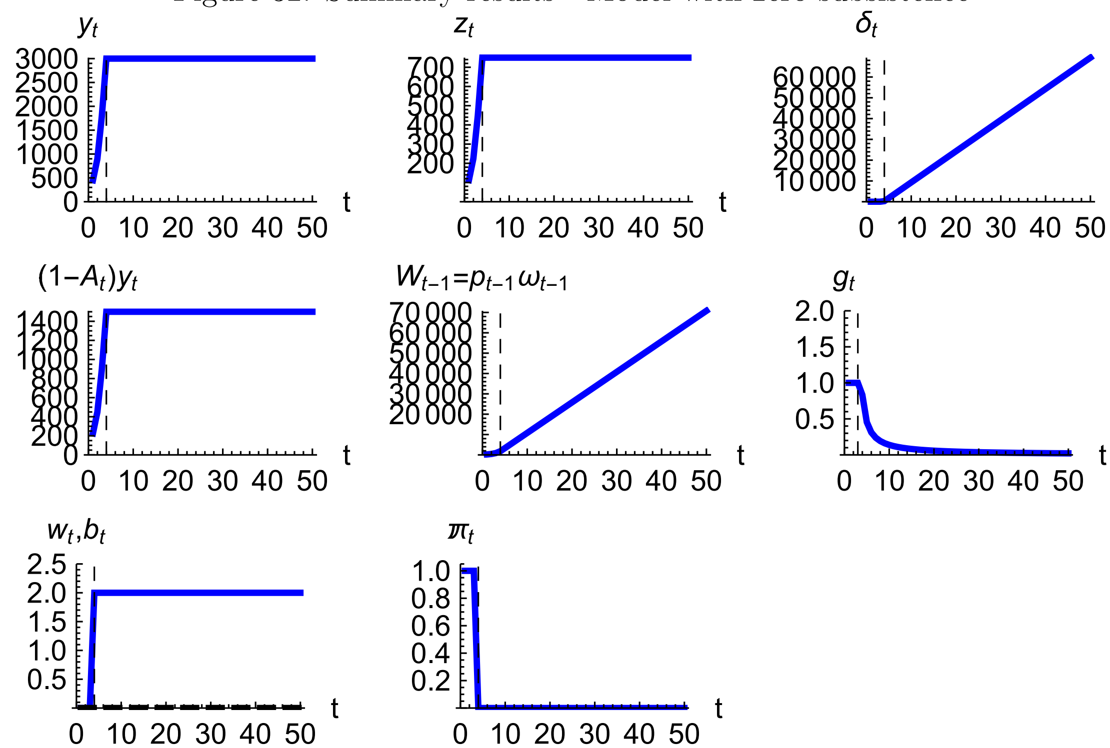
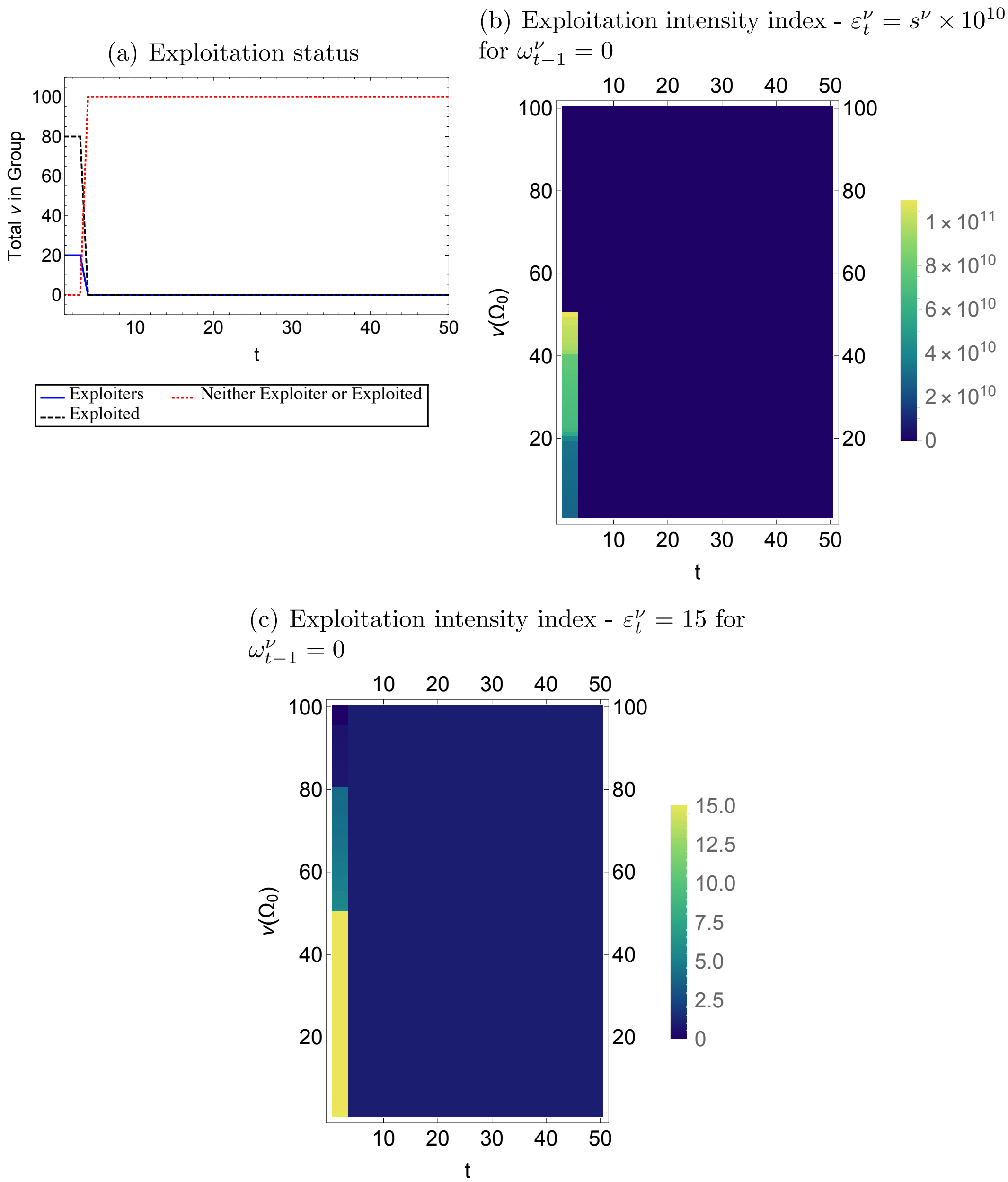


Figure 33: Exploitation - Model with zero subsistence



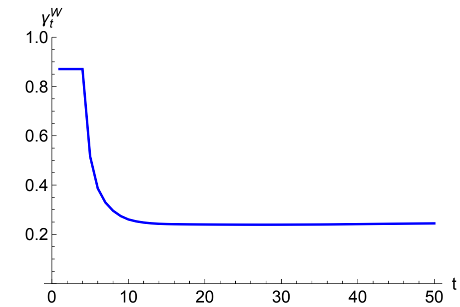
(a) Exploitation status

(b) Exploitation intensity index - $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for $\omega_{t-1}^\nu = 0$

(c) Exploitation intensity index - $\varepsilon_t^\nu = 15$ for $\omega_{t-1}^\nu = 0$

Figure 34: Distribution of wealth - Model with zero subsistence

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

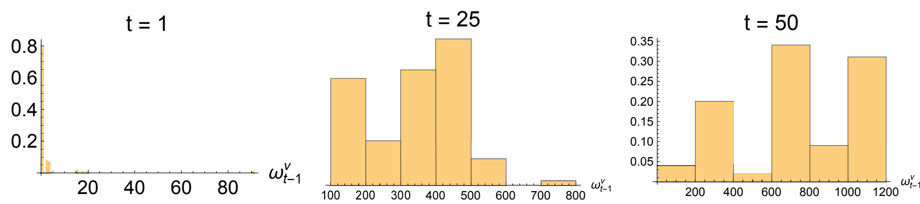
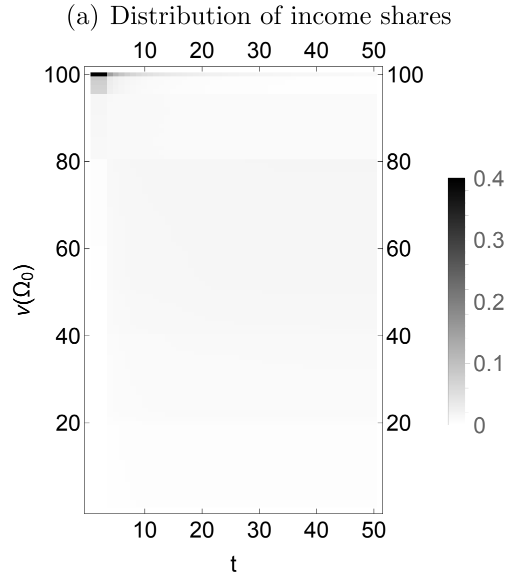
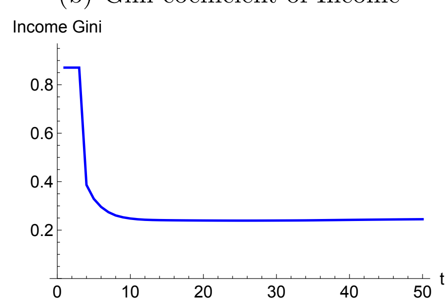


Figure 35: Distribution of income - Model with zero subsistence

(a) Distribution of income shares



(b) Gini coefficient of Income



8.2 The economy with zero subsistence and wealth taxes

This section presents the economy with Piketty-type wealth taxes for the case of $b = 0$. The results of the simulation over T can be found in figures 36-39. As figure 36 shows, setting the subsistence level to zero has no effect on the dynamics of the aggregate behaviour of the model. The only difference between the results immediately below and those of the corresponding simulation in the main paper is the time at which the economy become labour constrained.

Figure 37 shows the dynamics of exploitation status and intensity over the course of the simulation. As figure 37(a) shows, the structure of exploitation status is stable for the short period of time in which the economy is capital constrained, and exploitation disappears after the labour constraint is binding for all agents. Figure 37(b) shows the distribution of ε_t^ν across agents over t. Due to the zero subsistence level, a slight adjustment to the calculation of ε_t^ν is made to avoid a zero value of vc_t^ν. For the case in which agents have $\omega_{t-1}^\nu = 0$ and vc_t^ν would go to zero since $b = 0$ for all agents, $\varepsilon_t^\nu = s^\nu \times 10^{10}$. This captures the fact that agents with $\omega_{t-1}^\nu = 0$ experience intense exploitation relative to others, and it preserves the differences in exploitation intensity induced by the distribution of skills. Figure 37(c) takes a slightly different approach to setting ε_t^ν for agents with $\omega_{t-1}^\nu = 0$. In figure 37(c), for agents with $\omega_{t-1}^\nu = 0$, $\varepsilon_t^\nu = 50$. This is done to select an arbitrarily high level for ε_t^ν while not obfuscating the fact that agents who possess small amounts of wealth and high skill levels still experience exploitation. In both cases, exploitation intensity is diminished during the periods in which the economy is capital constrained due to the redistributive wealth taxes, and exploitation completely disappears once the economy is labour constrained. Figure 37(d) shows γ_t^ε for the calculation of ε_t^ν where $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for ν with $\omega_{t-1}^\nu = 0$. As expected, wealth taxes reduce inequality in exploitation intensity while the simulation is capital constrained and exploitation disappears entirely once the economy is labour constrained.

Figure 38 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth, and wealth inequality falls even more dramatically once the economy is labour constrained - as shown by the path of γ_t^W. Figure 39 shows the dynamics of the distribution of income over t.

Figure 36: Summary results - Zero subsistence and wealth taxes

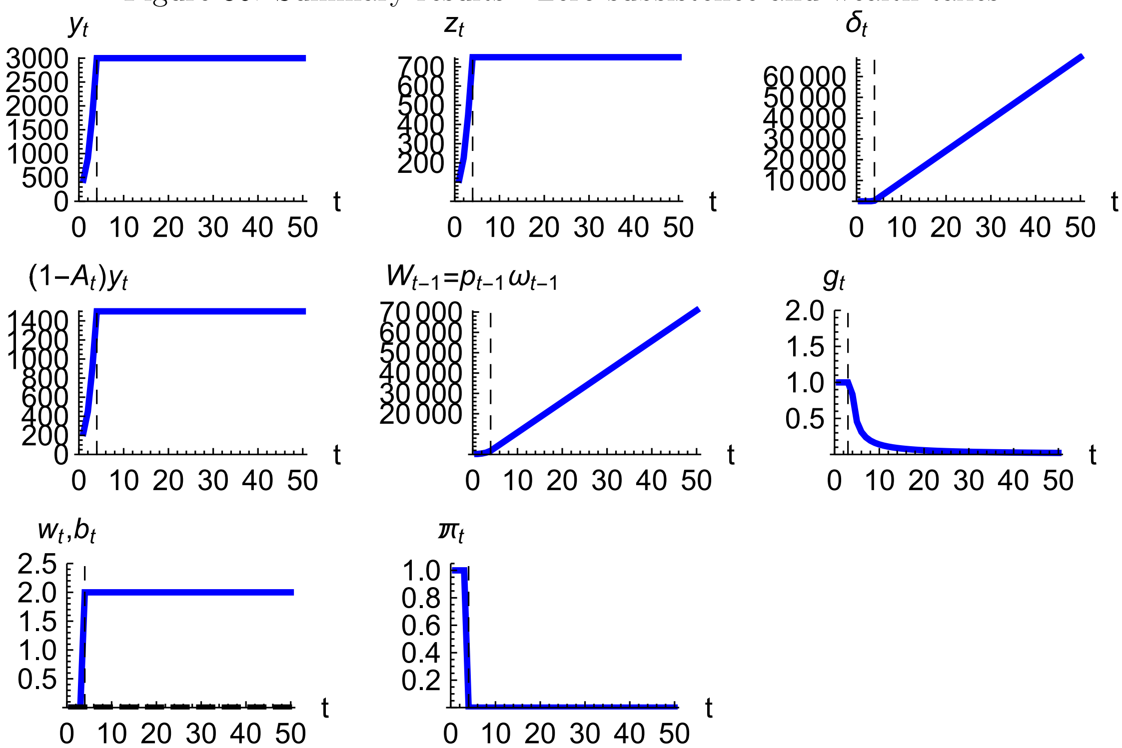
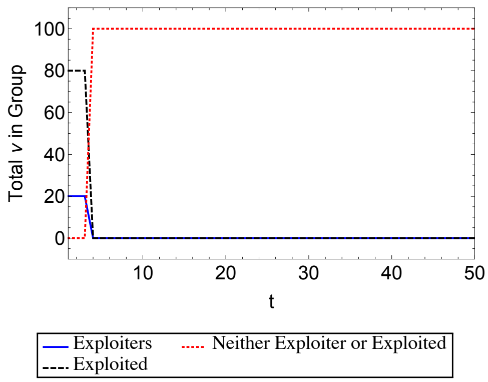
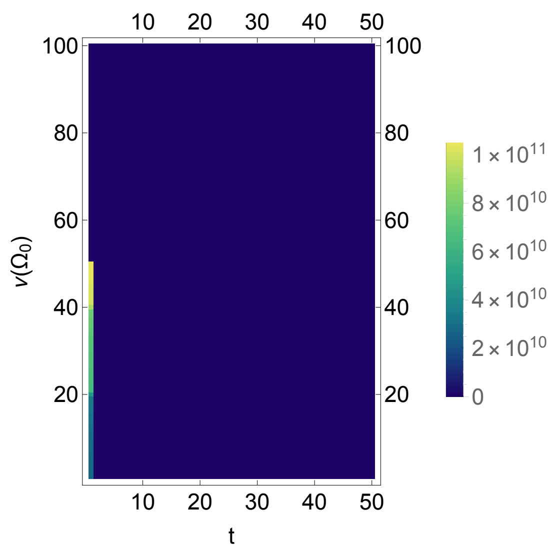


Figure 37: Exploitation - Zero subsistence and wealth taxes

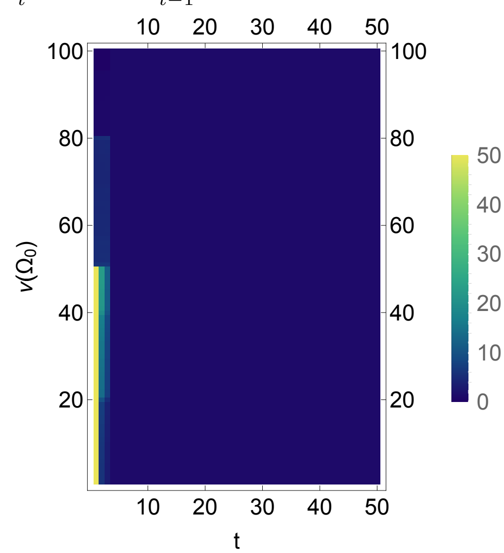
(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index - $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for $\omega_{t-1}^\nu = 0$



(c) Post-tax Exploitation intensity index - $\varepsilon_t^\nu = 50$ for $\omega_{t-1}^\nu = 0$



(d) Post-tax γ_t^ε - for $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for $\omega_{t-1}^\nu = 0$

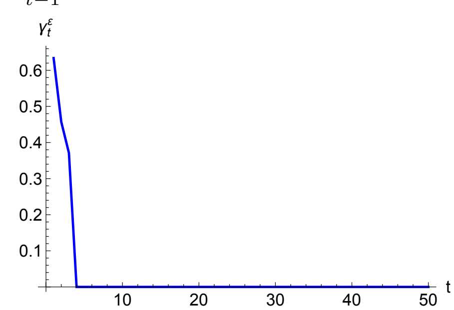
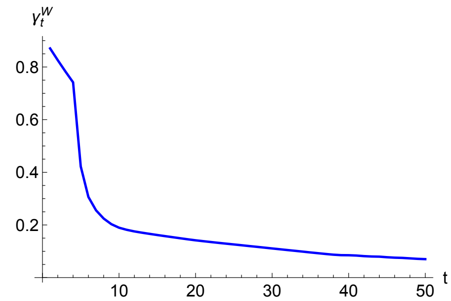


Figure 38: Distribution of wealth - Zero subsistence with wealth taxes

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

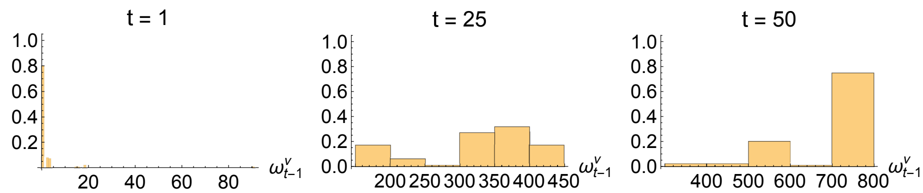
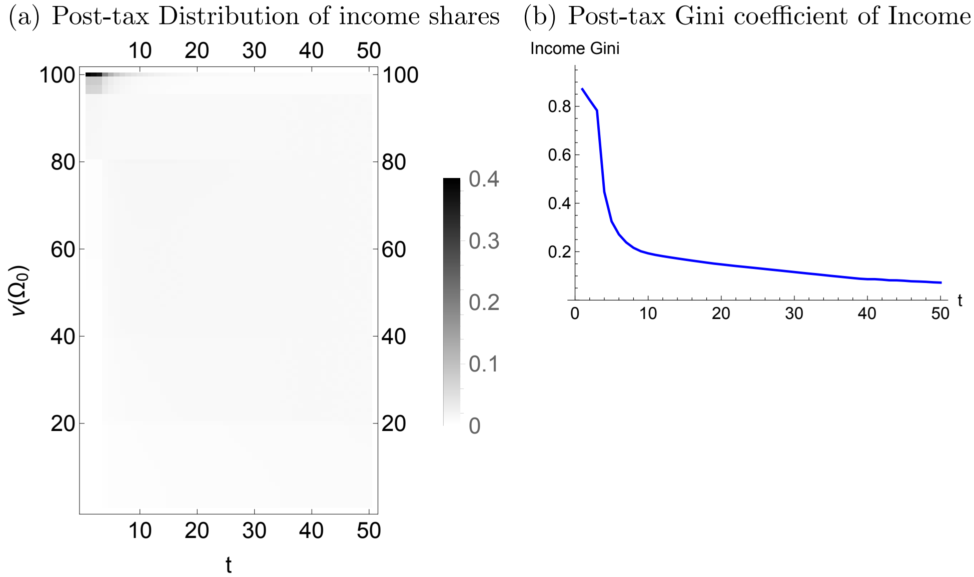


Figure 39: Distribution of income - Zero subsistence with wealth taxes

(a) Post-tax Distribution of income shares (b) Post-tax Gini coefficient of Income



8.3 The economy with zero subsistence and wealth equality

This section presents the economy with wealth taxes designed to achieve wealth equality, using Rule 1, for the case of $b = 0$. The results of the simulation over T can be found in figures 40-43. As figure 40 shows, setting the subsistence level to zero has no effect on the dynamics of the aggregate behaviour of the model. The only difference between the results immediately below and those of the corresponding simulation in the main paper is the time at which the economy become labour constrained.

Figure 41 shows the dynamics of exploitation status and intensity over the course of the simulation. As figure 41(a) shows, the structure of exploitation status is stable for the short period of time in which the economy is capital constrained, and exploitation disappears after the labour constraint is binding for all agents. Figure 41(b) shows the distribution of ε_t^ν across agents over t. Due to the zero subsistence level, a slight adjustment to the calculation of ε_t^ν is made to avoid a zero value of vc_t^ν. For the case in which agents have $\omega_{t-1}^\nu = 0$ and vc_t^ν would go to zero since $b = 0$ for all agents, $\varepsilon_t^\nu = s^\nu \times 10^{10}$. This captures the fact that agents with $\omega_{t-1}^\nu = 0$ experience intense exploitation relative to others, and it preserves the differences in exploitation intensity induced by the distribution of skills. Figure 41(c) takes a slightly different approach to setting ε_t^ν for agents with $\omega_{t-1}^\nu = 0$. In figure 41(c), for agents with $\omega_{t-1}^\nu = 0$, $\varepsilon_t^\nu = 30$. This is done to select an arbitrarily high level for ε_t^ν while not obfuscating the fact that agents who possess small amounts of wealth and high skill levels still experience exploitation. In both cases, exploitation intensity is diminished during the periods in which the economy is capital constrained due to the redistributive wealth taxes, and exploitation completely disappears once the economy is labour constrained. Figure 41(d) shows γ_t^ε for the calculation of ε_t^ν where $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for ν with $\omega_{t-1}^\nu = 0$. As expected, wealth taxes reduce inequality in exploitation intensity while the simulation is capital constrained and exploitation disappears entirely once the economy is labour constrained.

Figure 42 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth, and wealth inequality falls even more dramatically once the economy is labour constrained - as shown by the path of γ_t^W. Figure 43 shows the dynamics of the distribution of income over t.

Figure 40: Summary results - Zero subsistence and wealth equality

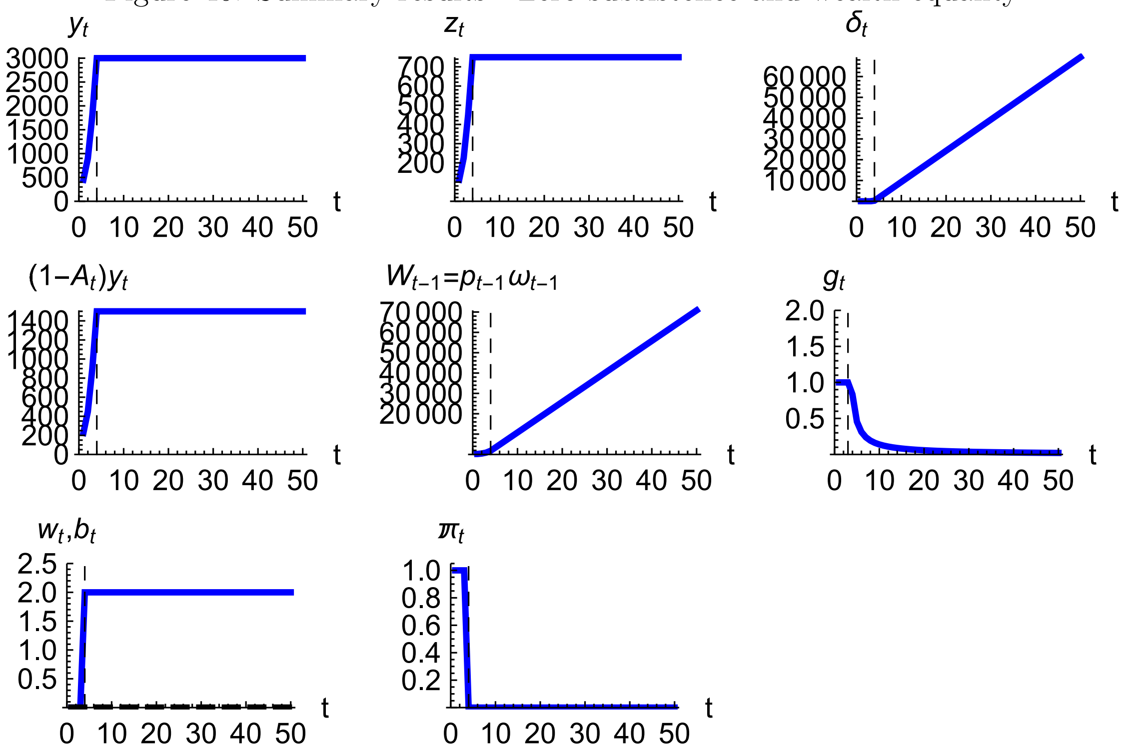
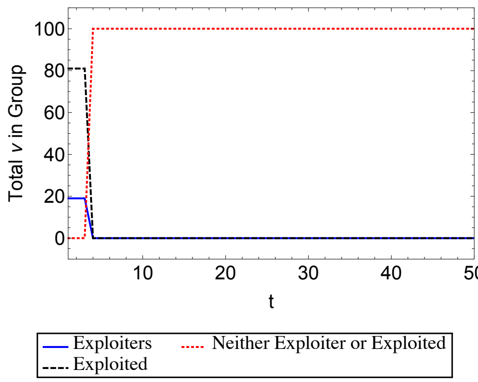
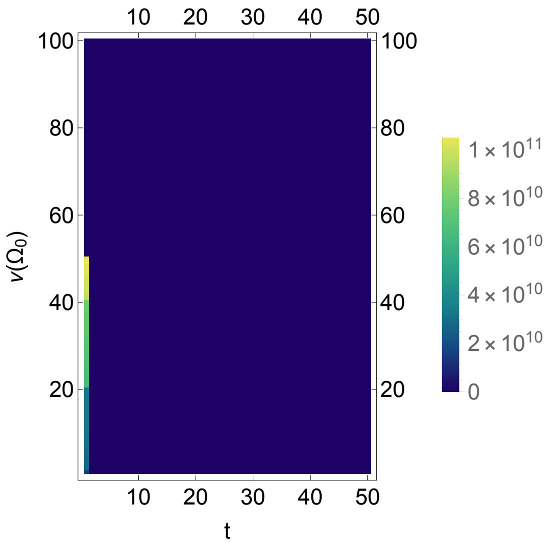


Figure 41: Exploitation - Zero subsistence and wealth equality

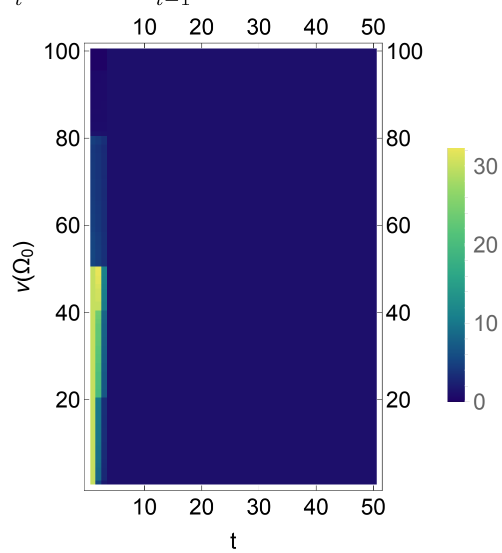
(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index - $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for $\omega_{t-1}^\nu = 0$



(c) Post-tax Exploitation intensity index - $\varepsilon_t^\nu = 30$ for $\omega_{t-1}^\nu = 0$



(d) Post-tax γ_t^ε - for $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for $\omega_{t-1}^\nu = 0$

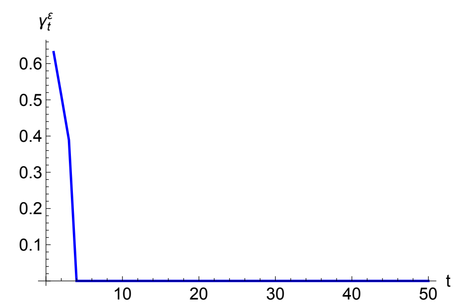
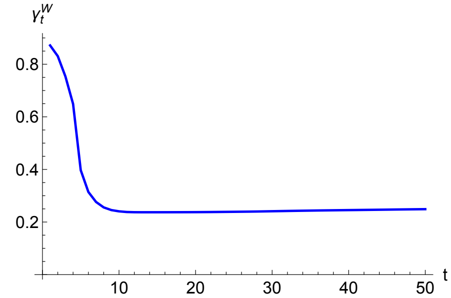


Figure 42: Distribution of wealth - Zero subsistence with wealth equality

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

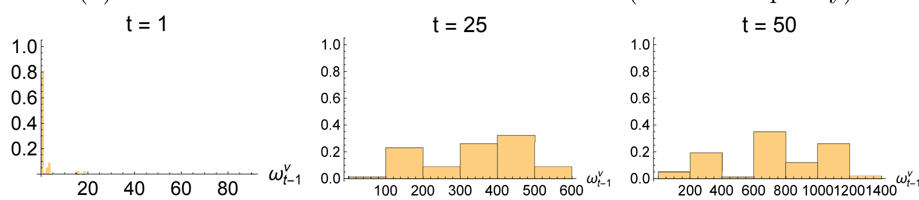
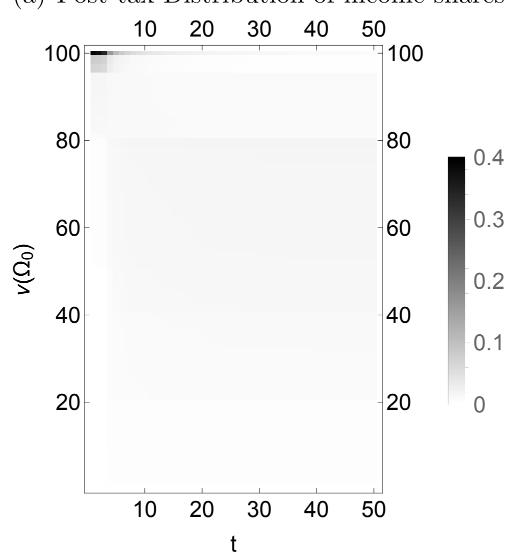
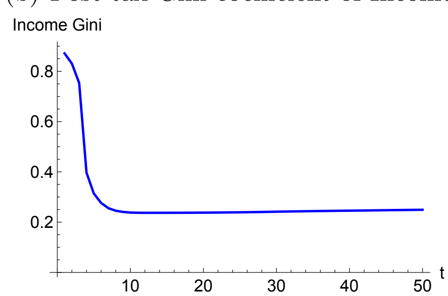


Figure 43: Distribution of income - Zero subsistence with wealth equality

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



8.4 The economy with zero subsistence and a socialist allocation

This section presents the economy with wealth taxes designed to achieve a socialist allocation, using Rule 2, for the case of $b = 0$. The results of the simulation over T can be found in figures 44-47. As figure 44 shows, setting the subsistence level to zero has no effect on the dynamics of the aggregate behaviour of the model. The only difference between the results immediately below and those of the corresponding simulation in the main paper is the time at which the economy become labour constrained.

Figure 45 shows the dynamics of exploitation status and intensity over the course of the simulation. As figure 45(a) shows, the number of exploited agents diminishes slightly for the short period of time in which the economy is capital constrained, and exploitation disappears after the labour constraint is binding for all agents. Figure 45(b) shows the distribution of ε_t^ν across agents over t. Due to the zero subsistence level, a slight adjustment to the calculation of ε_t^ν is made to avoid a zero value of vc_t^ν. For the case in which agents have $\omega_{t-1}^\nu = 0$ and vc_t^ν would go to zero since $b = 0$ for all agents, $\varepsilon_t^\nu = s^\nu \times 10^{10}$. This captures the fact that agents with $\omega_{t-1}^\nu = 0$ experience intense exploitation relative to others, and it preserves the differences in exploitation intensity induced by the distribution of skills. Figure 45(c) takes a slightly different approach to setting ε_t^ν for agents with $\omega_{t-1}^\nu = 0$. In figure 45(c), for agents with $\omega_{t-1}^\nu = 0$, $\varepsilon_t^\nu = 30$. This is done to select an arbitrarily high level for ε_t^ν while not obfuscating the fact that agents who possess small amounts of wealth and high skill levels still experience exploitation. In both cases, exploitation intensity is diminished during the periods in which the economy is capital constrained due to the redistributive wealth taxes, and exploitation completely disappears once the economy is labour constrained. Figure 45(d) shows γ_t^ε for the calculation of ε_t^ν where $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for ν with $\omega_{t-1}^\nu = 0$. As expected, wealth taxes reduce inequality in exploitation intensity while the simulation is capital constrained and exploitation disappears entirely once the economy is labour constrained.

Figure 46 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth, and wealth inequality falls even more dramatically once the economy is labour constrained - as shown by the path of γ_t^W. Figure 47 shows the dynamics of the distribution of income over t.

Figure 44: Summary results - Zero subsistence and socialist allocation

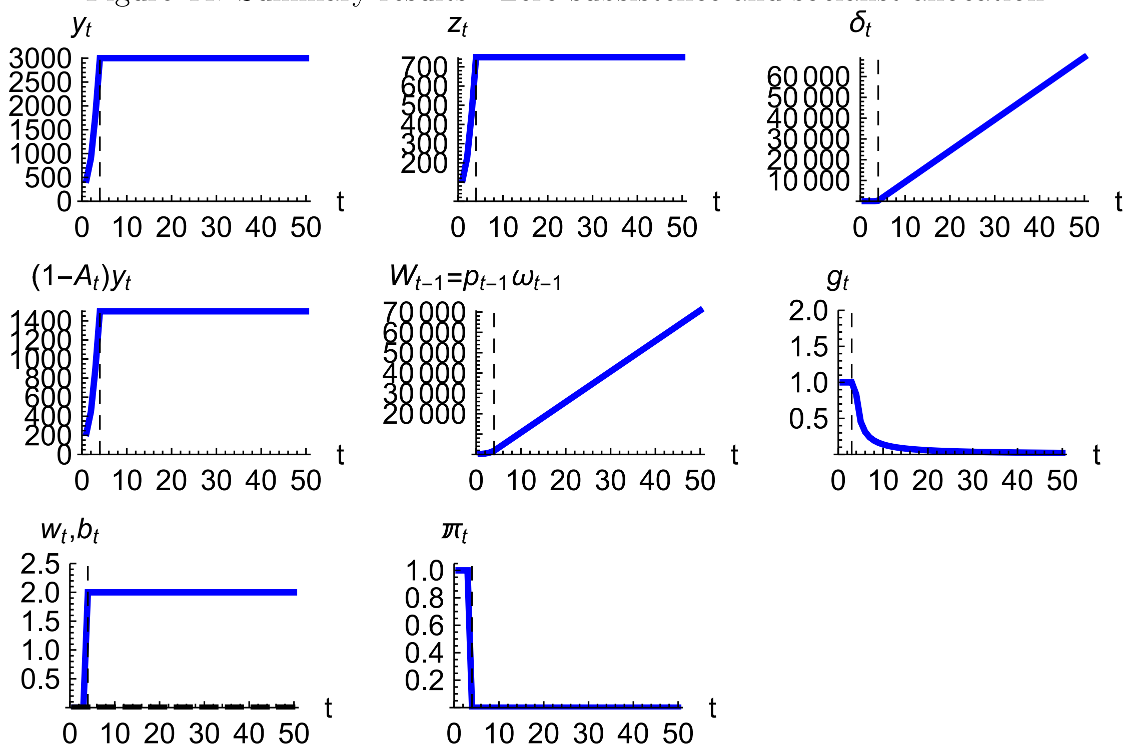
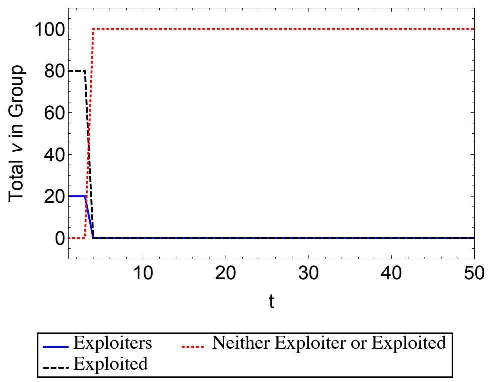
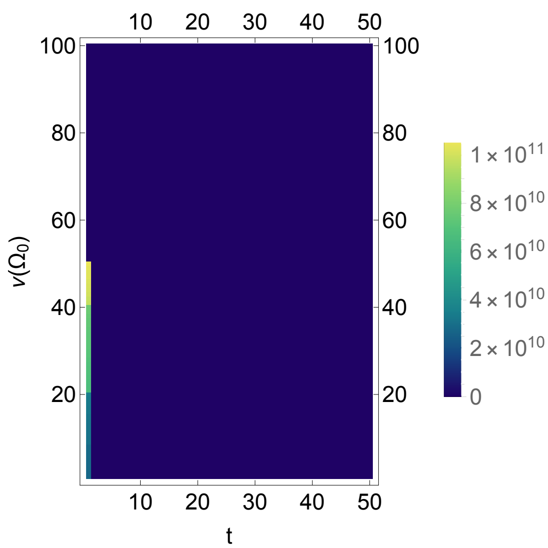


Figure 45: Exploitation - Zero subsistence and socialist allocation

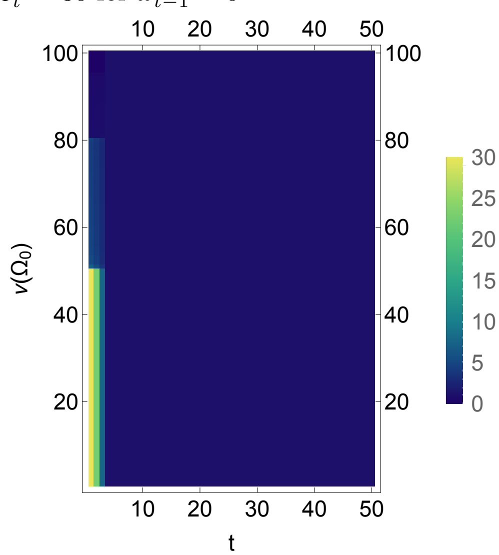
(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index - $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for $\omega_{t-1}^\nu = 0$



(c) Post-tax Exploitation intensity index - $\varepsilon_t^\nu = 30$ for $\omega_{t-1}^\nu = 0$



(d) Post-tax γ_t^ε - for $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for $\omega_{t-1}^\nu = 0$

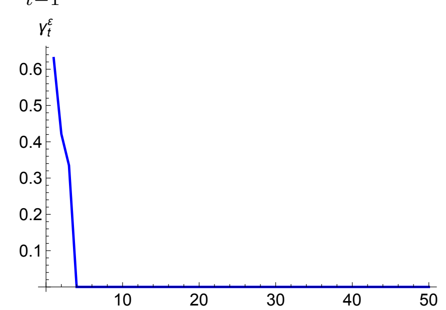
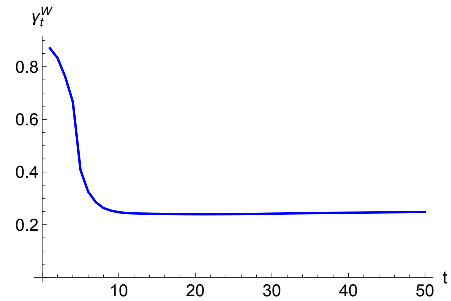


Figure 46: Distribution of wealth - Zero subsistence with socialist allocation

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

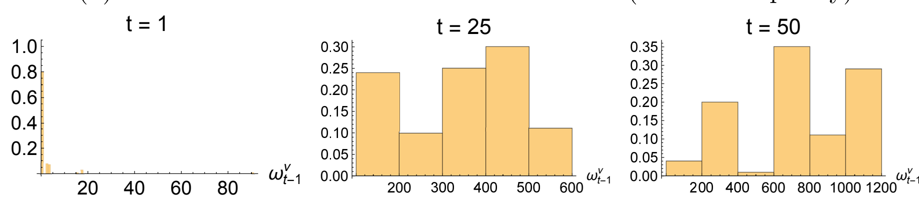
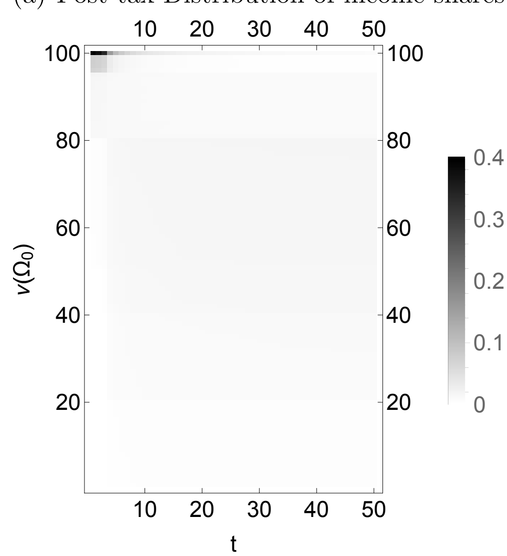
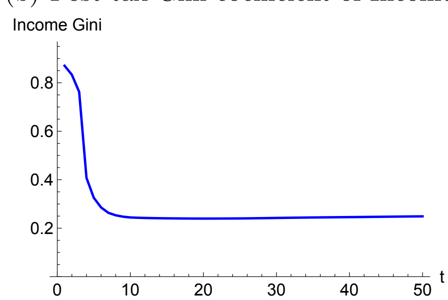


Figure 47: Distribution of income - Zero subsistence with socialist allocation

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



8.5 Class in the model with zero subsistence

The figures below display the dynamics of class over the simulations with $b = 0$. Class status is determined according to Corollary 1 of Theorem 3. The figures below also show the CECP according to Theorem 4.

Figure 48: Class - Model with zero subsistence

(a) Class status (b) CECP

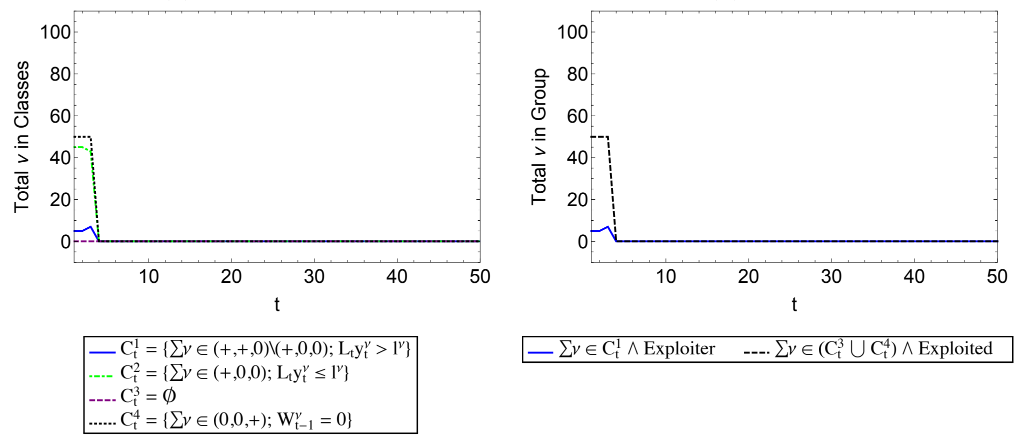


Figure 49: Class - Model with zero subsistence and wealth taxes

(a) Class status (b) CECP

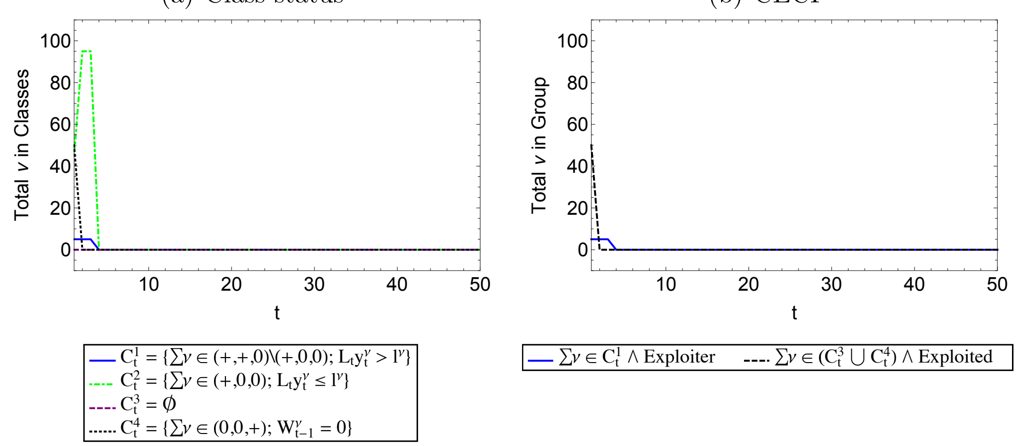
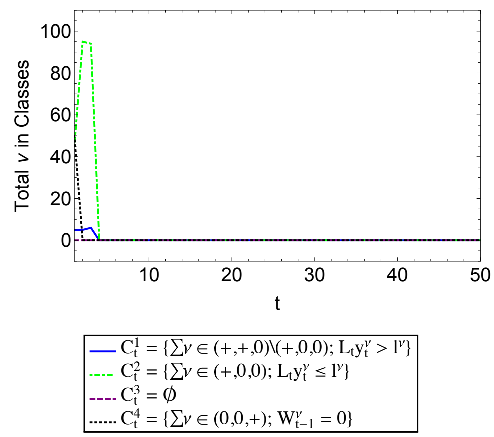


Figure 50: Class - Model with zero subsistence and wealth equality

(a) Class status



(b) CECP

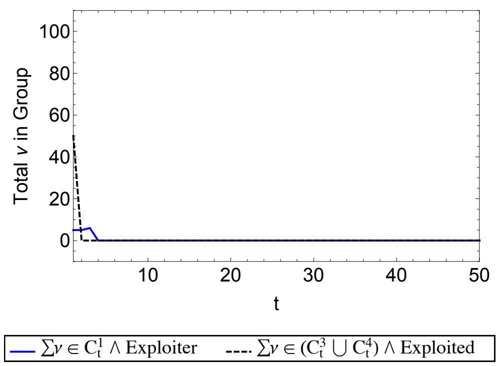
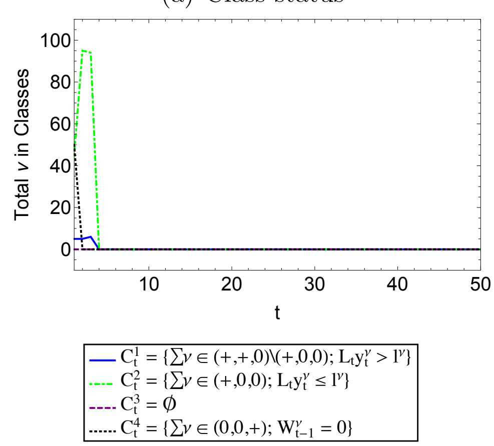
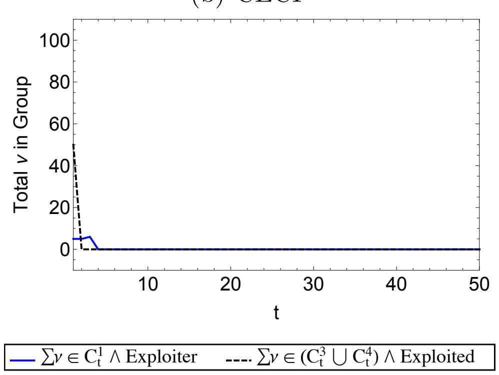


Figure 51: Class - Model with zero subsistence and socialist allocation

(a) Class status



(b) CECP



9 Random heterogeneous consumption preferences

This section extends the model of the paper to include heterogeneous consumption preferences for all agents. During each t, agents consume $h^\nu b \Lambda_t^\nu$, where $h^\nu \in (0,1)$ is randomly assigned to each agent prior to the start of the simulation and is constant over t. The simulations below use the same parameters from the paper, and h^ν are the same across simulations. The initial distributions of wealth and skills are determined using the same method as in the paper.

9.1 Baseline model with random heterogeneous consumption

This section presents the baseline model with heterogenous consumption. The results of the simulation over T can be found in figures 52-55. As figure 52 shows, varying consumption across agents has no qualitative impact on the dynamics of the aggregate behaviour of the model. The only difference between the results immediately below and those of the corresponding simulation in the main paper is the time at which the economy become labour constrained. The lower overall consumption, and all agents accumulating to some degree, causes the economy to become labour constrained after only a few time periods, due to the faster rate of accumulation.

Figure 53 shows the dynamics of exploitation status and intensity over the course of the simulation. Widespread accumulation across agents causes the number of exploited agents to quickly fall while the economy is capital constrained, as shown in figure 53(a). Figure 53(b) shows the distribution of ε_t^ν across agents over t. There is greater variation in exploitation intensity across agents given the broad heterogeneity in consumption and accumulation. This is most evident in the cases where different agents who begin the simulation with $\omega_0^\nu = 0$ reach lower levels of exploitation intensity at different t due to their different consumption preferences.

Figure 54 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality falls and stabilises as the economy becomes labour constrained. Figure 55 shows the dynamics of the distribution of income over t. As expected, heterogeneous consumption induces wider variation in income shares over t due to different individual accumulation patterns.

Figure 52: Summary results - Model with heterogeneous consumption

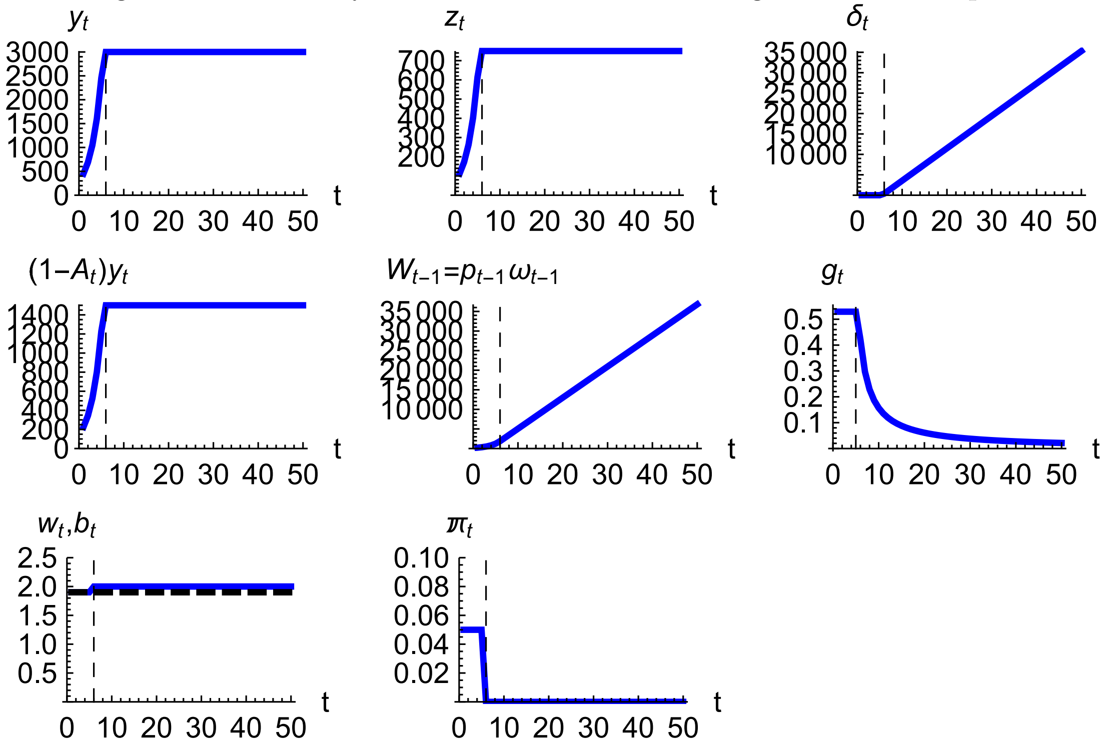


Figure 53: Exploitation - Model with heterogeneous consumption

(a) Exploitation status

(b) Exploitation intensity index

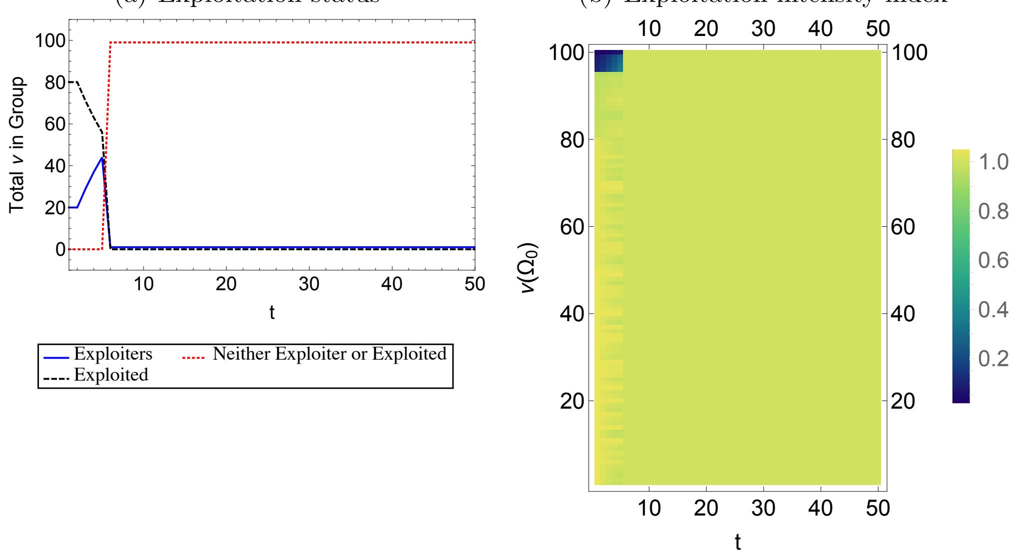
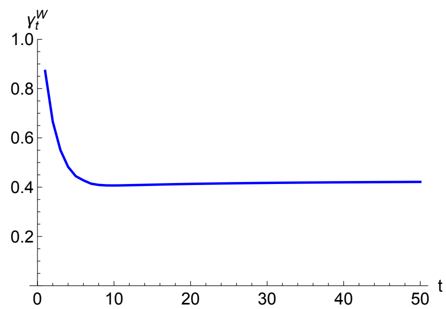


Figure 54: Distribution of wealth - Model with heterogeneous consumption

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

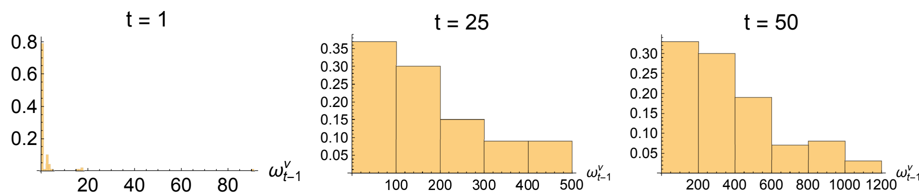
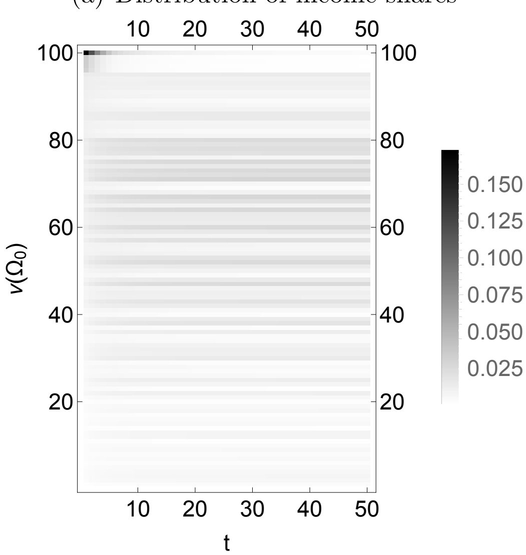
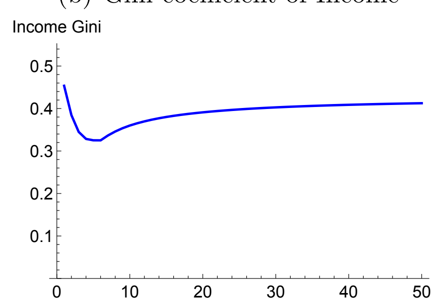


Figure 55: Distribution of income - Model with heterogeneous consumption

(a) Distribution of income shares



(b) Gini coefficient of Income



9.2 The economy with heterogeneous consumption and wealth taxes

This section presents the economy with Piketty-type wealth taxes for the case of heterogeneous consumption. The results of the simulation over T can be found in figures 56-59. As figure 56 shows, modifying agents' consumption preferences has no impact on the aggregate dynamics of the model. The only difference between the results immediately below and those of the corresponding simulation in the main paper is the time at which the economy become labour constrained.

Figure 57 shows the dynamics of exploitation status and intensity over the course of the simulation. Figure 57(a) shows that the number of exploited agents quickly falls until the economy becomes labour constrained and exploitation disappears entirely. Figure 57(b) shows the distribution of ε_t^{ν} across agents over t, which shows that agents experience different exploitation intensity depending on their individual accumulation of wealth. Figure 57(c) shows that wealth taxes quickly reduce γ_t^{ε} while the simulation is capital constrained and exploitation disappears entirely once the economy is labour constrained.

Figure 58 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth, and wealth inequality falls even more dramatically once the economy is labour constrained - as shown by the path of γ_t^W. Figure 59 shows the dynamics of the distribution of income over shares and the Gini coefficient of income over t, both of which exhibit behaviour consistent the other figures and the variation in agents' consumption.

Figure 56: Summary results - Heterogeneous consumption and wealth taxes

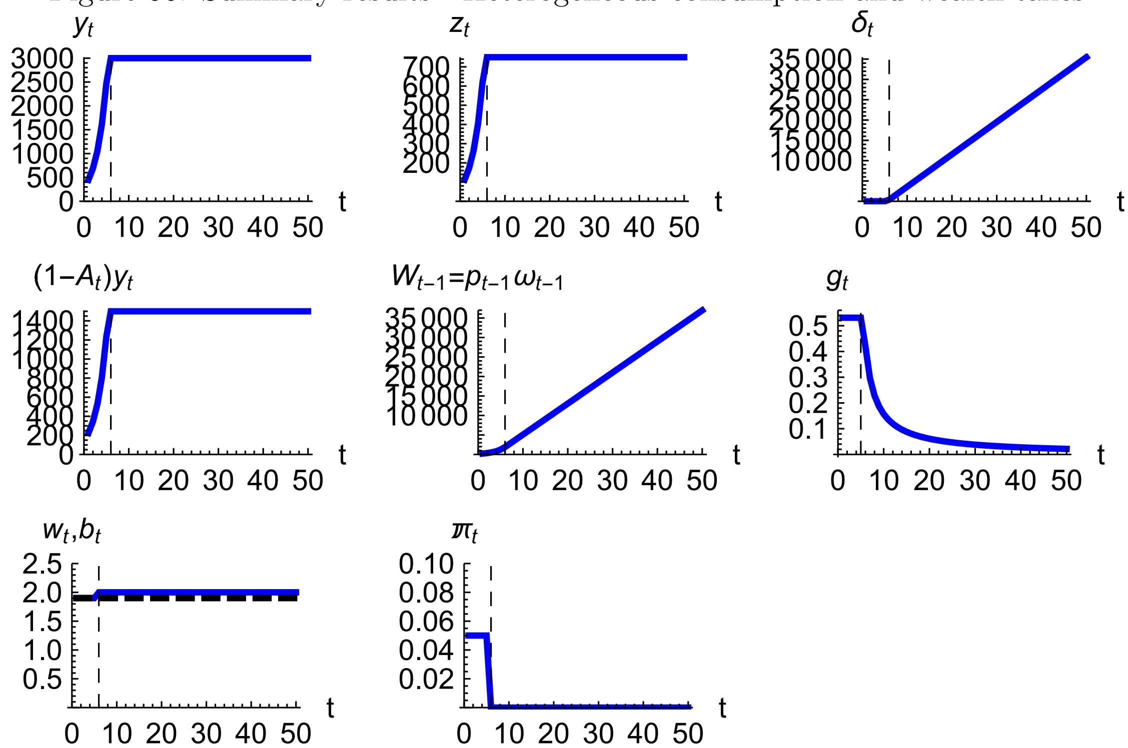
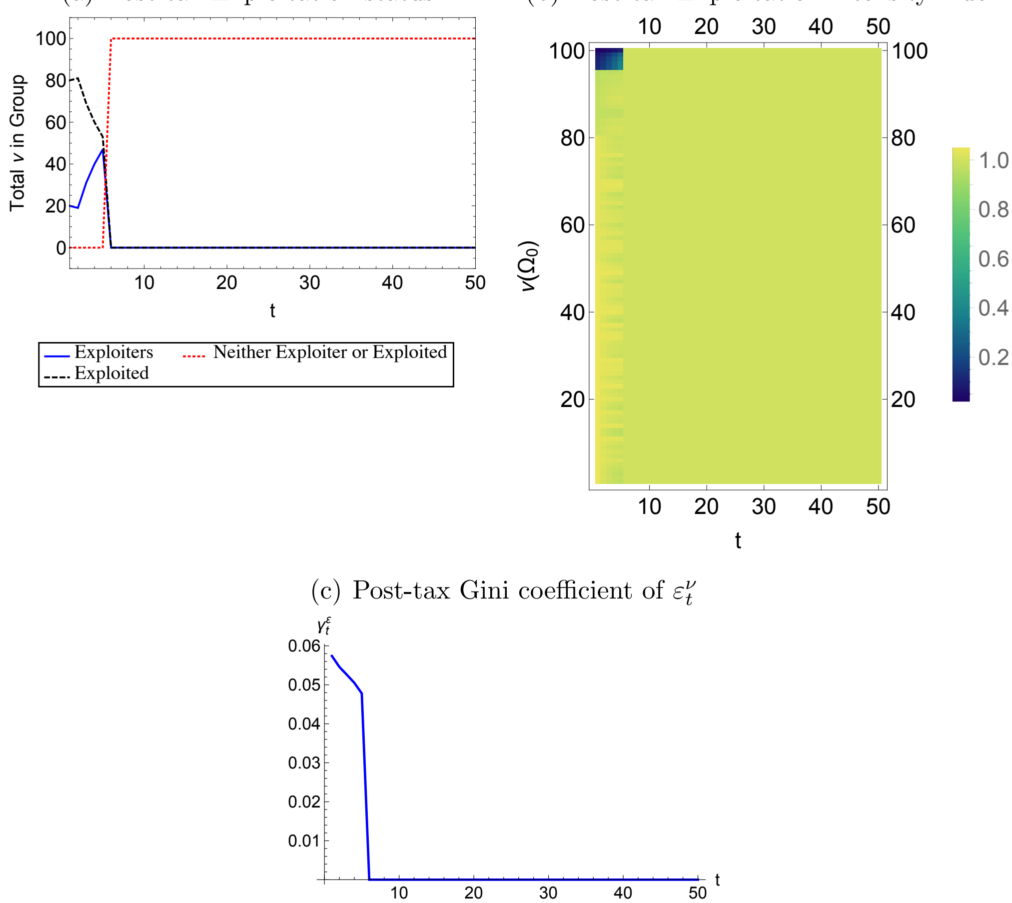


Figure 57: Exploitation - Heterogeneous consumption and wealth taxes



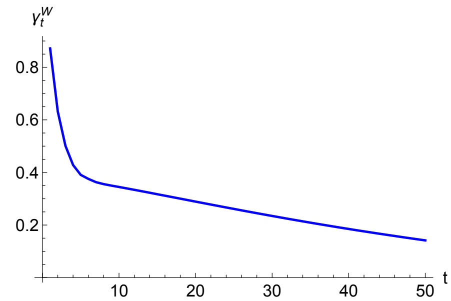
(a) Post-tax Exploitation status

(b) Post-tax Exploitation intensity index

(c) Post-tax Gini coefficient of ε_t^ν

Figure 58: Distribution of wealth - Heterogeneous consumption with wealth taxes

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

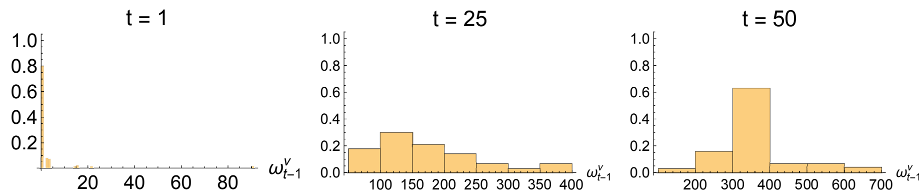
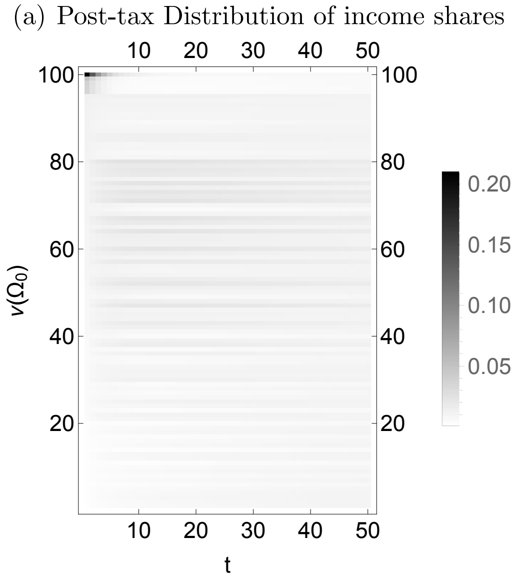
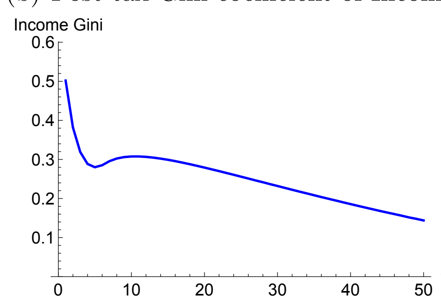


Figure 59: Distribution of income - Heterogeneous consumption with wealth taxes

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



9.3 The economy with heterogeneous consumption and wealth equality

This section presents the economy with wealth taxes designed to achieve wealth equality, using Rule 1, for the case of heterogeneous consumption. The results of the simulation over T can be found in figures 60-63. Figure 60 shows that heterogeneous consumption preferences do not impact the aggregate behaviour of the model. The only difference between the results immediately below and those of the corresponding simulation in the main paper is the time at which the economy become labour constrained.

Figure 61 shows the dynamics of exploitation status and intensity over the course of the simulation. Figure 61(a) shows that the number of exploited agents quickly falls due to wealth taxes until the economy becomes labour constrained and exploitation disappears. Figure 61(b) shows the distribution of ε_t^ν across agents over t. Figure 61(c) shows γ_t^ε, which as expected, falls until exploitation disappears once the economy becomes labour constrained.

Figure 62 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth. Once the economy is labour constrained, wealth inequality then begins to rise slightly since Rule 1 is no longer operating. Figure 63 shows the dynamics of the distribution of income over t, which exhibits behaviour consistent the other figures.

Figure 60: Summary results - Heterogeneous consumption and wealth equality

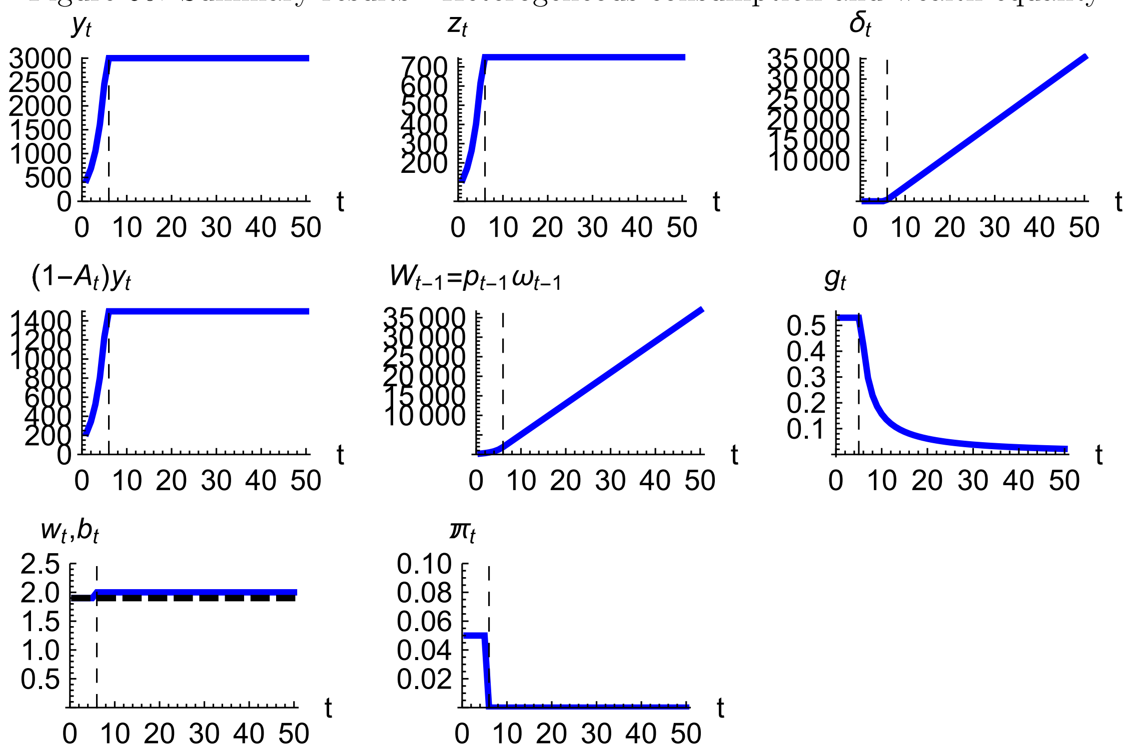
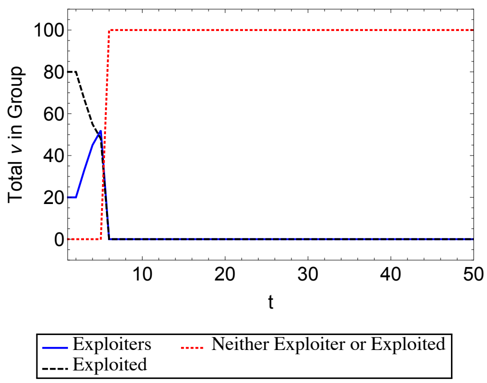
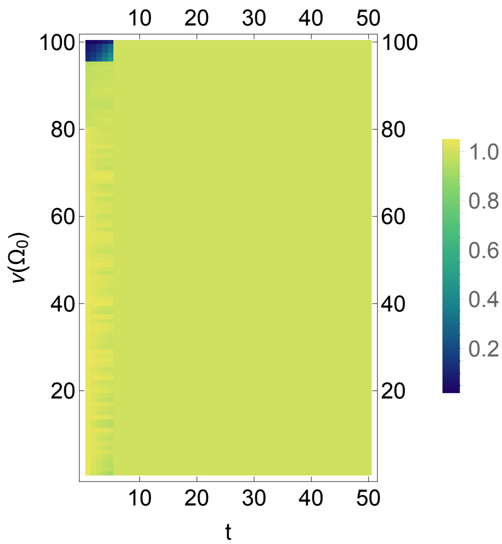


Figure 61: Exploitation - Heterogeneous consumption and wealth equality

(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index - $\varepsilon_t^\nu = s^\nu \times 10^{10}$ for $\omega_{t-1}^\nu = 0$



(c) Post-tax Gini coefficient of ε_t^ν

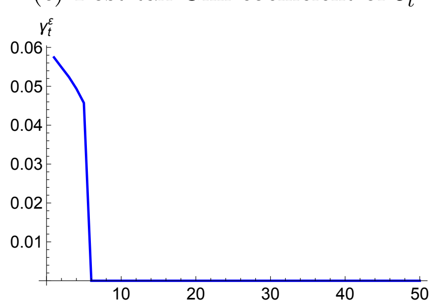
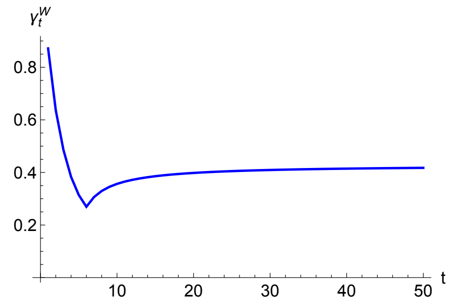


Figure 62: Distribution of wealth - Heterogeneous consumption with wealth equality

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

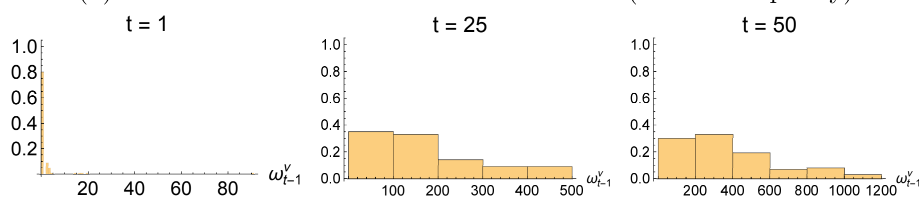
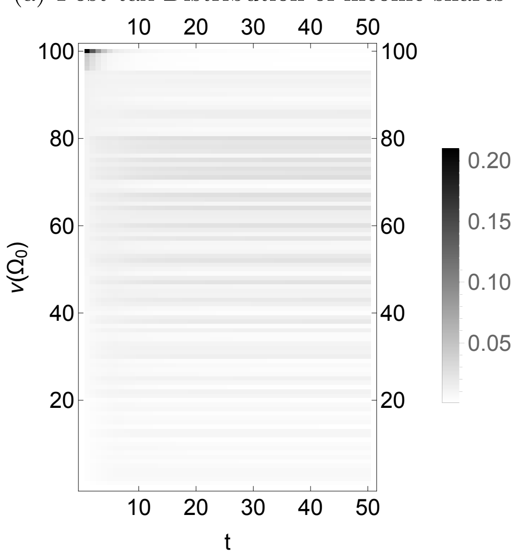
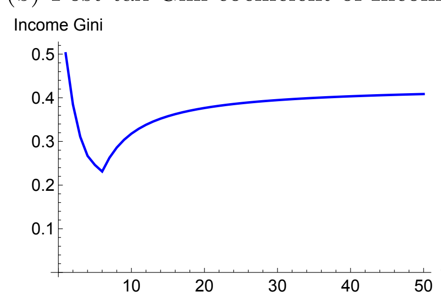


Figure 63: Distribution of income - Heterogeneous consumption with wealth equality

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



9.4 The economy with heterogeneous consumption and a socialist allocation

This section presents the economy with wealth taxes designed to achieve a socialist allocation, using Rule 2, for the case of heterogeneous consumption. The results of the simulation over T can be found in figures 64-67. Figure 64 shows that heterogeneous consumption has no impact on the aggregate behaviour of the model. The only difference between the results immediately below and those of the corresponding simulation in the main paper is the time at which the economy become labour constrained.

Figure 65 shows the dynamics of exploitation status and intensity over the course of the simulation. As figure 65(a) shows, the number of exploited agents diminishes slightly for the short period of time in which the economy is capital constrained, and exploitation disappears after the labour constraint is binding for all agents. Figure 65(b) shows the distribution of ε_t^{ν} across agents over t. Figure 65(c) shows γ_t^{ε}, which declines due to Rule 2 and eventually goes to zero once the economy is labour constrained.

Figure 66 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth and reaches a point consistent with a socialist allocation once the economy is labour constrained - as shown by the path of γ_t^{W}. Figure 67 shows the dynamics of the distribution of income over t, which exhibits behaviour consistent the other figures.

Figure 64: Summary results - Heterogeneous consumption and socialist allocation

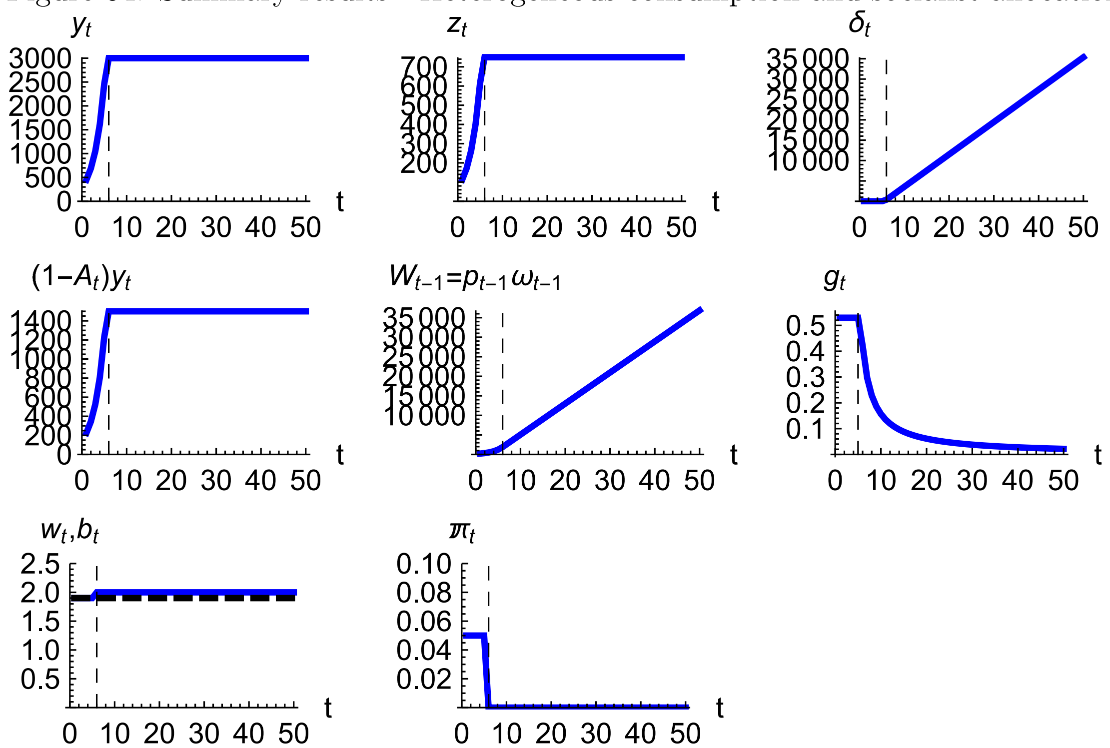
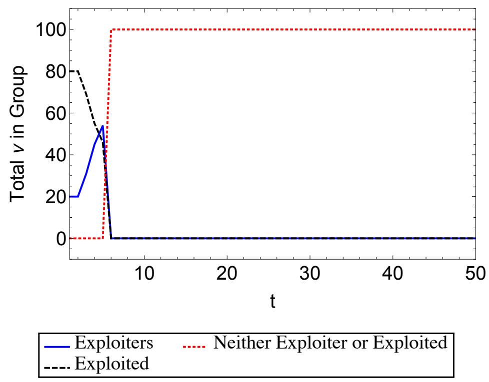
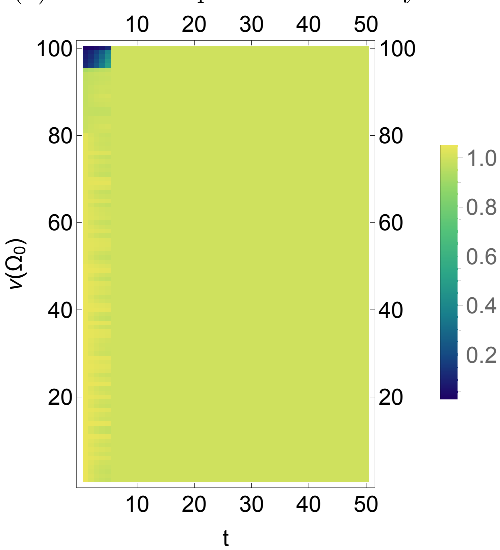


Figure 65: Exploitation - Heterogeneous consumption and socialist allocation

(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index



(c) Post-tax Gini coefficient of ε_t^ν

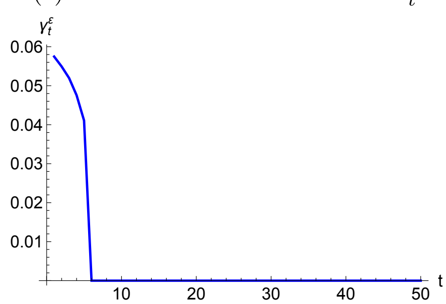
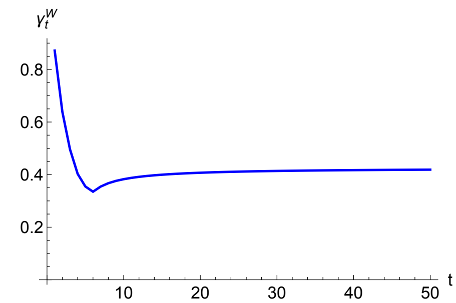


Figure 66: Distribution of wealth - Heterogeneous consumption with socialist allocation

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

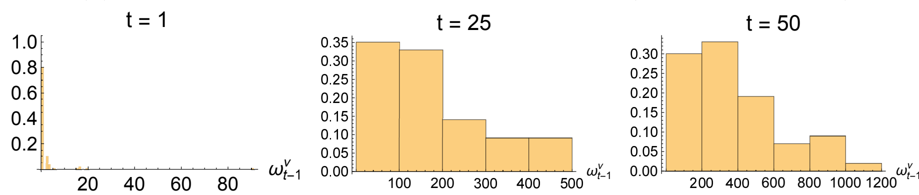
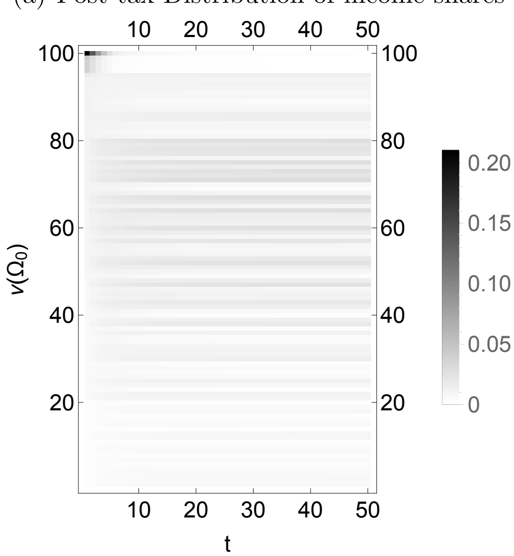
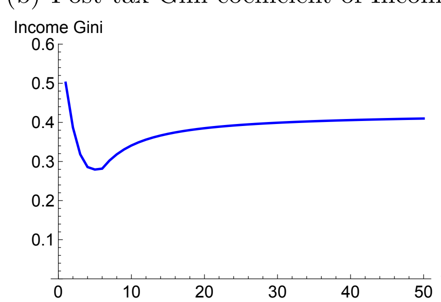


Figure 67: Distribution of income - Heterogeneous consumption with socialist allocation

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



9.5 Class in the model with heterogeneous consumption

The figures below display the dynamics of class over the simulations with heterogeneous consumption. Class status is determined according to Corollary 1 of Theorem 3. The figures below also show the CECP according to Theorem 4.

Figure 68: Class - Heterogeneous consumption with zero subsistence

(a) Class status (b) CECP

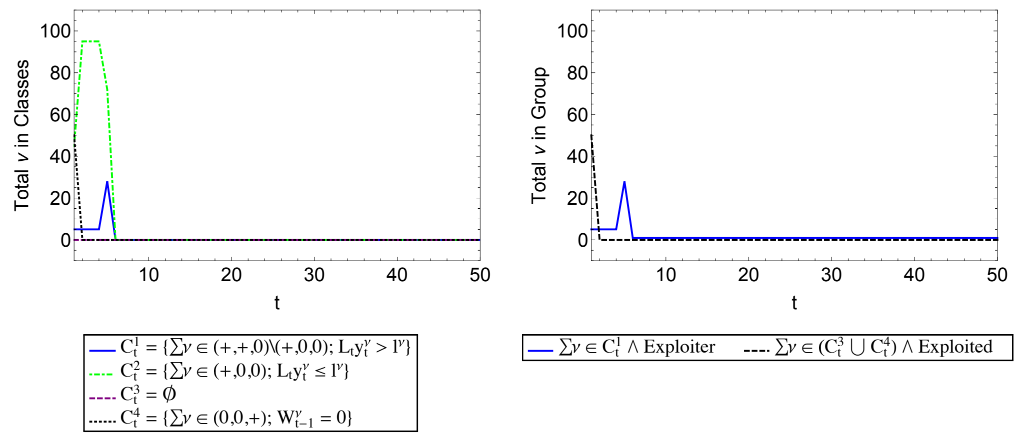


Figure 69: Class - Model with heterogeneous consumption and wealth taxes

(a) Class status (b) CECP

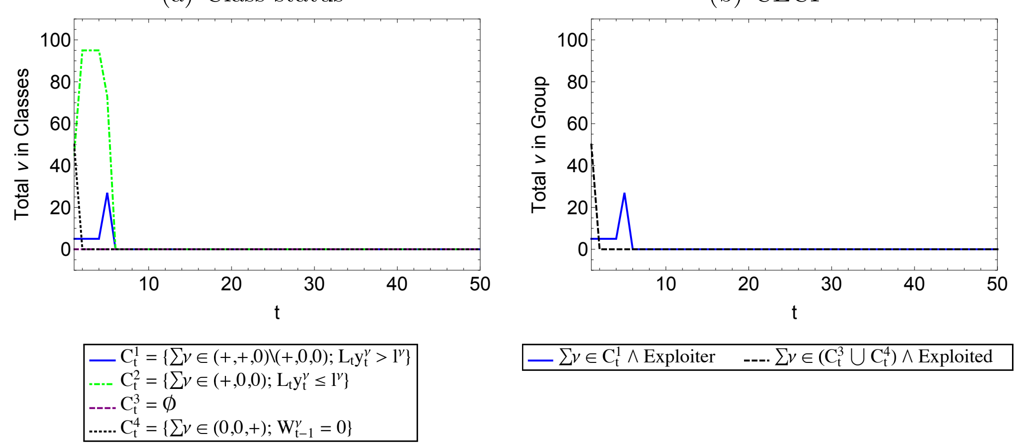


Figure 70: Class - Model with heterogeneous consumption and wealth equality

(a) Class status (b) CECP

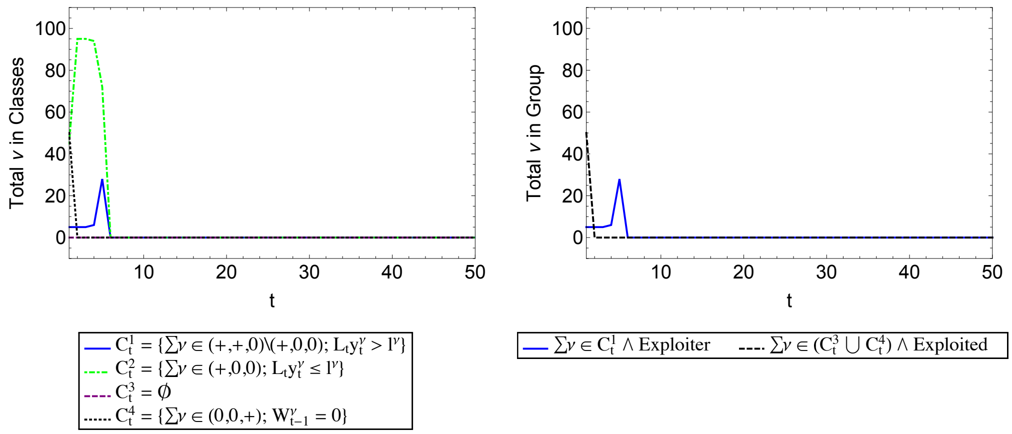
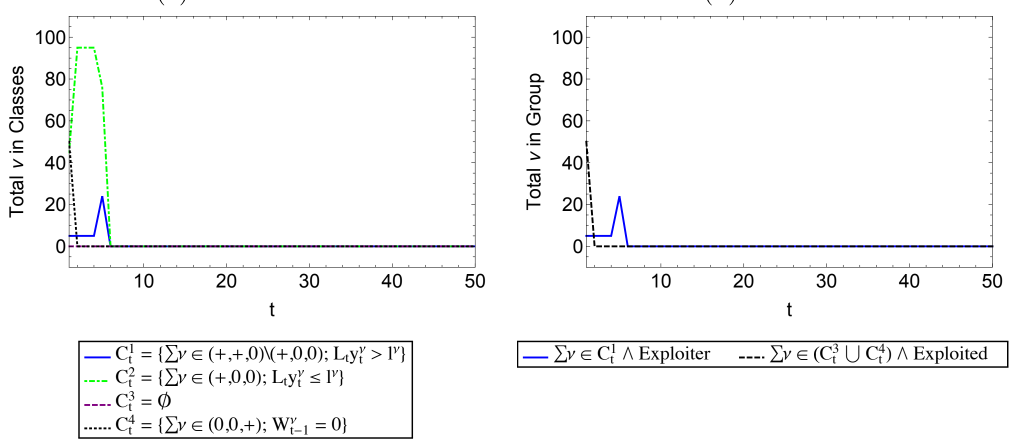


Figure 71: Class - Model with heterogeneous consumption and socialist allocation

(a) Class status (b) CECP



10 Consumption decreasing in wealth

This section analyses an extension of the simulation in the paper with heterogeneous consumption preferences as a decreasing function of wealth. Agents solve MP_t^ν as in the paper, except rather than consume $b\Lambda_t^\nu$ they consume some portion $h_t^\nu \in (0,1]$ of their net income $\pi_t \omega_{t-1}^\nu + b\Lambda_t^\nu$. Thus, it is possible that some agents will consume out of their capital income, but agents will maintain enough wealth to continue accumulating. It is also possible that agents with high skills and low levels of wealth will wind up accumulating more relative to other simulations since they will not consume all of their labour income. This modification entails that constraint (1) of MP_t^ν is now

$$p_t x_t^\nu + [p_t - w_t L]\, y_t^\nu + w_t z_t^\nu + p_t \delta_t^\nu = p_t \omega_t^\nu + h_t^\nu(\pi_t p_t \omega_{t-1}^\nu + b\Lambda_t^\nu).$$

The distribution of h_t^ν at any t is determined as follows. If agents have zero wealth $h_t^\nu = 1.0$; if agents have wealth up and including the median wealth $h_t^\nu = 0.9$; if agents have wealth above the median and up to and including the 90th percentile of wealth $h_t^\nu = 0.8$; if agents have wealth greater than the 90th percentile up to and including the 99th percentile $h_t^\nu = 0.7$; and agents at the top one percent of the wealth distribution have $h_t^\nu = 0.4$. This distribution of consumption preferences is determined for each t, thus if agents move up or down the wealth distribution their consumption behaviour can change.

The simulations that follow use the same parameters from the simulations in the paper, and the initial distributions of wealth and skills are generated using the same method as in the paper.

10.1 The economy with consumption decreasing in wealth

This section presents the baseline model with consumption decreasing in wealth. The results of the simulation over T can be found in figures 72-75. Figure 72 shows the aggregate results of the model. The only notable difference between figure 72 and the corresponding simulation from the paper is that $g_t \neq \pi_t$ while the economy is capital constrained, and g_t decreases slightly up until the point at which the economy becomes labour constrained. This behaviour is due to some agents consuming out of their capital income, which in other simulations is accumulated as wealth.

Figure 73 shows the dynamics of exploitation status and intensity over the course of the simulation. The distribution of h_t^ν allows more agents to accumulate than in other simulations, thus the number of exploited agents falls until the simulation becomes labour constrained, as shown in figure 73(a). Figure 73(b) shows the distribution of ε_t^ν across agents over t. Allowing more agents to accumulate leads to the exploitation intensity of agents who begin the simulation with little wealth and moderate-to-high skill levels falling until the economy becomes labour constrained.

Figure 74 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality falls due to broader accumulation across agents. Figure 75 shows the dynamics of the distribution of income over t. As expected, the modification to consumption induces wider variation in income shares over t due to different individual accumulation patterns.

Figure 72: Summary results - Model with consumption decreasing in wealth

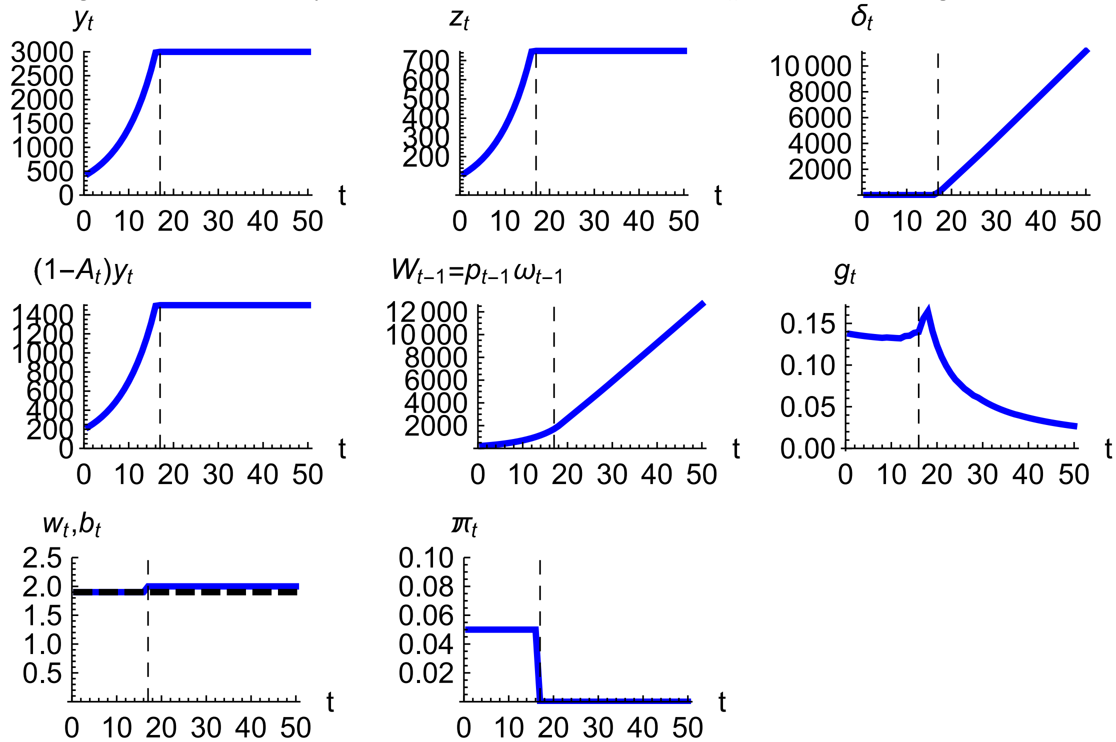
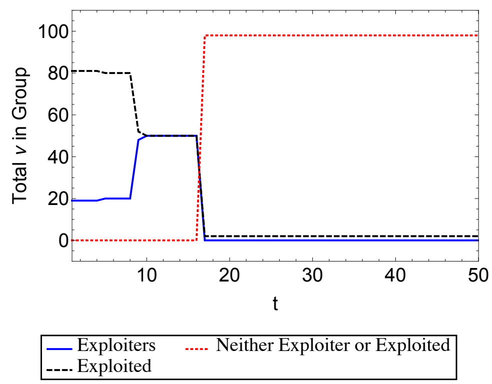


Figure 73: Exploitation - Model with consumption decreasing in wealth

(a) Exploitation status



(b) Exploitation intensity index

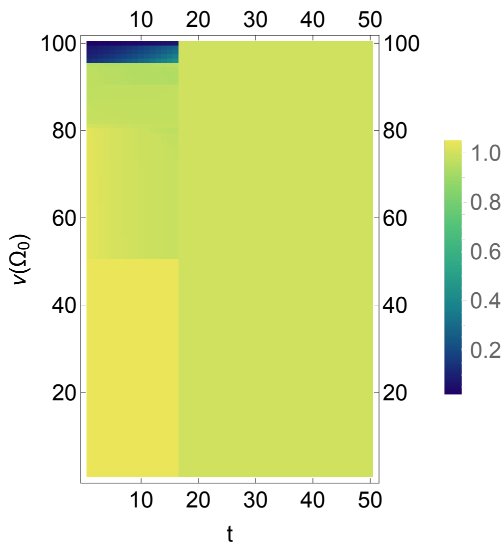
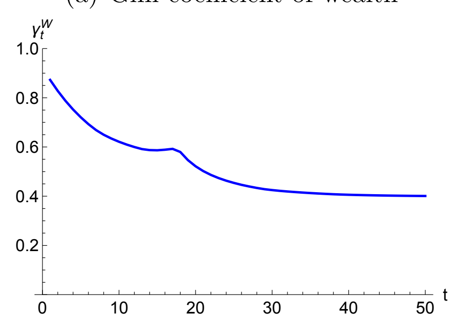


Figure 74: Distribution of wealth - Model with consumption decreasing in wealth

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

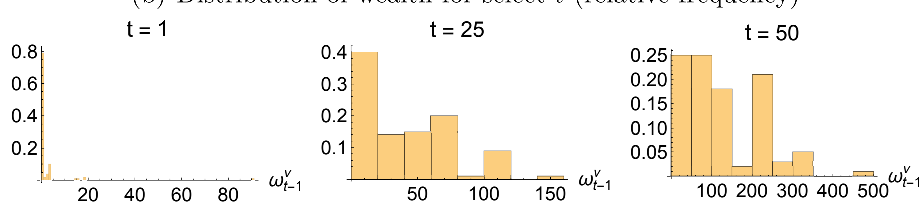
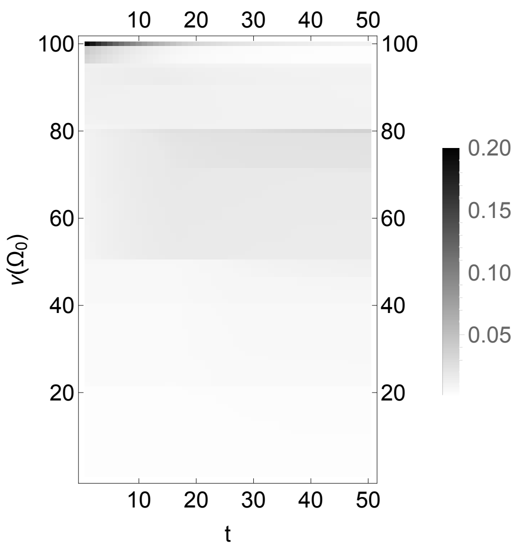
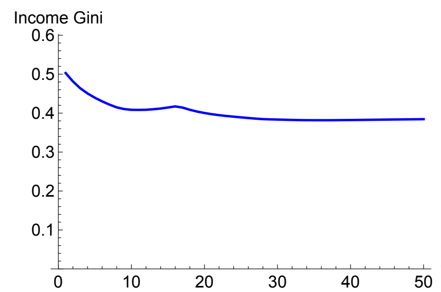


Figure 75: Distribution of income - Model with consumption decreasing in wealth

(a) Distribution of income shares



(b) Gini coefficient of Income



10.2 The economy with consumption decreasing in wealth and wealth taxes

This section presents the economy with Piketty-type wealth taxes for the case of consumption decreasing in wealth. The results of the simulation over T can be found in figures 76-79. Figure 76 shows that the only difference in the aggregate results of the model is that g_t shows a slight upward trend until the economy becomes labour constrained.

Figure 77 shows the dynamics of exploitation status and intensity over the course of the simulation. Figure 77(a) shows a relatively stable structure of exploitation. Figure 77(b) shows the distribution of ε_t^{ν} across agents over t. Figure 77(c) shows that wealth taxes quickly reduce γ_t^{ε} while the simulation is capital constrained and exploitation disappears entirely once the economy is labour constrained.

Figure 78 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth, and wealth inequality continues to fall once the economy is labour constrained - as shown by the path of γ_t^{W}. Figure 79 shows the dynamics of the distribution of income over shares and the Gini coefficient of income over t, both of which exhibit behaviour consistent the other figures.

Figure 76: Summary results - Consumption decreasing in wealth and wealth taxes

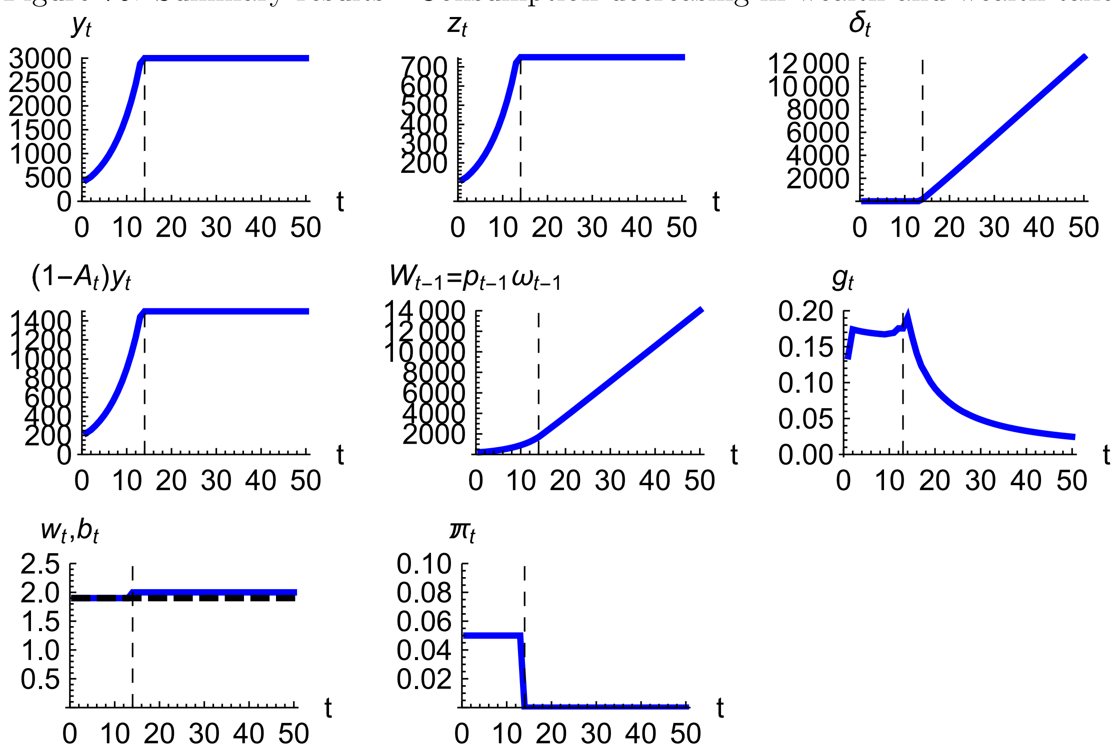
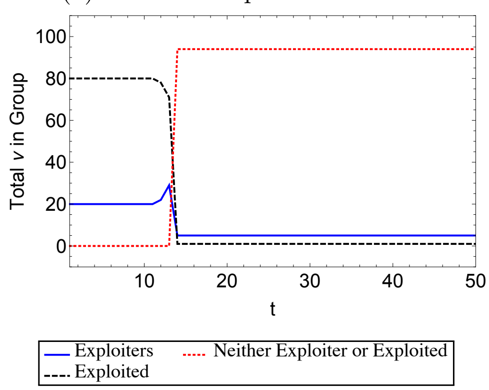
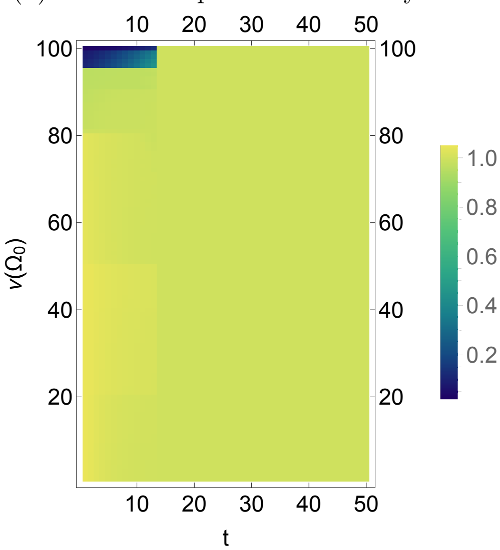


Figure 77: Exploitation - Consumption decreasing in wealth and wealth taxes

(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index



(c) Post-tax Gini coefficient of ε_t^ν

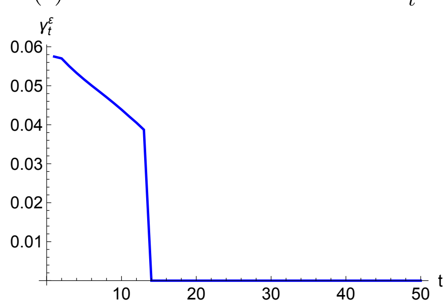
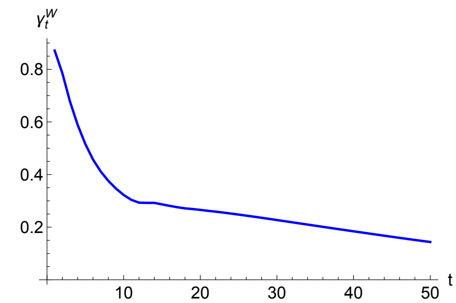


Figure 78: Distribution of wealth - Consumption decreasing in wealth with wealth taxes

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

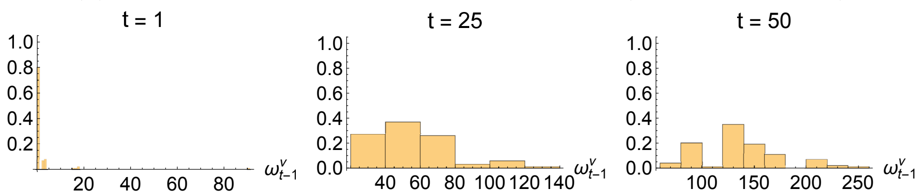
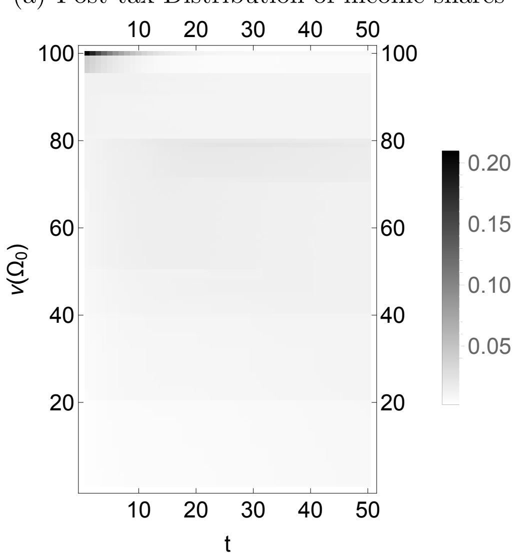
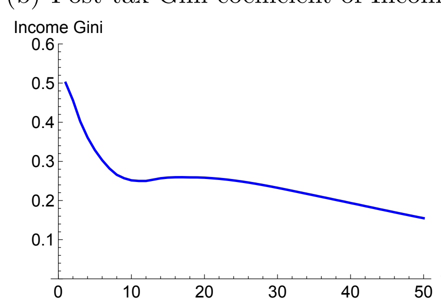


Figure 79: Distribution of income - Consumption decreasing in wealth with wealth taxes

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



10.3 The economy with consumption decreasing in wealth and wealth equality

This section presents the economy with wealth taxes designed to achieve wealth equality, using Rule 1, for the case of consumption decreasing in wealth. The results of the simulation over T can be found in figures 80-83. Figure 80 shows that g_t slightly increases until the point at which the economy becomes labour constrained while the other aggregate results remain similar to other simulations.

Figure 81 shows the dynamics of exploitation status and intensity over the course of the simulation. Figure 81(a) shows that the number of exploited agents quickly falls due to wealth taxes until the economy becomes labour constrained and exploitation disappears. Figure 81(b) shows the distribution of ε_t^{ν} across agents over t. Figure 81(c) shows γ_t^{ε}, which as expected, falls until exploitation disappears once the economy becomes labour constrained.

Figure 82 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth. Once the economy is labour constrained, wealth inequality then begins to rise slightly since Rule 1 is no longer operating. Figure 83 shows the dynamics of the distribution of income over t.

Figure 80: Summary results - Consumption decreasing in wealth and wealth equality

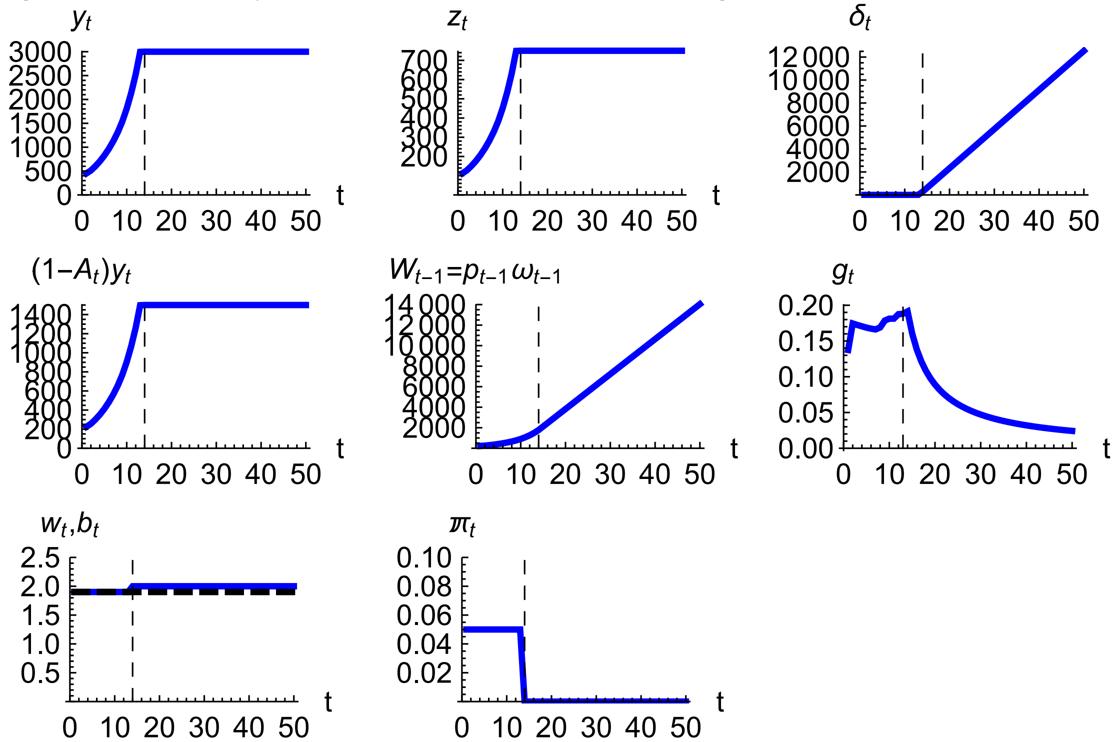
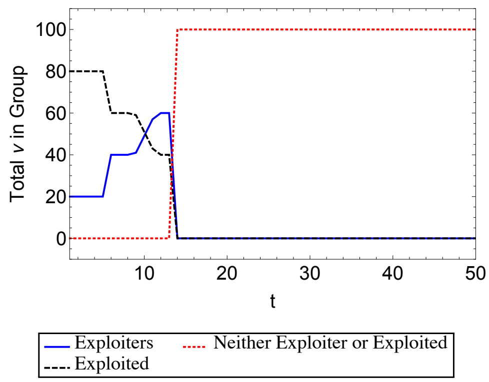
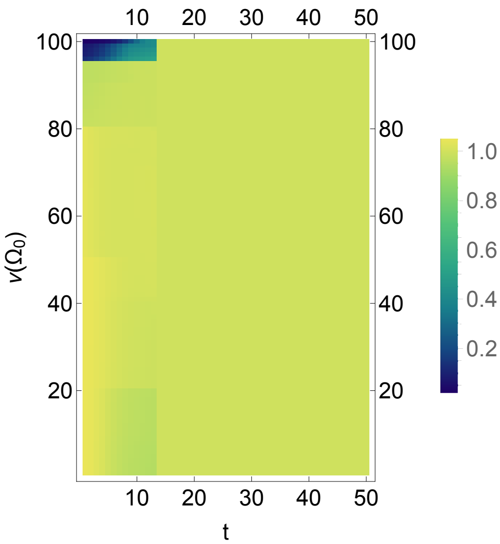


Figure 81: Exploitation - Consumption decreasing in wealth and wealth equality

(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index



(c) Post-tax Gini coefficient of ε_t^{ν}

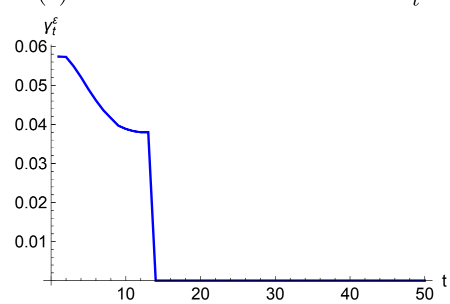
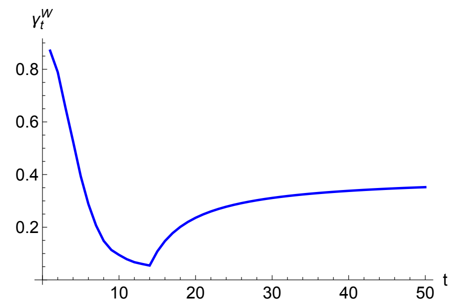


Figure 82: Distribution of wealth - Consumption decreasing in wealth with wealth equality

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

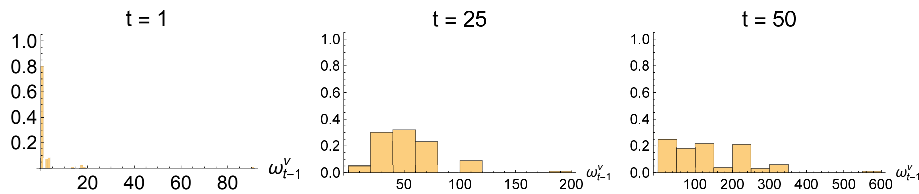
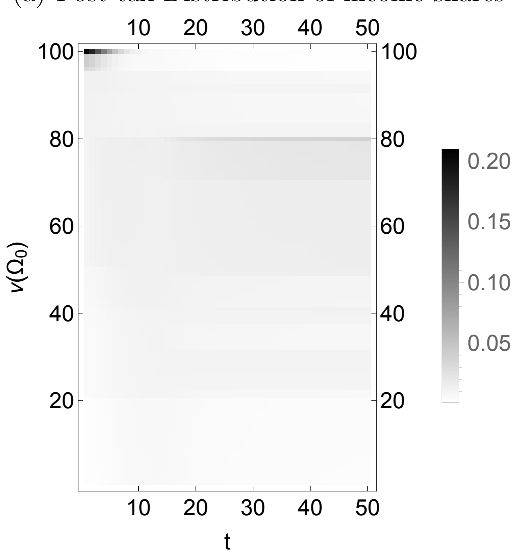
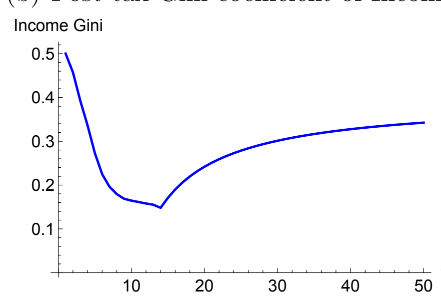


Figure 83: Distribution of income - Consumption decreasing in wealth with wealth equality

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



10.4 The economy with consumption decreasing in wealth and a socialist allocation

This section presents the economy with wealth taxes designed to achieve a socialist allocation, using Rule 2, for the case of consumption decreasing in wealth. The results of the simulation over T can be found in figures 84-87. Figure 84 shows that specifying consumption behaviour to be decreasing in wealth induces $g_t < \pi_t$ while the simulation is capital constrained, and that g_t is increasing over this same timeframe.

Figure 85 shows the dynamics of exploitation status and intensity over the course of the simulation. As figure 85(a) shows, the number of exploited agents diminishes slightly for the short period of time in which the economy is capital constrained, and exploitation disappears after the labour constraint is binding for all agents. Figure 85(b) shows the distribution of ε_t^{ν} across agents over t. Figure 85(c) shows γ_t^{ε}, which declines due to Rule 2 and eventually goes to zero once the economy is labour constrained.

Figure 86 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth and reaches a point consistent with a socialist allocation once the economy is labour constrained - as shown by the path of γ_t^W. Figure 87 shows the dynamics of the distribution of income over t.

Figure 84: Summary results - Consumption decreasing in wealth and socialist allocation

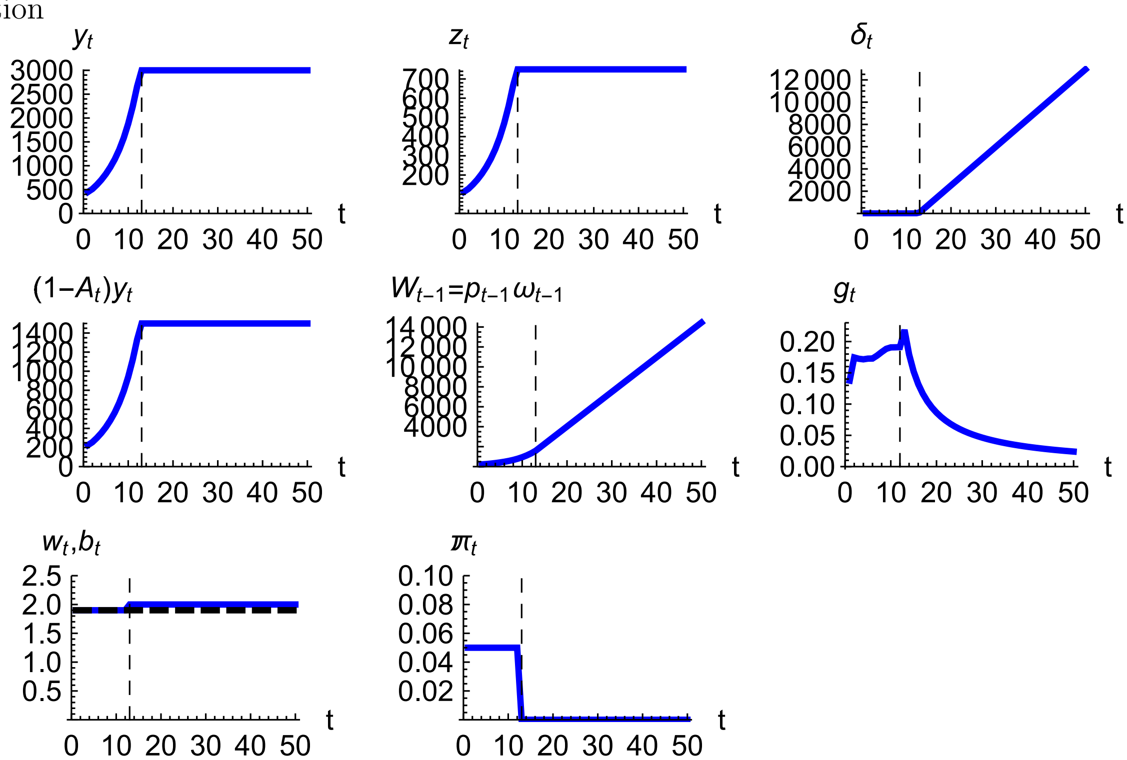
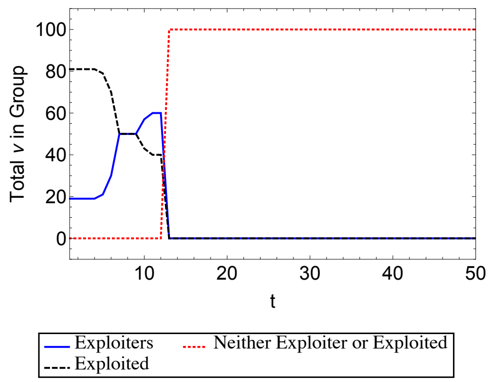
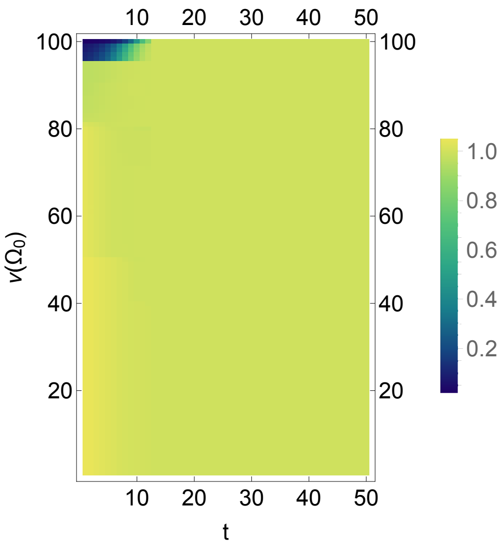


Figure 85: Exploitation - Consumption decreasing in wealth and socialist allocation

(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index



(c) Post-tax Gini coefficient of ε_t^ν

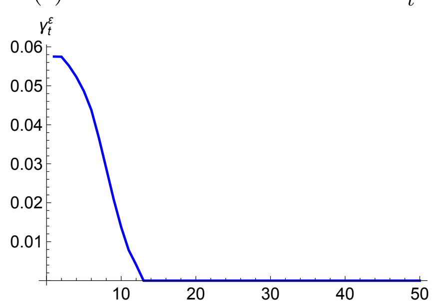
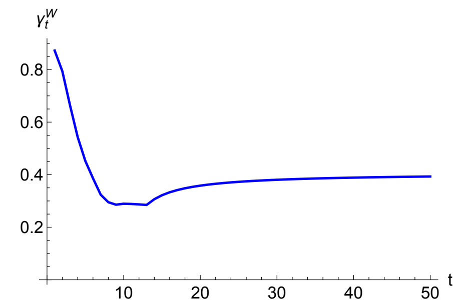


Figure 86: Distribution of wealth - Consumption decreasing in wealth with socialist allocation

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

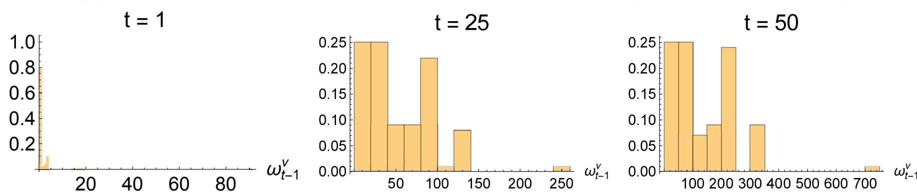
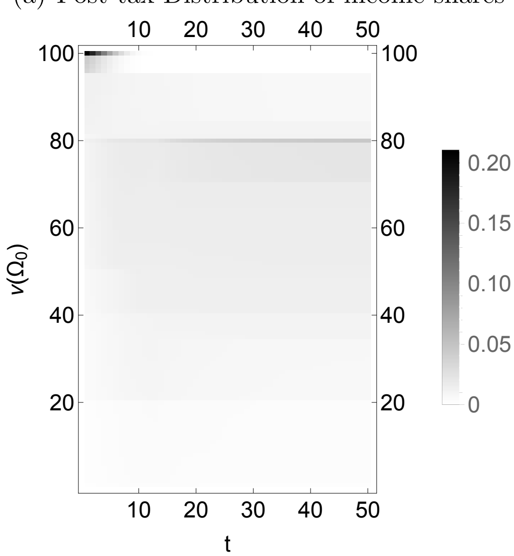
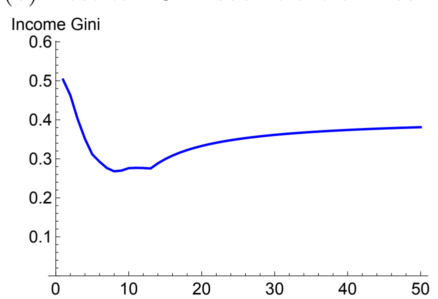


Figure 87: Distribution of income - Consumption decreasing in wealth with socialist allocation

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



10.5 Class in the model with consumption decreasing in wealth

The figures below display the dynamics of class over the simulations with consumption decreasing in wealth. Class status is determined according to Corollary 1 of Theorem 3. The figures below also show the CECP according to Theorem 4.

Figure 88: Class - Model with consumption decreasing in wealth

(a) Class status (b) CECP

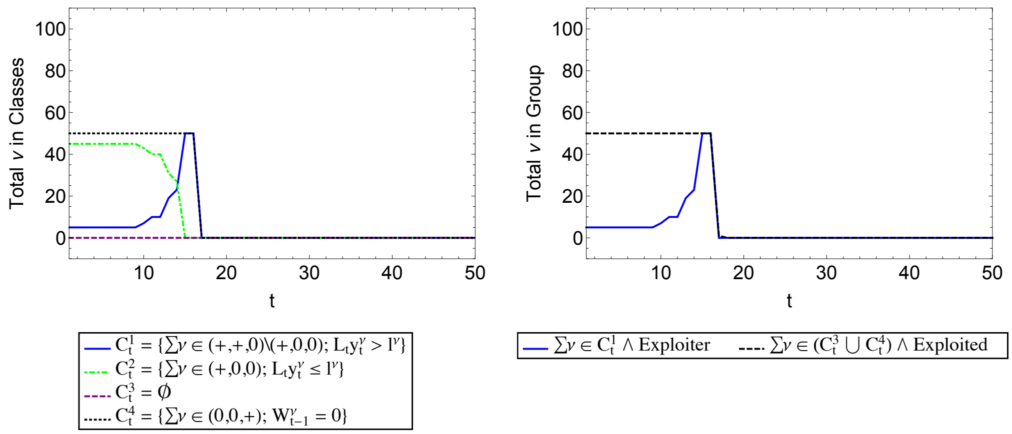


Figure 89: Class - Consumption decreasing in wealth and wealth taxes

(a) Class status (b) CECP

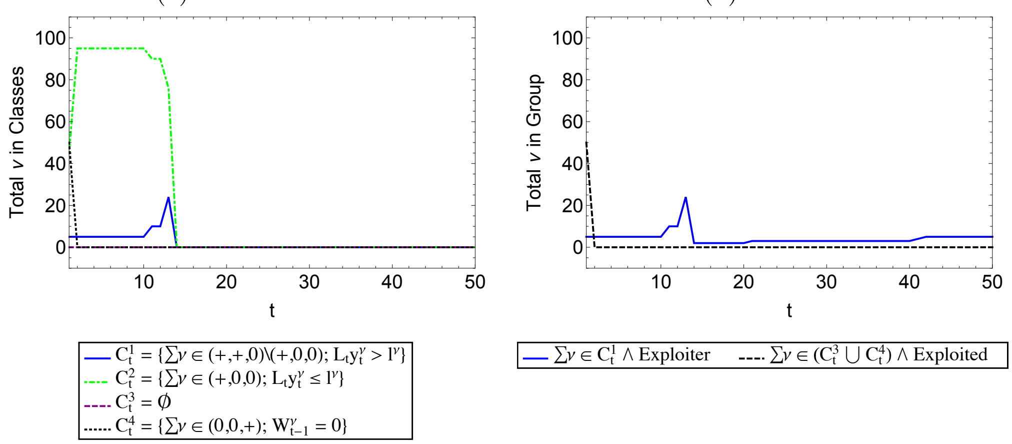


Figure 90: Class - Consumption decreasing in wealth and wealth equality

(a) Class status (b) CECP

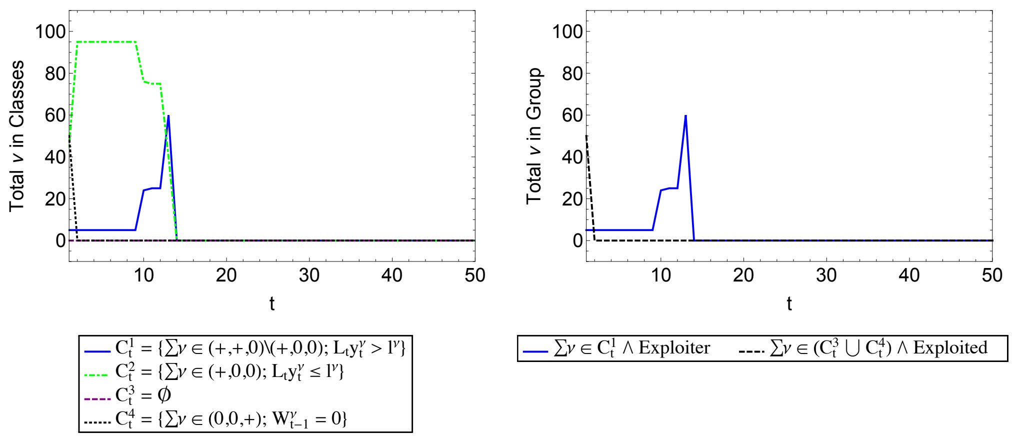
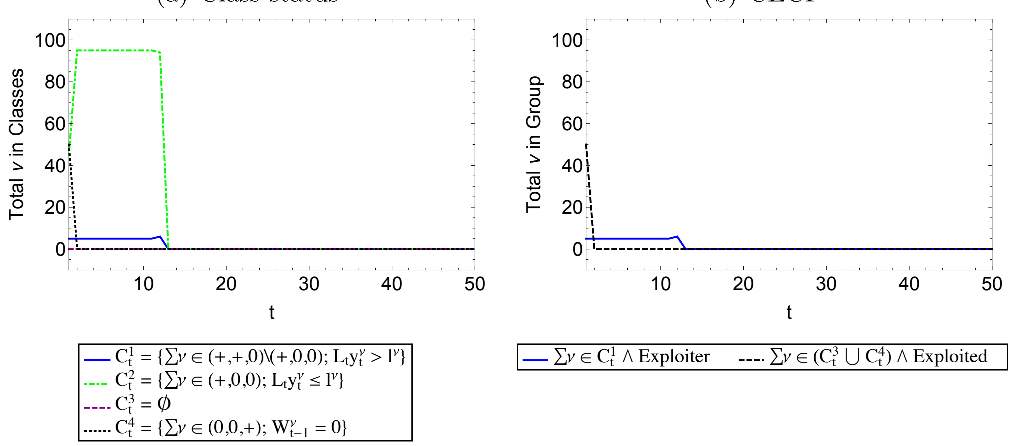


Figure 91: Class - Consumption decreasing in wealth and socialist allocation

(a) Class status (b) CECP



11 Heterogeneous labour supply and consumption decreasing in wealth

This section explores an extension of the previous model where agents have heterogeneous preferences over the proportion of their effective labour they supply when the economy is capital constrained. In the simulations that follow agents initial endowments of skills and wealth are determined in the same way as all other simulations but agents are now randomly assigned a disturbance ℓ^ν that modifies the proportion of effective labour supplied at every t while the simulation is capital constrained. ℓ^ν are between 0.98 and 1.02 so the disturbances to labour supplied z_t^ν are small, and ℓ^ν are determined at the start of the simulation such that the total labour supplied (demanded) at any t is equal to the total labour supplied (demanded) in the benchmark model, thus all equilibrium conditions are maintained. After randomly determining $\ell^\nu \in \{0.98, 1.02\}$ the ℓ^ν of the agent with the highest skill level is adjusted such that there is no over- or undershooting of initial labour demand, ensuring that Definition 1(c) holds. This sets ℓ^ν as proportions of labour supplied across all t while the simulation is capital constrained. ℓ^ν are constant across t and the same ℓ^ν are used across all of the simulations in this section. All simulations below use the parameters from the paper.

In the simulations below, during any t, if $\widehat{w}_t > b$ then $\Lambda_t^\nu = l^\nu$, if $\widehat{w}_t < b$ then $\Lambda_t^\nu = 0$, and if $\widehat{w}_t = b$ then Λ_t^ν is determined by $\ell^\nu \frac{LA^{-1}\omega_{t-1}}{l} l^\nu$ for all ν, assuming $x_t^\nu = 0$ as in the paper. With Λ_t^ν decided, agents then choose their activity levels ξ_t^ν subject to the typical constraints, except their consumption is determined according to h_t^ν from the previous section[2] so that constraint (1) of MP_t^ν is now

$$p_t x_t^\nu + [p_t - w_t L]\, y_t^\nu + w_t z_t^\nu + p_t \delta_t^\nu = p_t \omega_t^\nu + h_t^\nu (\pi_t p_t \omega_{t-1}^\nu + b\Lambda_t^\nu).$$

These modifications to labour supply and consumption are made to introduce behaviour into the model that is closer to the kind of heterogenous decisions and interactions that are common in agent-based modeling approaches to economic phenomena. As can be seen in the results below, this increase in the degree of heterogeneity in agents' beahviour has a distinct impact on the performance of the economy and individual agents' experience of exploitation.

[2] As in the previous section, if agents have zero wealth $h_t^\nu = 1.0$; if agents have wealth up and including the median wealth $h_t^\nu = 0.9$; if agents have wealth above the median and up to and including the 90th percentile of wealth $h_t^\nu = 0.8$; if agents have wealth greater than the 90th percentile up to and including the 99th percentile $h_t^\nu = 0.7$; and agents at the top one percent of the wealth distribution have $h_t^\nu = 0.4$.

11.1 The economy with random labour supply

This section presents the baseline model with random labour supply and consumption decreasing in wealth. The results of the simulation over T can be found in figures 92-95. Figure 92 shows the aggregate results of the model. The notable difference between the results below and other models is the behaviour of g_t. The path of g_t is similar to that of the previous section yet more accentuated, as shown in the close-up provided in the bottom-right panel of figure 92. The behaviour of g_t shows that accumulation varies according to how much agents are willing to work and how much they consume depending on where they fall in the distribution of wealth.

Figure 93 shows the dynamics of exploitation status and intensity over the course of the simulation. The interaction of ℓ^ν and h_t^ν across agents over t allows some high-skilled agents who do not consume all of their income to experience $\varepsilon_t^\nu < 1$.

Figure 94 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality falls due to broader accumulation across agents. Figure 75 shows the dynamics of the distribution of income over t. As expected, the modification to consumption induces wider variation in income shares over t due to different individual accumulation and labour supply patterns.

Figure 92: Summary results - Model with random labour supply

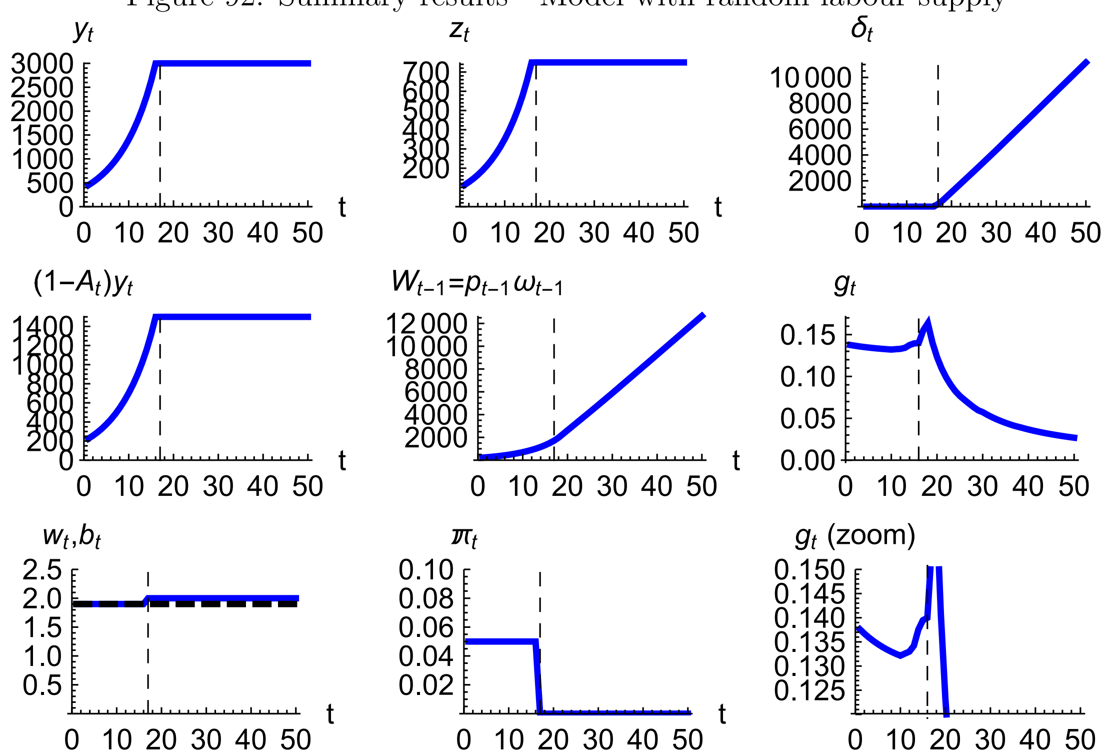
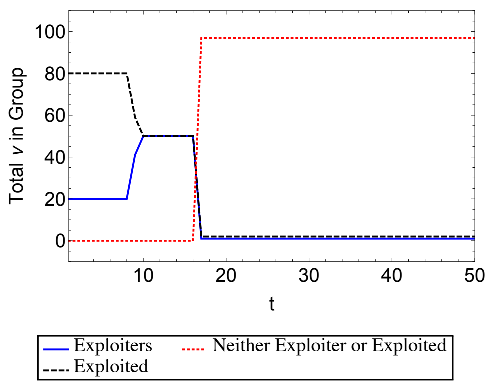


Figure 93: Exploitation - Model with random labour supply

(a) Exploitation status



(b) Exploitation intensity index

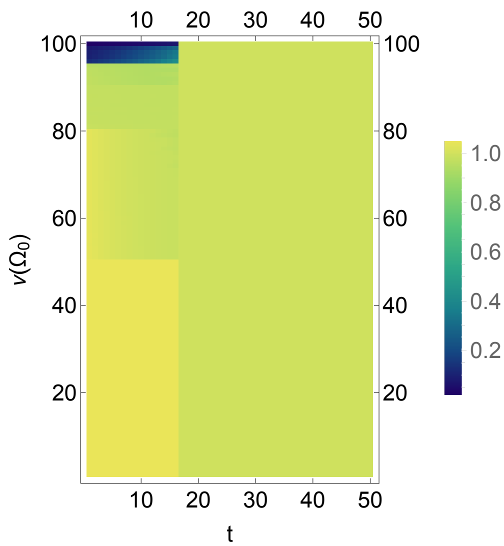
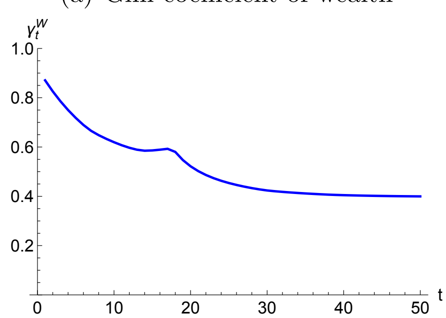


Figure 94: Distribution of wealth - Model with random labour supply

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

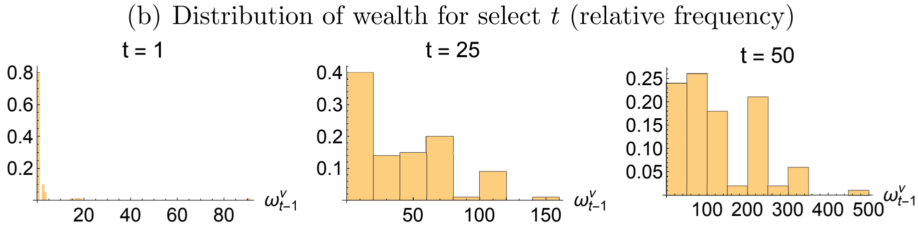
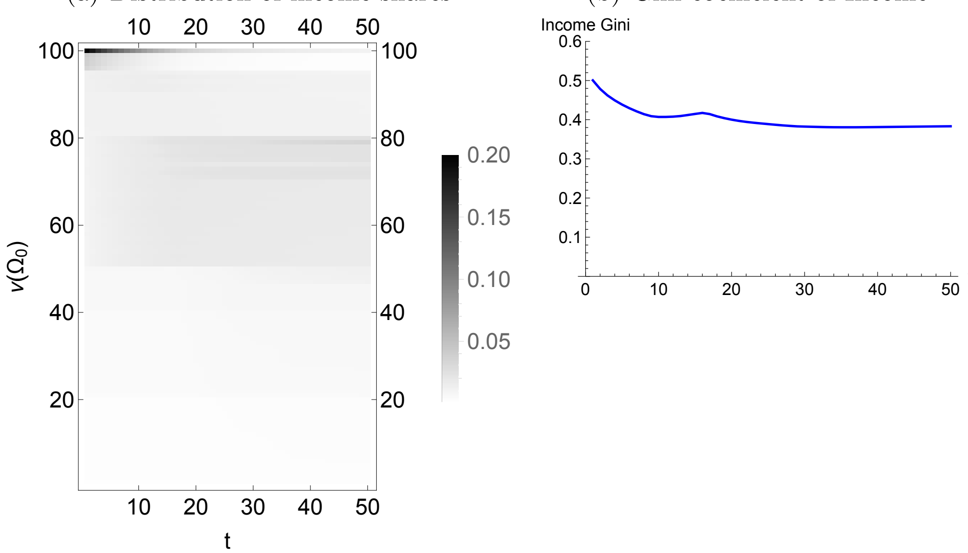


Figure 95: Distribution of income - Model with random labour supply



11.2 The economy with random labour supply and wealth taxes

This section presents the economy with Piketty-type wealth taxes for the case of random labour supply and consumption decreasing in wealth. The results of the simulation over T can be found in figures 96-99. Figure 96 shows the aggregate results of the model, which are consistent with the previous simulation. The lower-right panel of figure 96 provides a closeup of g_t during the capital constrained portion of the simulation. The interaction of heterogeneous decisions of labour supply, heterogeneous consumption, and redistributive wealth taxes lead to an upward trend in the accumulation, with a slight dip prior to the simulation becoming labour constrained caused by increases in consumption as agents move up the distribution of wealth. Overall, g_t is higher than in the previous simulation while the economy is capital constrained. This higher accumulation rate can be seen as a benefit of redistributive wealth taxes and broader accumulation across the economy.

Figure 97 shows the dynamics of exploitation status and intensity over the course of the simulation. Figure 97(a) shows that, despite wealth taxes, the structure of exploitation is relatively stable with the number of exploited agents only falling right before the economy becomes labour constrained. Figure 97(b) shows the distribution of ε_t^ν across agents over t. Exploitation intensity varies across agents depending on how much labour they supply and their accumulation. As figure 97(b) shows, some agents who begin the simulation with zero wealth start to experience $\varepsilon_t^\nu < 1$ due to transfers of wealth through taxes and their relatively low labour supply. Figure 97(c) shows that wealth taxes reduce γ_t^ε while the simulation is capital constrained and exploitation disappears entirely once the economy is labour constrained.

Figure 98 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth, and wealth inequality continues to fall once the economy is labour constrained - as shown by the path of γ_t^W. Figure 99 shows the dynamics of the distribution of income over shares and the Gini coefficient of income over t. As expected the pattern of income shares over time is varied across agents due to the heterogeneity of ℓ^ν, and the Gini coefficient of income decreases over both phases of the simulation.

Figure 96: Summary results - Random labour supply and wealth taxes

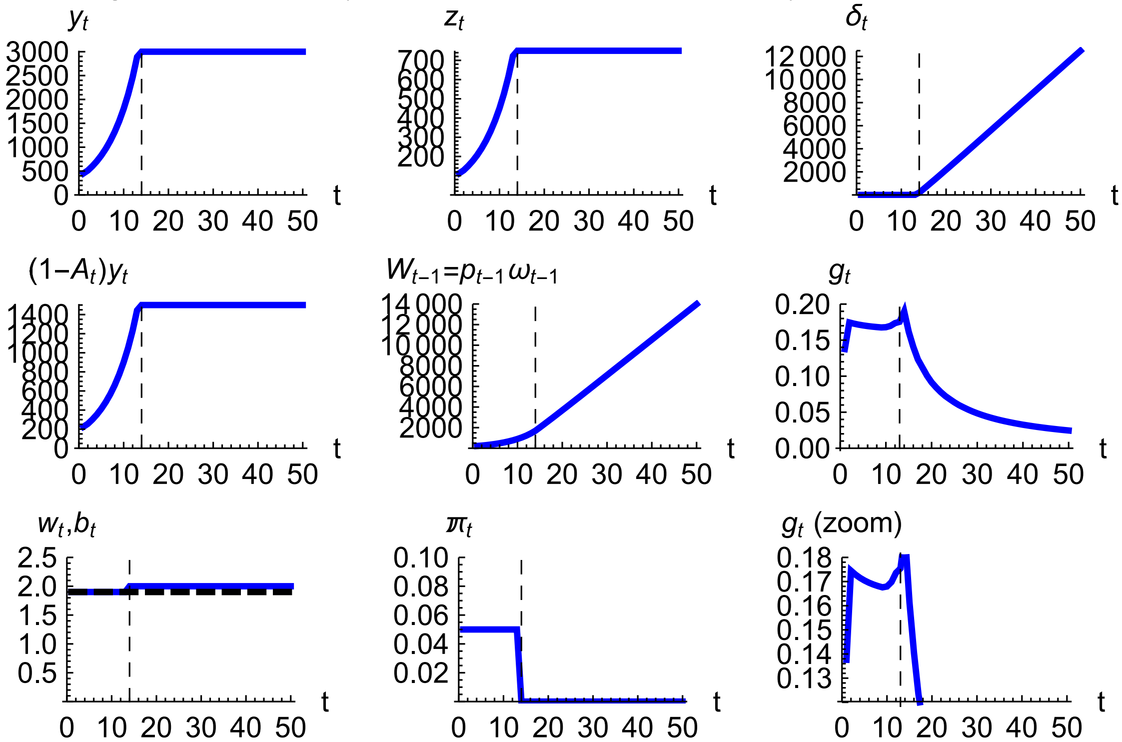
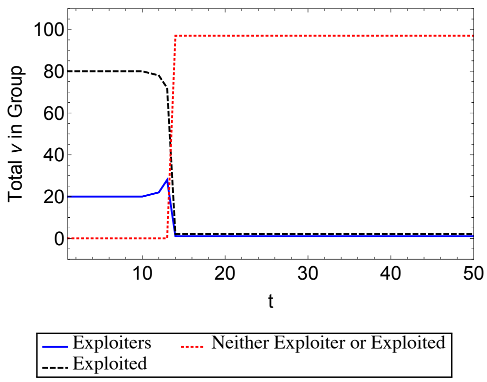
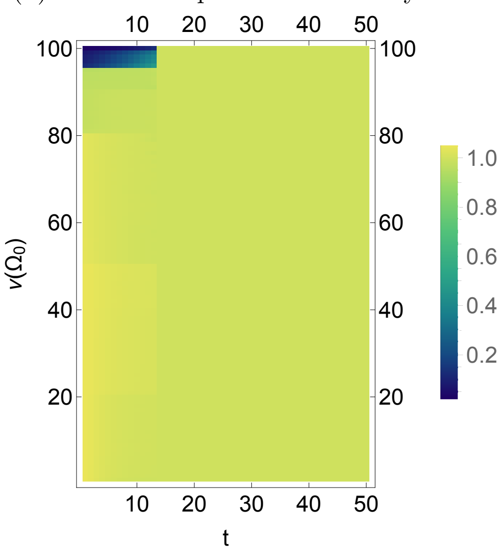


Figure 97: Exploitation - Random labour supply and wealth taxes

(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index



(c) Post-tax Gini coefficient of ε_t^{ν}

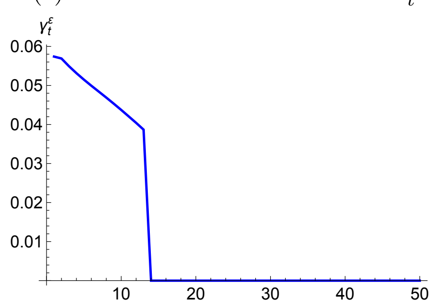
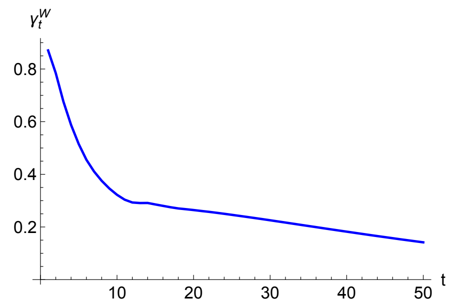


Figure 98: Distribution of wealth - Random labour supply with wealth taxes

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

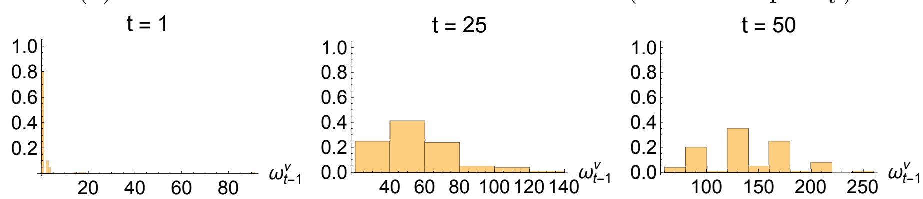
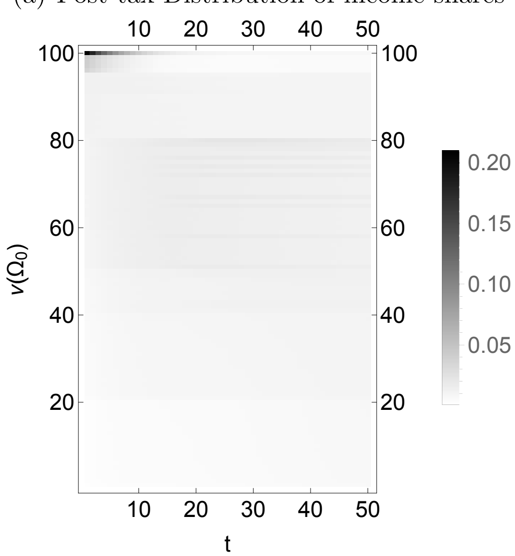
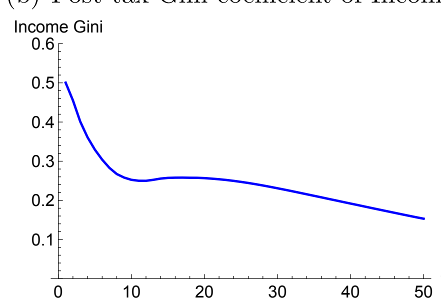


Figure 99: Distribution of income - Random labour supply with wealth taxes

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



11.3 The economy with random labour supply and wealth equality

This section presents the economy with wealth taxes designed to achieve wealth equality, using Rule 1, for the case of heterogeneous labour supply and consumption decreasing in wealth. The results of the simulation over T can be found in figures 100-103. Figure 100 shows aggregate results consistent with earlier versions of the simulation and an interesting path for g_t. As the lower-right panel of figure 100 shows, g_t initially increases and then falls during the capital constrained portion of the simulation. This path is due to the interaction of taxes to achieve wealth equality, heterogeneous consumption, and heterogeneous labour supply. As agents accumulate, they increase their consumption which detracts from the capital available for accumulation. Shortly after g_t falls, it then begins to rise again as the redistribution of wealth and widespread accumulation across agents dominates the increased consumption resulting from more agents accumulating.

Figure 101 shows the dynamics of exploitation status and intensity over the course of the simulation. Figure 101(a) shows that the number of exploited agents quickly falls due to wealth taxes until the economy becomes labour constrained and exploitation disappears. Figure 101(b) shows the distribution of ε_t^ν across agents over t. As expected, there is variation in ε_t^ν due to the variation in ℓ^ν and the redistribution according to Rule 1. As figure 101(b) shows, it is even possible for some agents who begin the simulation with zero wealth to experience $\varepsilon_t^\nu < 1$ at later t due to the wealth transfers they receive and their relatively low ℓ^ν. Figure 101(c) shows γ_t^ε, which falls until exploitation disappears once the economy becomes labour constrained.

Figure 102 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth. Once the economy is labour constrained, wealth inequality then begins to rise slightly since Rule 1 is no longer operating. Figure 103 shows the dynamics of the distribution of income over t.

Figure 100: Summary results - Random labour supply and wealth equality

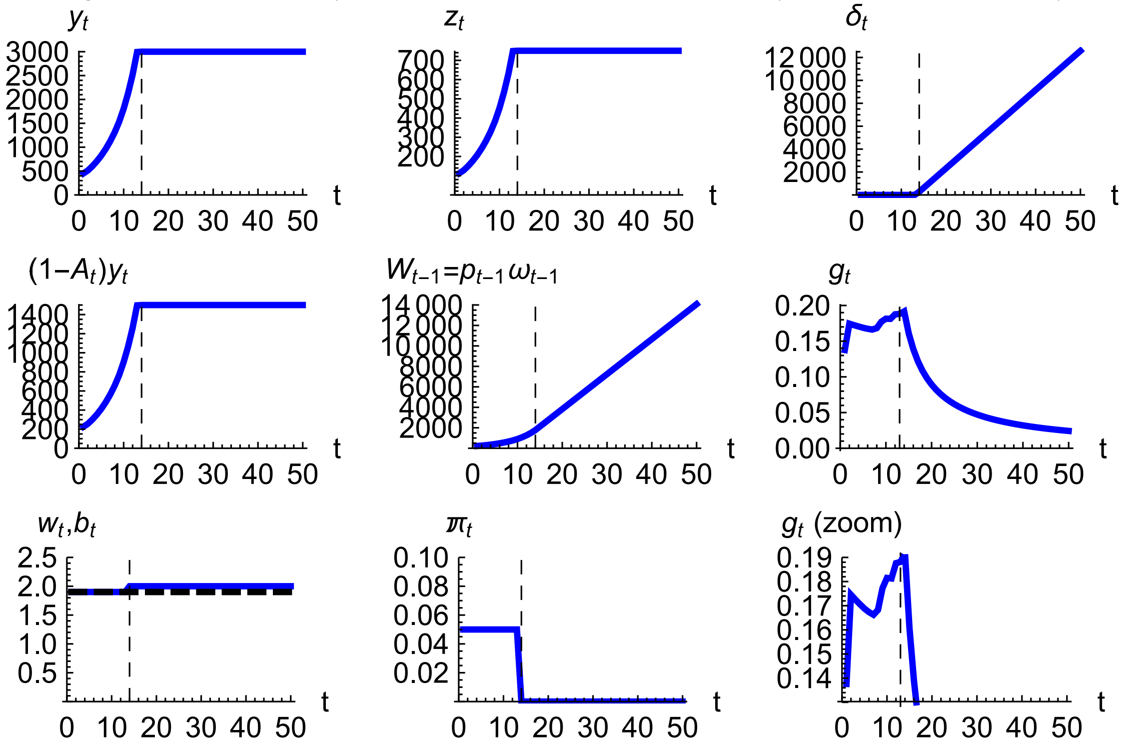
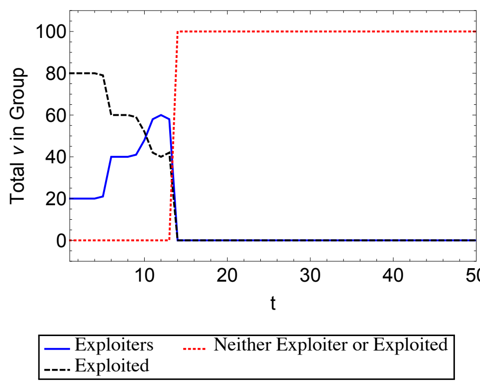
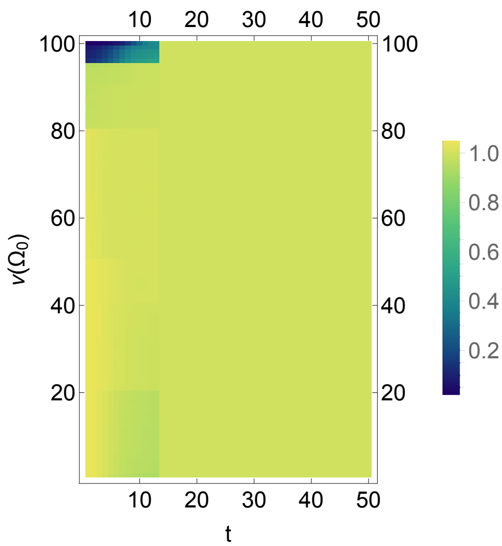


Figure 101: Exploitation - Random labour supply and wealth equality

(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity index



(c) Post-tax Gini coefficient of ε_t^ν

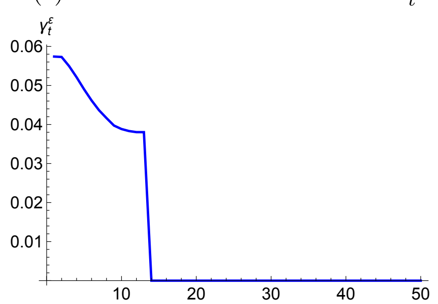
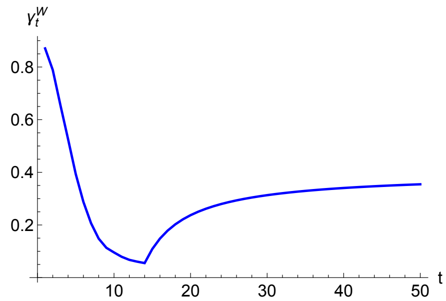


Figure 102: Distribution of wealth - Random labour supply with wealth equality

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

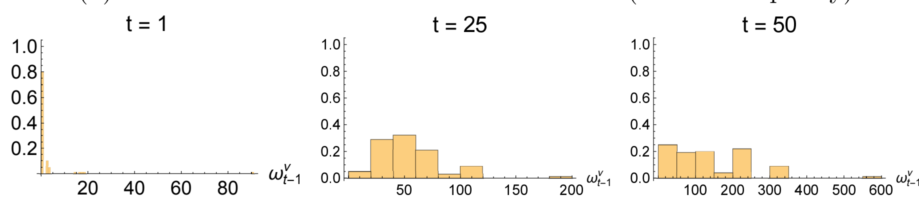
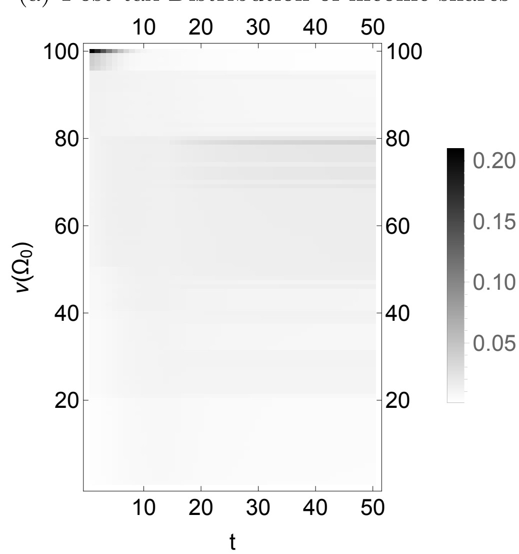
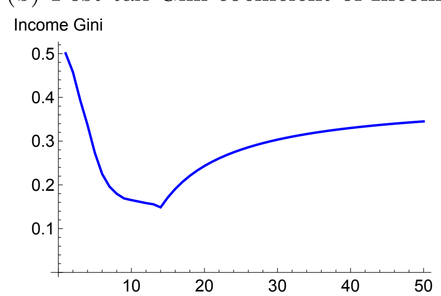


Figure 103: Distribution of income - Random labour supply with wealth equality

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



11.4 The economy with random labour supply and a socialist allocation

This section presents the economy with wealth taxes designed to achieve a socialist allocation, using Rule 2, for the case of heterogeneous labour supply and consumption decreasing in wealth. The results of the simulation over T can be found in figures 104-107. Figure 104 shows aggregate results consistent with earlier versions of the model, and an upward trajectory for g_t during the capital constrained portions of the simulation. The lower-right panel of figure 104 shows a relatively steady rise in g_t and until the simulation is labour constrained.

Figure 105 shows the dynamics of exploitation status and intensity over the course of the simulation. As figure 105(a) shows, the number of exploited agents diminishes due to Rule 2 and exploitation disappears after the labour constraint is binding for all agents. Figure 105(b) shows the distribution of ε_t^{ν} across agents over t. As expected, ε_t^{ν} varies across agents according to how much labour they supply, yet exploitation is quickly alleviated for agents who begin the simulation with $\varepsilon_t^{\nu} > 1$. Figure 105(c) shows γ_t^{ε}, which declines due to Rule 2 and eventually goes to zero once the economy is labour constrained.

Figure 106 shows the dynamics of the distribution of wealth over t. As expected, wealth inequality immediately begins to decrease due to the redistribution of wealth and reaches a point consistent with a socialist allocation once the economy is labour constrained - as shown by the path of γ_t^W. Figure 107 shows the dynamics of the distribution of income over t.

Figure 104: Summary results - Random labour supply and socialist allocation

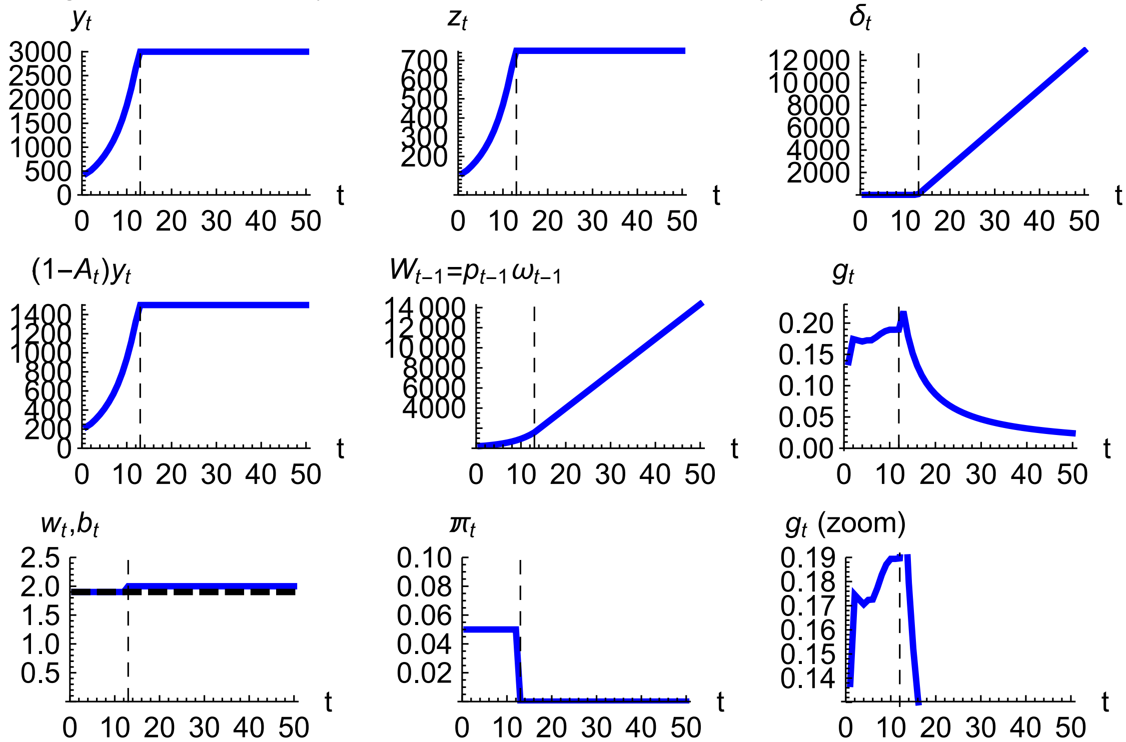
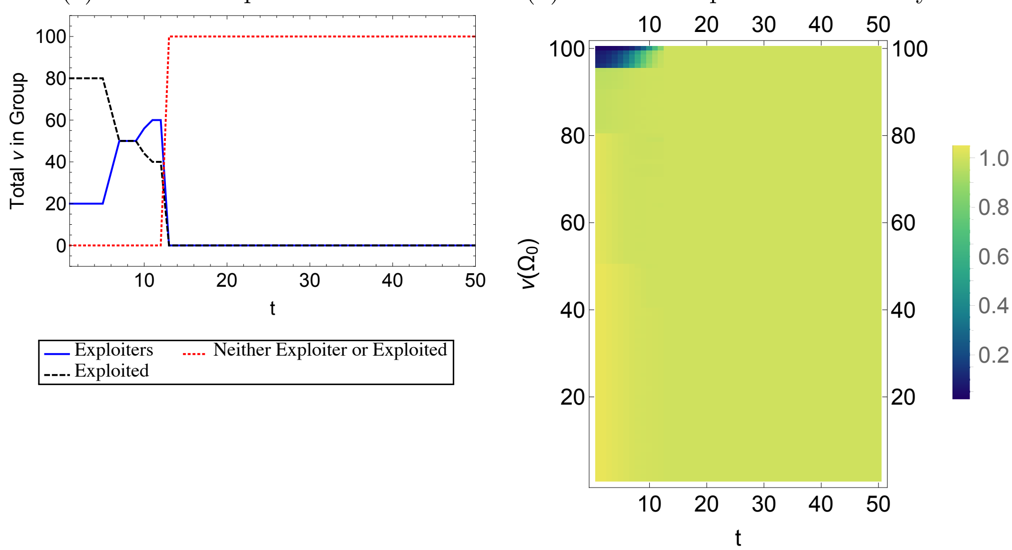


Figure 105: Exploitation - Random labour supply and socialist allocation

(a) Post-tax Exploitation status (b) Post-tax Exploitation intensity index



(c) Post-tax Gini coefficient of ε_t^{ν}

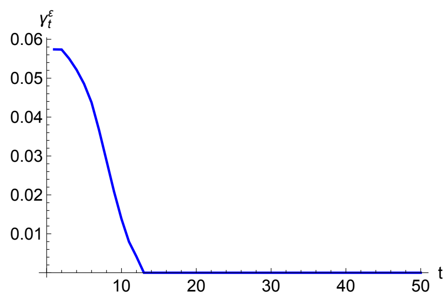
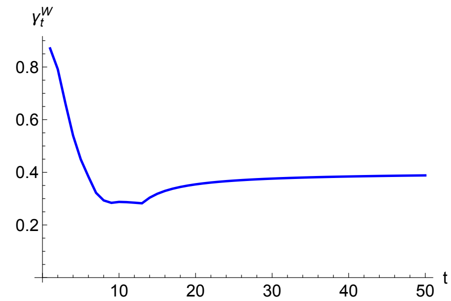


Figure 106: Distribution of wealth - Random labour supply with socialist allocation

(a) Post-tax Gini coefficient of wealth



(b) Post-tax Distribution of wealth for select t (relative frequency)

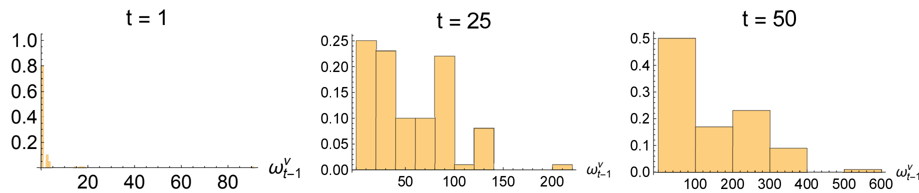
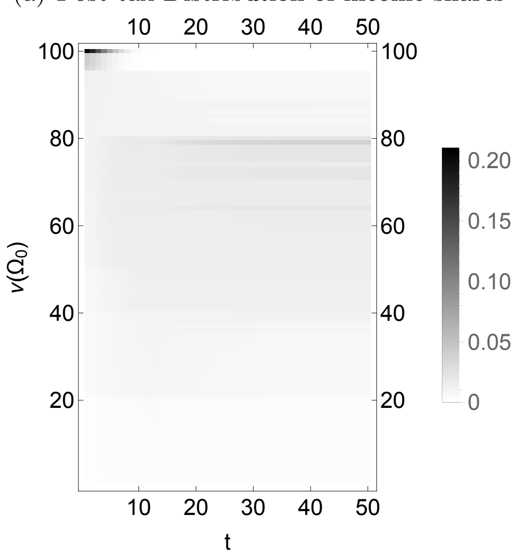
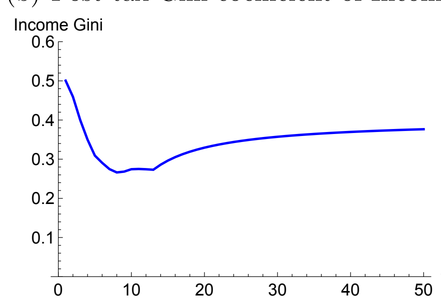


Figure 107: Distribution of income - Random labour supply with socialist allocation

(a) Post-tax Distribution of income shares



(b) Post-tax Gini coefficient of Income



11.5 Class in the model with random labour supply

The figures below display the dynamics of class over the simulations with random labour supply and consumption decreasing in wealth. Class status is determined according to Corollary 1 of Theorem 3. The figures below also show the CECP according to Theorem 4.

Figure 108: Class - Model with random labour supply

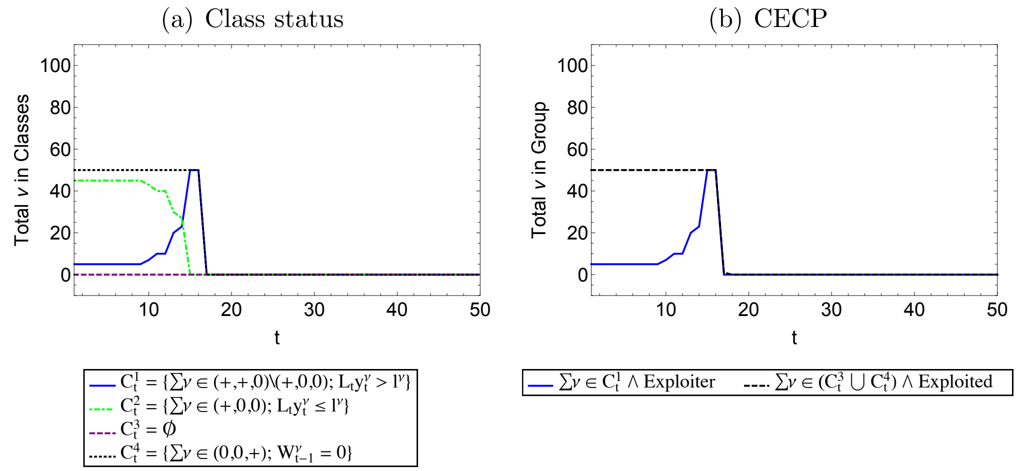


Figure 109: Class - Random labour supply and wealth taxes

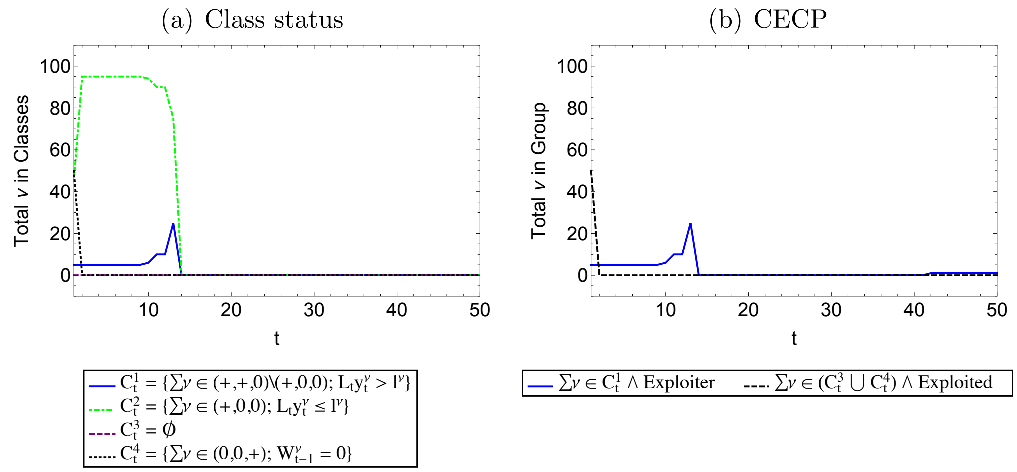


Figure 110: Class - Random labour supply and wealth equality

(a) Class status (b) CECP

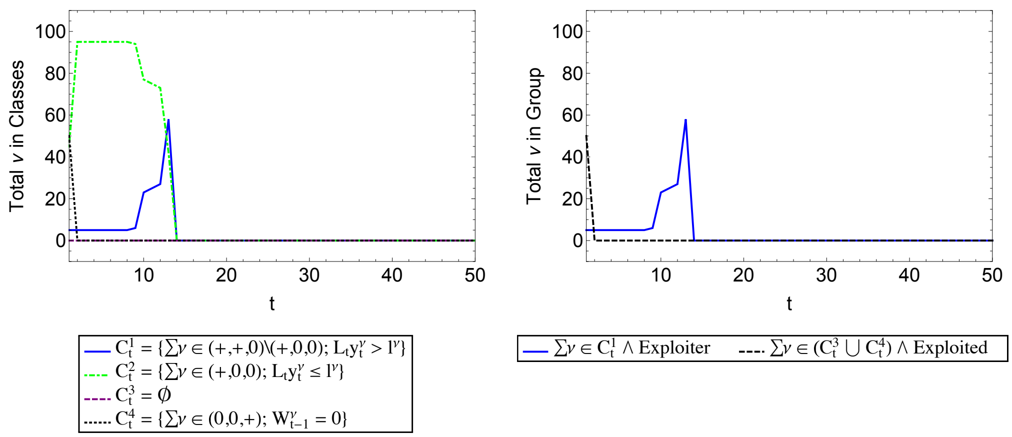
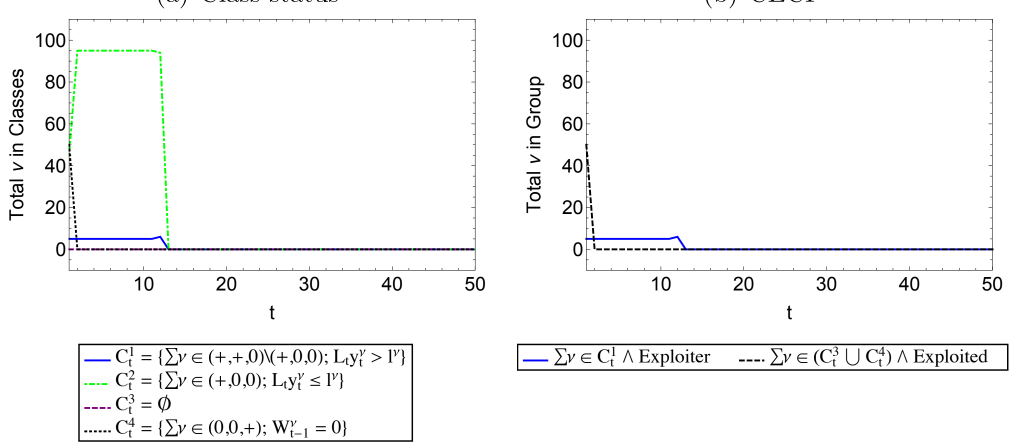


Figure 111: Class - Random labour supply and socialist allocation

(a) Class status (b) CECP



12 Homogeneous labour

In this section we analyse the special case of homogeneous labour. In the simulations that follow, all $\nu \in \mathcal{N}_t$ have uniform labour endowments $l^\nu = 1$ over all t. This robustness check is conducted to show that including heterogeneous labour as the benchmark scenario does not alter the macro-behaviour of the economy or any of the equilibrium conditions detailed in the paper.

12.1 The dynamics of the model with homogeneous labour

This section presents the special case of the model in which labour is homogeneous. The parameters are: $N = 100$, $A = 0.5$, $L = 0.25$, $b = 1.9$, and $l^\nu = s^\nu \zeta^\nu = 1$, for all $\nu \in \mathcal{N}$. The simulation runs for $T = 50$ periods.

The results of the simulation over T can be found in Figures 112-113(b). Figure 112 reports the aggregate activity levels (y_t, z_t, δ_t), aggregate net output $(1 - A)\, y_t$, wealth W_{t-1}, the growth rate of capital g_t, $\widehat{w}_t$ and b, and π_t. In all panels, the dashed vertical line denotes the period in which the economy becomes labour constrained.

Figure 112: Summary results - Model with homogeneous labour

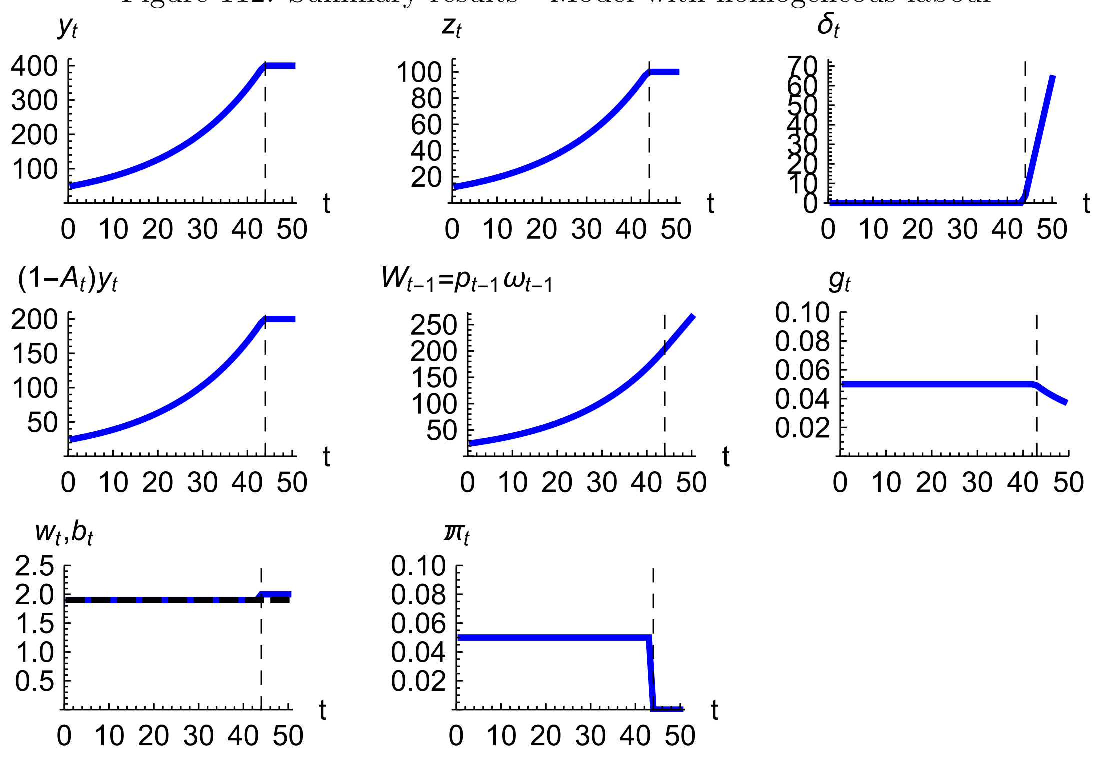


Figure 113 reports the dynamics of exploitation. Figure 113(a) provides a headcount of the agents who are exploiters, exploited, or neither. Clearly, the exploitation status of agents is constant while the economy is capital constrained and exploitation ceases to exist once it becomes labour constrained. Figure 113(b) provides a complete description of the distribution of the exploitation intensity index, ε_t^ν, over the course of the simulation. Prior to the economy becoming labour constrained, the distribution of ε_t^ν is constant over time: there is no tendency for exploitation to diminish. When the economy becomes labour constrained, profits and exploitation disappear, and one can observe that $\varepsilon_t^\nu = 1$, all $\nu \in \mathcal{N}$.

Figure 113(b) displays a relatively low dispersion of the exploitation index. This is due to the fact that, unlike in actual economies, all agents perform the same amount of labour and, in the capital constrained phase, the given parameterisation (in particular the rather high value of b) yields a low profit rate. Different values of the parameters, or a heterogeneous allocation of labour (perhaps inversely proportional to wealth, in order to reflect class differences) lead to a much higher dispersion. Note that in the special case of homogeneous labour agents' effective labour and labour time converge so that there is no difference between the skill-adjusted and time-adjusted exploitation intensity indices ε_t^ν and e_t^ν.

Figure 113: Exploitation - Model with homogeneous labour

(a) Exploitation status (b) Exploitation intensity index

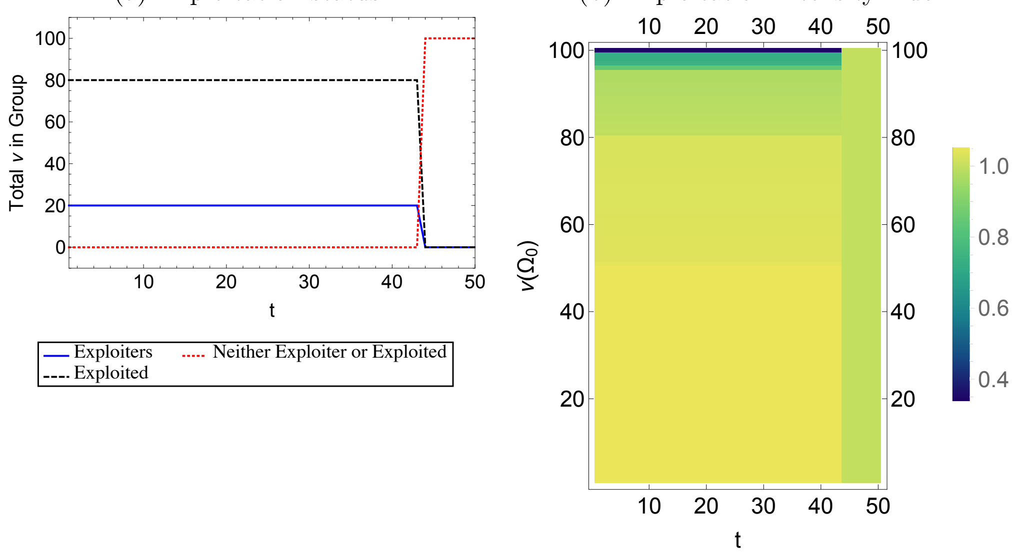
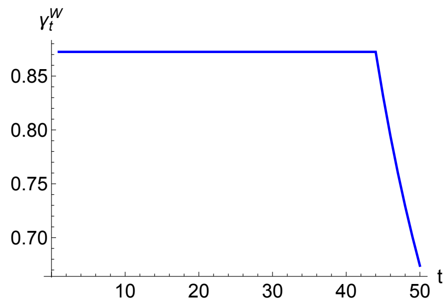


Figure 114(a) shows the Gini coefficient of wealth. The index remains constant as long as exploitation exists, because all agents in the middle and upper classes accumulate at the same rate, and so their relative positions in the wealth distribution remain unchanged, even though they become increasingly wealthier than propertyless agents. Once the economy becomes labour constrained and exploitation ceases, the Gini coefficient monotonically decreases and asymptotically approaches zero. The same pattern emerges in Figure 114(b) where the whole wealth distribution is shown for select t (before *and* after the end of exploitation).

Figure 114: Distribution of wealth - Model with homogeneous labour

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

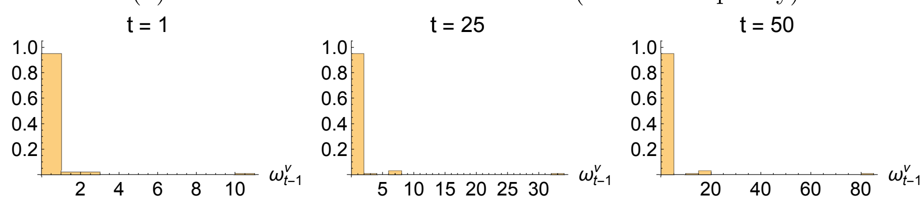
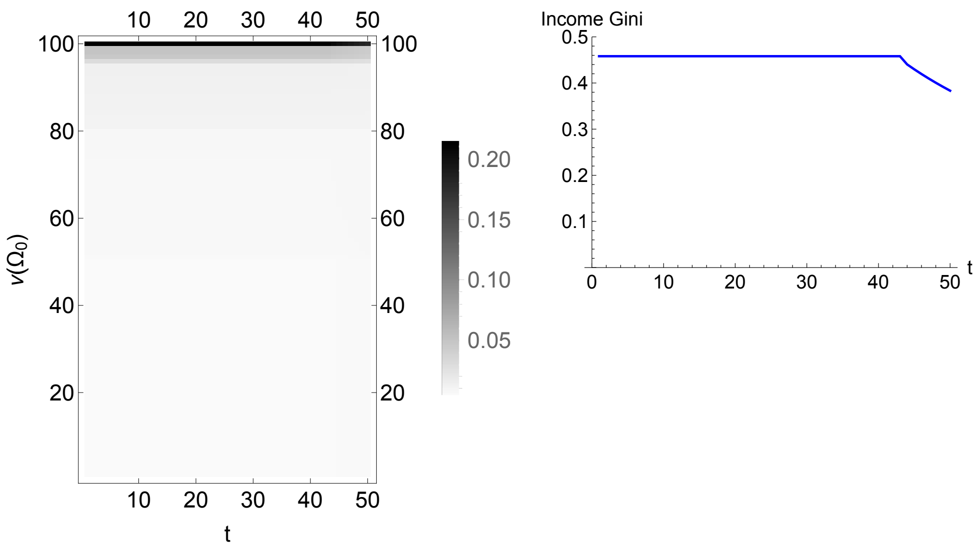


Figure 115 displays data on the distribution of income $(1 + \pi_t)\omega_{t-1}^{\nu} + \widehat{w}_t\Lambda_t^{\nu}$ across agents. The distribution of income is static until the simulation becomes labour constrained, after which the distribution is uniform.

Figure 115: Distribution of income - Model with homogeneous labour

(a) Distribution of income shares (b) Gini coefficient of Income



12.2 Model with homogeneous labour and wealth taxes

The simulation that follows extends the basic with homogeneous labour $l^{\nu} = 1$ to include Piketty-type wealth taxes. This exercise is conducted to further emphasize the fact that heterogenous labour and redistributive wealth taxes are neutral additions in terms of the aggregate behaviour of the models. The tax scheme and determination of $(\tau_t^{\nu})_{\nu \in \mathcal{N}_t}$ are the same as in the paper. The determination of Ω_0 is the same as in the paper as well. The simulation begins with the same benchmark parameters as the simulation in section 5 of the paper: $N = 100$, $A = 0.5$, $L = 0.25$, $b = 1.9$, and $l = 100$. $l > LA^{-1}\omega_0$ holds and the economy is initially capital constrained, starting far from the knife-edge.

Figure 116 reports the summary results for the model with homogeneous labour and wealth taxes. The simulation shows steady growth of activity levels $(y_t, z_t, \delta_t^{\nu})$, net output $(1 - A)y_t$, and aggregate wealth until the simulation becomes labour constrained.

Figure 116: Summary results - Model with homogeneous labour and wealth taxes

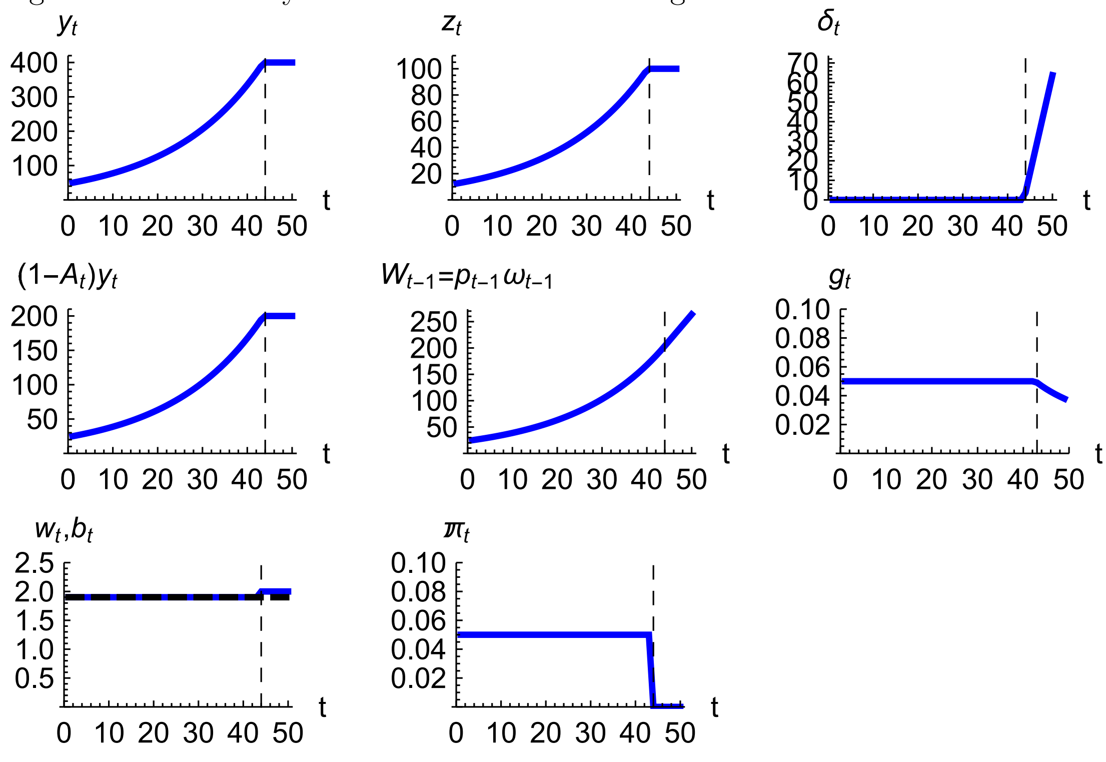


Figure 117 reports the dynamics of exploitation over the course of the simulation. Similar to the model with homogeneous labour, there is a consistent structure of exploitation as long as the simulation remains capital constrained. Figure 117(a) shows that as the simulation evolves and taxes redistribute wealth, the number of exploiters decreases as the number of exploited agents rises. Figures 117(b) displays the distribution of ε_t^ν over t. In figure 117(b) there is a clear pattern of exploitation up until the point at which the economy becomes labour constrained. Agents who are exploited experience $\varepsilon_t^\nu > 1$ consistently until labour is abundant, and agents who are exploiters experience $\varepsilon_t^\nu < 1$ for all t while the economy is capital constrained. However, during the period of time in which the simulation is capital constrained the exploitation intensity experienced by agents at the top of the wealth distribution increases, thereby bringing all agents closer together in terms of their experience of exploitation. Uniformity in ε_t^ν is not achieved until the simulation is labour constrained.

Figure 117: Exploitation status - Model with homogeneous labour and wealth taxes

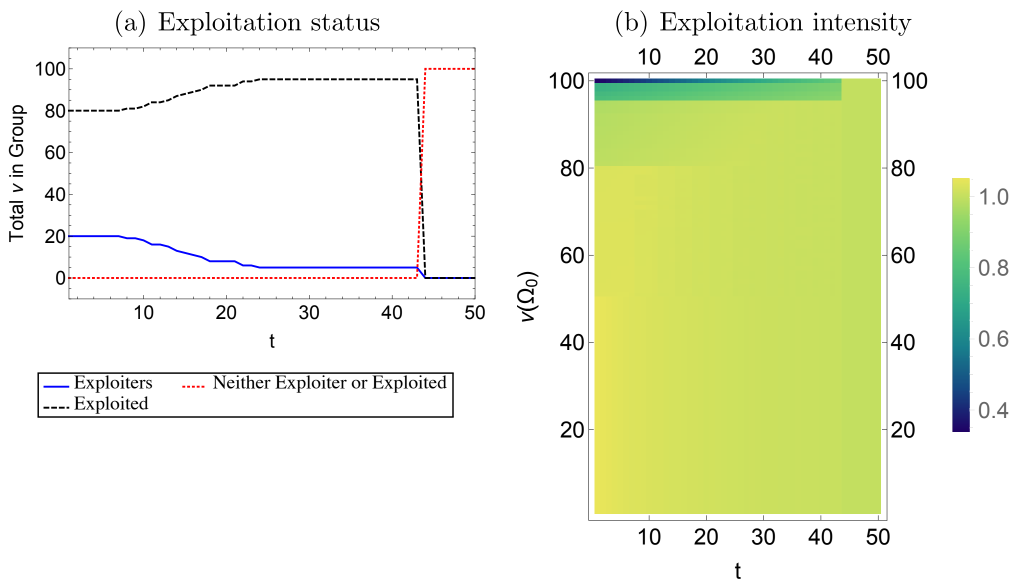
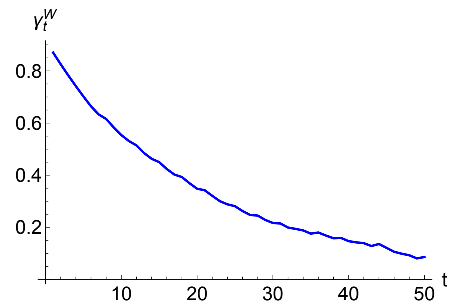


Figure 118(a) shows that the Gini coefficient of wealth steadily, and rapidly, decreases over t. This clearly shows how effective even small tax rates on wealth can be in reducing inequality. Figure 118(b) shows the distribution of wealth for select t.

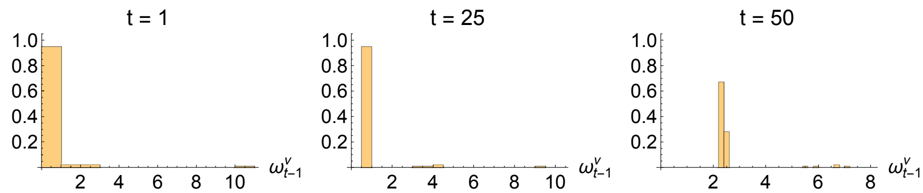
Figure 119 shows how the distribution of income $(1 + \pi_t)\omega_{t-1}^\nu + \widehat{w}_t\Lambda_t^\nu$ changes over the course of the simulation due to the taxes on wealth. Figure 119(a) shows the distribution of income shares across agents for all t. As the simulation unfolds and accumulation progresses, income becomes less unequal due to the redistribution of wealth via taxes. Once the simulation is labour constrained there is no income inequality since the income of all agents is $\widehat{w}_t\Lambda_t^\nu$ with $\widehat{w}_t$ such that $\pi_t = 0$ and uniform Λ_t^ν due to $s^\nu = 1$ for all ν. Figure 119(b) shows the evolution of the Gini coefficient

Figure 118: Distribution of wealth - Model with homogeneous labour and wealth taxes

(a) Gini coefficient of wealth



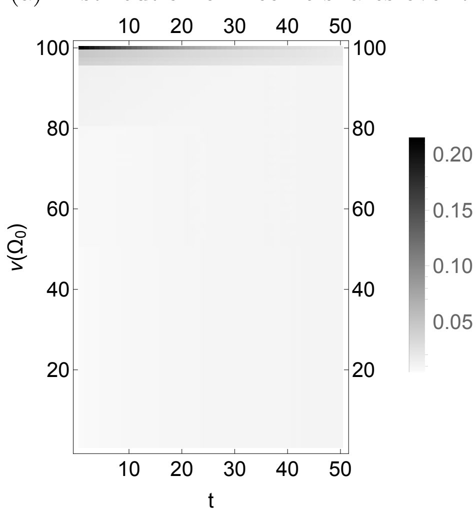
(b) Distribution of wealth for select t (relative frequency)



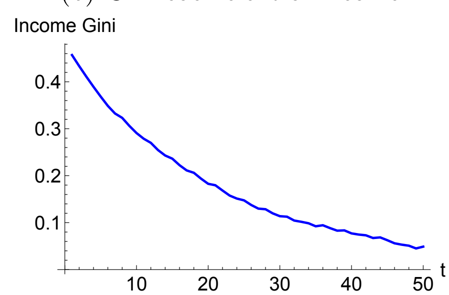
of income over t. As expected, the redistribution of wealth bolsters the non-labour income of agents who begin the simulation with little to no wealth, thereby rendering income more equal over time until the simulation becomes labour constrained.

Figure 119: Distribution of Income - Model with wealth taxes

(a) Distribution of income shares over t



(b) Gini coefficient of income



12.3 Model with homogeneous labour and wealth equality

The simulation below applies Rule 1 to the model with homogeneous labour. The initial parameters are $N = 100$, $A = 0.5$, $L = 0.25$, and $b = 1.9$. Figure 120 shows the aggregate results for the simulation. Consistent with other simulations, the implementation of Rule 1 has no impact on the macro-behaviour of the economy.

Figure 120: Summary results - Model with homogeneous labour and wealth equality

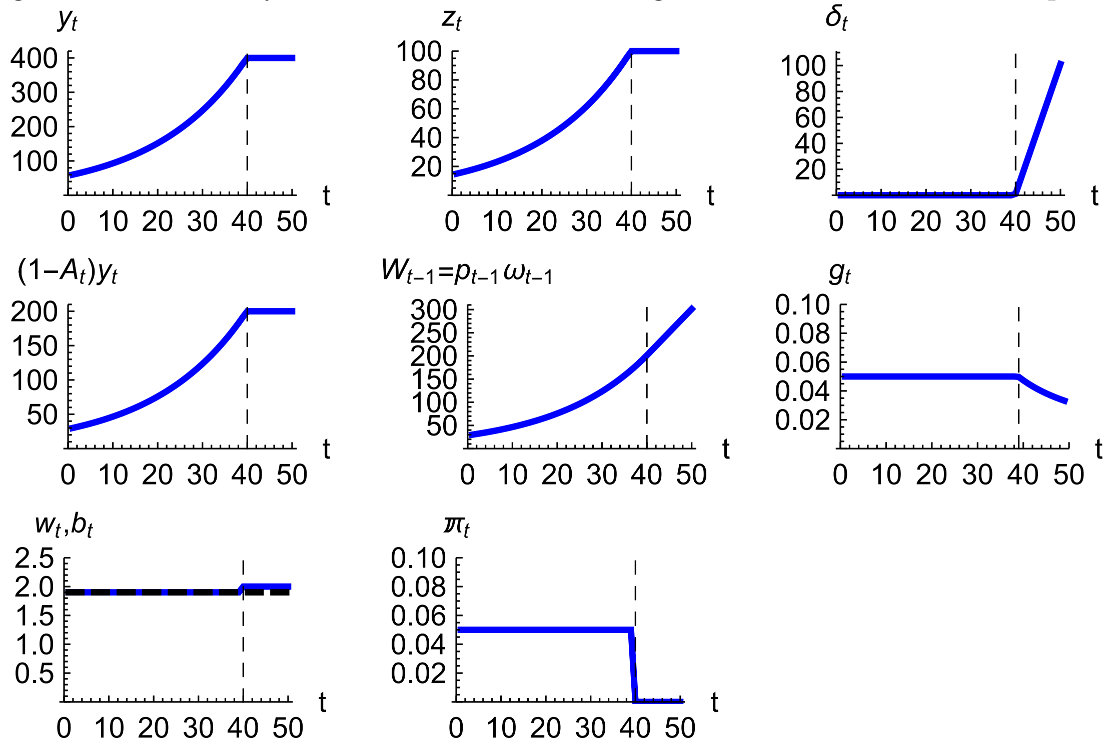


Figure 121 shows the exploitation status of agents. As Rule 1 is applied and wealth is redistributed, the number of exploited and exploiter agents become close as the change in the wealth distribution renders there ever little difference between exploited and exploiter agents. Figure 121(b) shows the exploitation intensity across agents. As these figures show, the redistribution of wealth quickly induces uniformity in exploitation intensity.

Figure 122 displays the dynamics of the distribution of wealth over the simulation. Figure 122(a) shows that the post- and pre-tax γ_t^W quickly drops to zero with the application of Rule 1. Figure 122(b) shows the post-tax distribution of wealth for select t.

Figure 123 shows the dynamics of the distribution of income over the simulation. Figure 123(a) shows the distribution of shares income over the simulation, which

Figure 121: Exploitation status - Model with homogeneous labour and wealth equality

(a) Post-tax Exploitation status (b) Post-tax Exploitation intensity

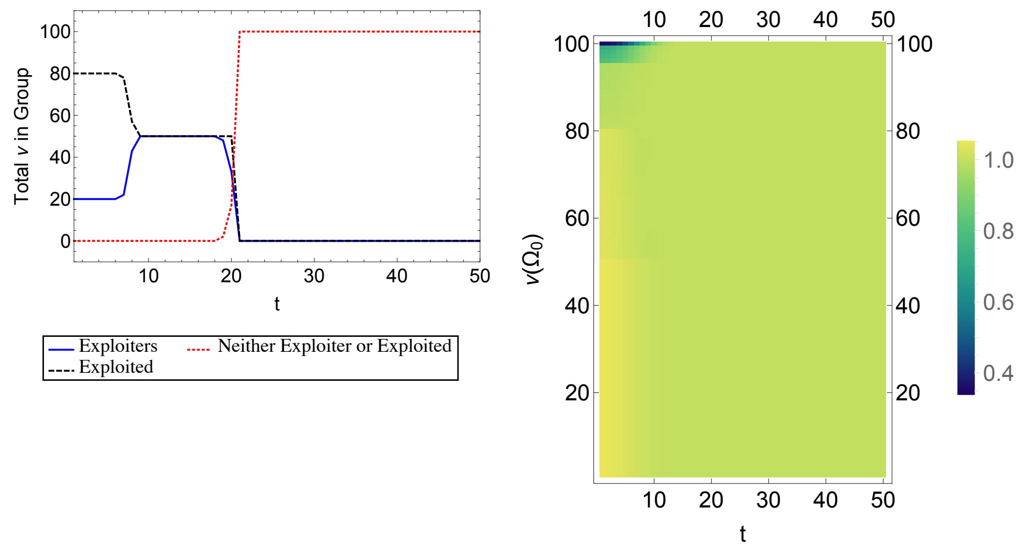
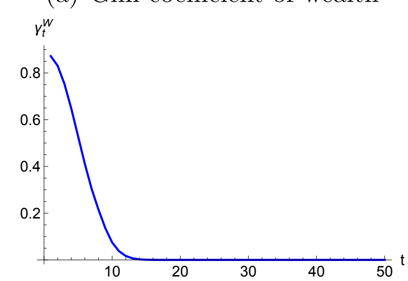
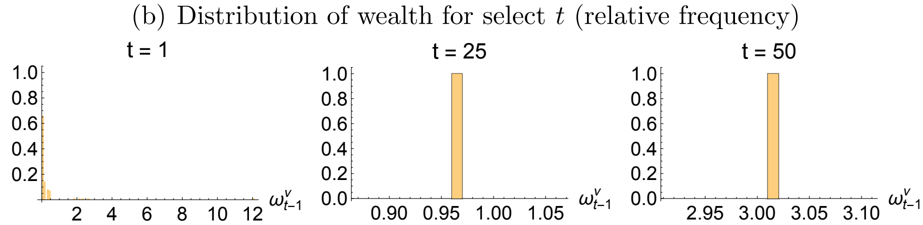


Figure 122: Distribution of wealth - Model with homogeneous labour and wealth equality

(a) Gini coefficient of wealth



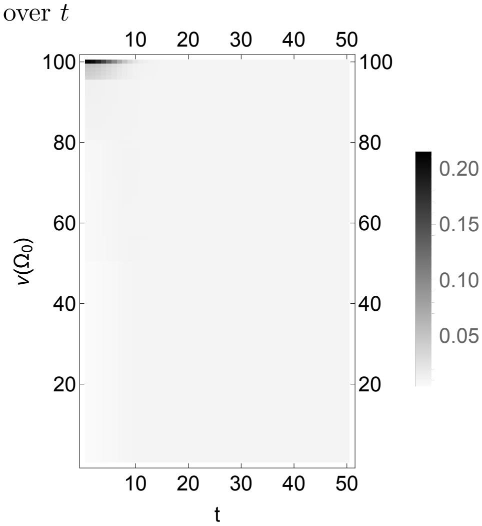
(b) Distribution of wealth for select t (relative frequency)



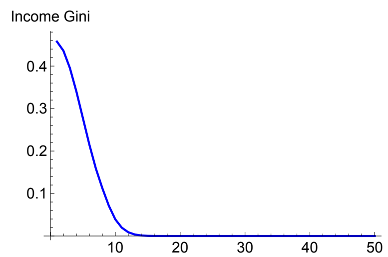
quickly collapses as wealth becomes uniform. Figure 123(b) shows the Gini coefficient of income.

Figure 123: Distribution of Income - Model with wealth equality

(a) Distribution of post-tax income shares over t



(b) Gini coefficient of income



12.4 Model with homogeneous labour and socialist allocation

The simulation below applies Rule 2 to the model with homogeneous labour. The initial parameters are $N = 100$, $A = 0.5$, $L = 0.25$, and $b = 1.9$. Figure 124 shows the aggregate results for the simulation. Consistent with other simulations, the implementation of Rule 2 has no impact on the macro-behaviour of the economy.

Figure 124: Summary results - Model with homogeneous labour and socialist allocation

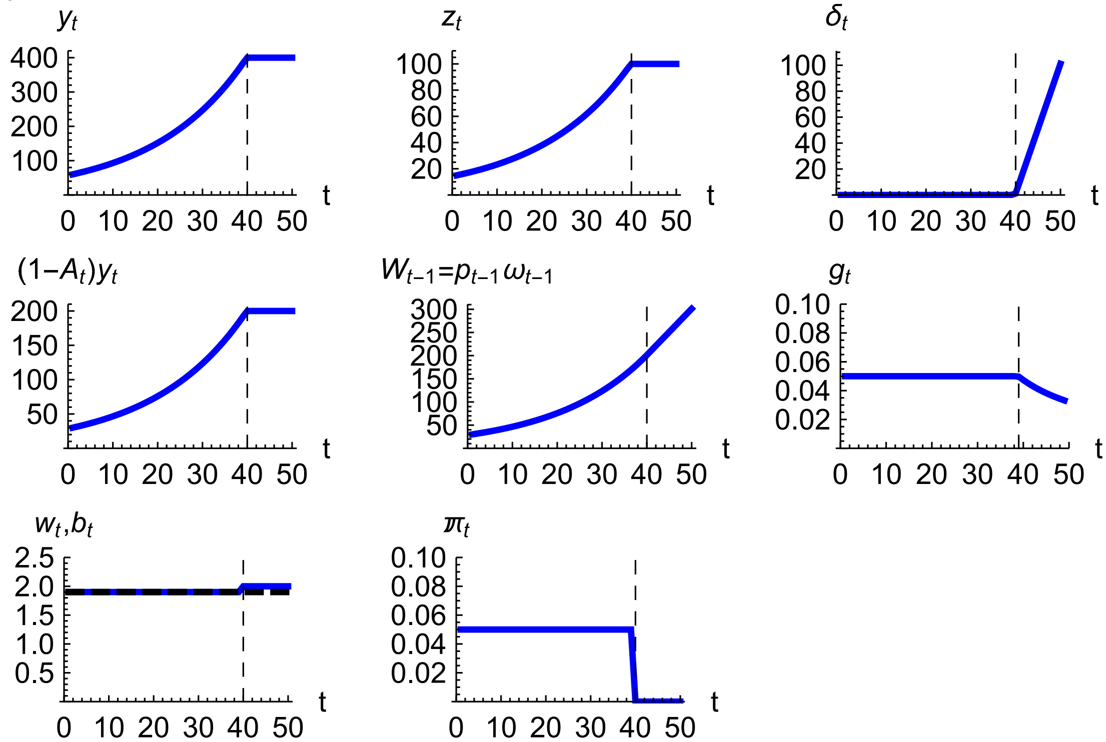


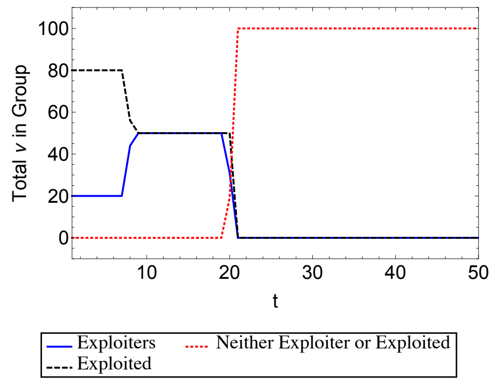
Figure 125 shows the exploitation status of agents. As Rule 2 is applied and wealth is redistributed, the number of exploited and exploiter agents become close as the change in the wealth distribution renders there ever little difference between exploited and exploiter agents. Figure 125(b) shows the exploitation intensity across agents. As these figures show, the redistribution of wealth quickly induces uniformity in exploitation intensity.

Figure 126 displays the dynamics of the distribution of wealth over the simulation. Figure 126(a) shows that the post-tax γ_t^W quickly drops to zero with the application of Rule 2. Figure 126(b) shows the post-tax distribution of wealth for select t.

Figure 127 shows the dynamics of the distribution of income over the simulation. Figure 127 shows the distribution of shares of post-tax income over the simulation,

Figure 125: Exploitation status - Model with homogeneous labour and socialist allocation

(a) Post-tax Exploitation status



(b) Post-tax Exploitation intensity

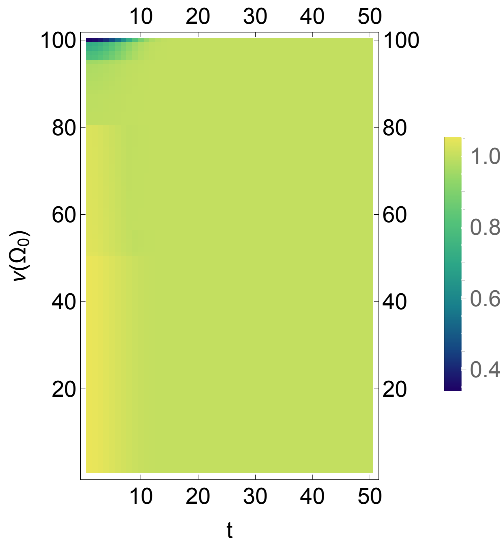
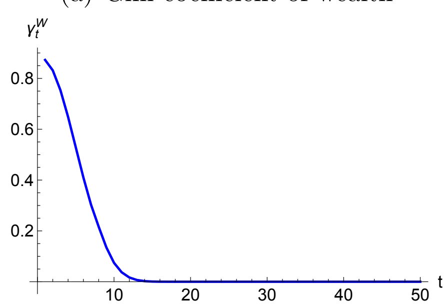
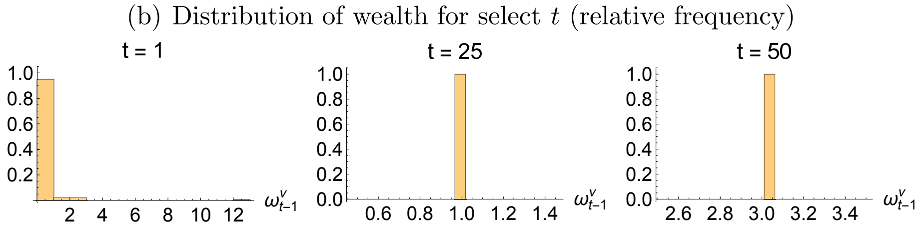


Figure 126: Distribution of wealth - Model with homogeneous labour and socialist allocation

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

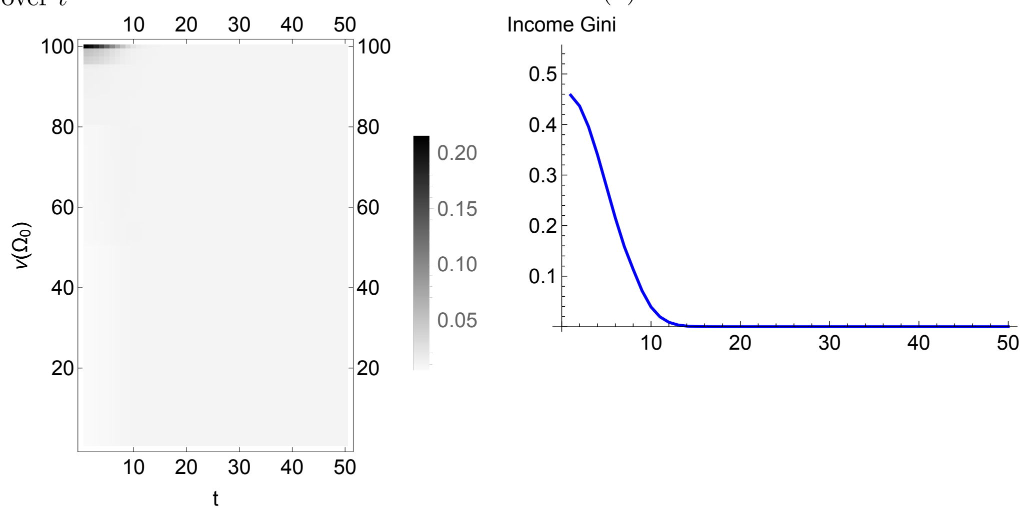


which quickly collapses as wealth becomes uniform. Figure 127(b) shows the post-tax Gini coefficient of income.

Figure 127: Distribution of Income - Model with homogeneous labour and socialist allocation

(a) Distribution of post-tax income shares over t

(b) Gini coefficient of income



12.5 Class in the model with homogeneous labour

The figures below display the dynamics of class over the simulations with homogeneous labour. Class status is determined according to Corollary 1 of Theorem 3. The figures below also show the CECP according to Theorem 4.

Figure 128: Class - Model with homogeneous labour

(a) Class status (b) CECP

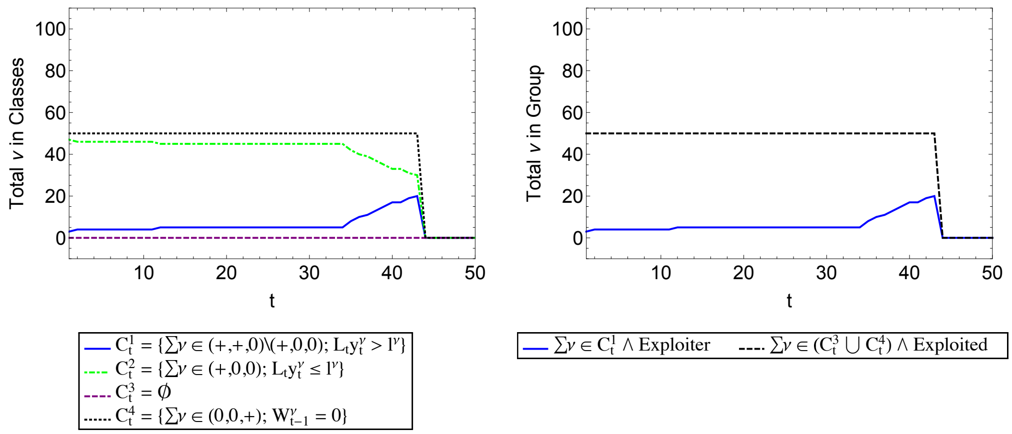


Figure 129: Class - Model with homogeneous labour and wealth taxes

(a) Class status (b) CECP

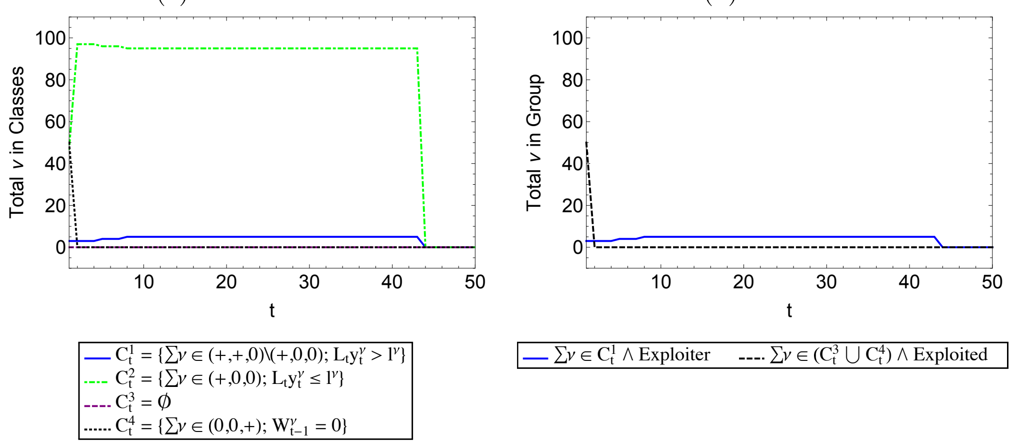


Figure 130: Class - Model with homogeneous labour and wealth equality

(a) Class status (b) CECP

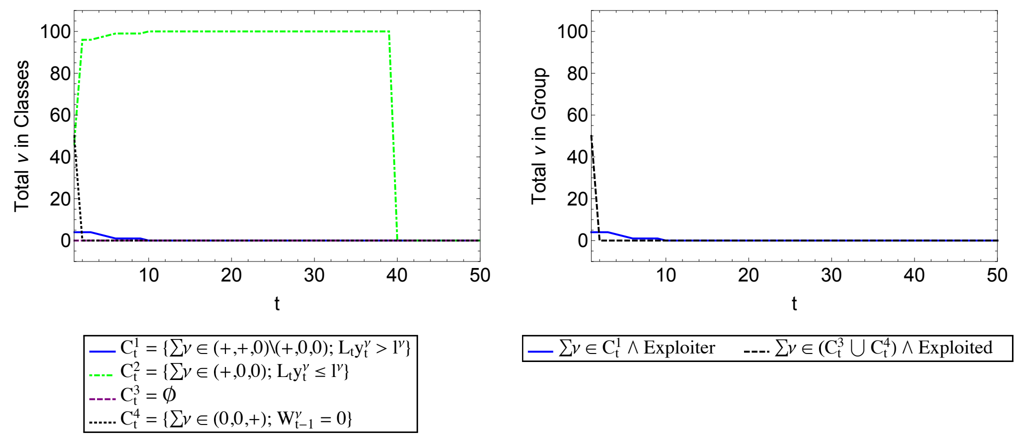
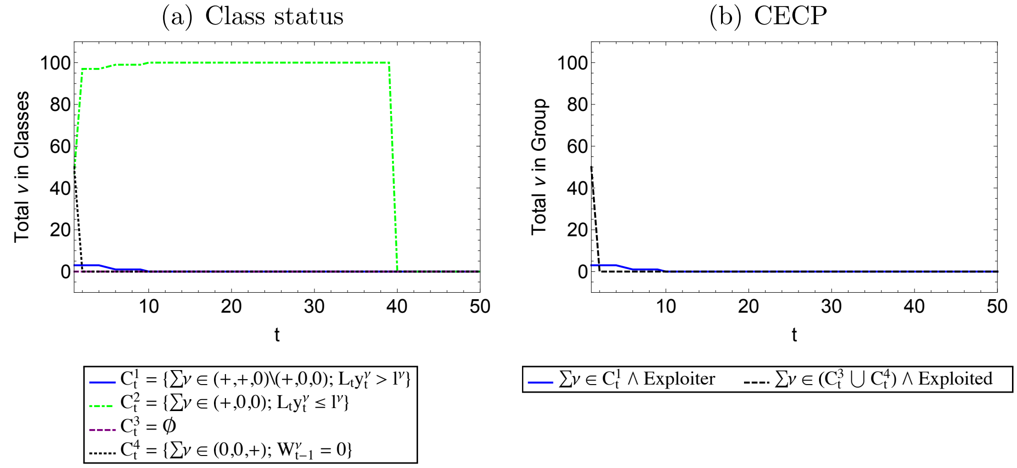


Figure 131: Class - Model with homogeneous labour and socialist allocation

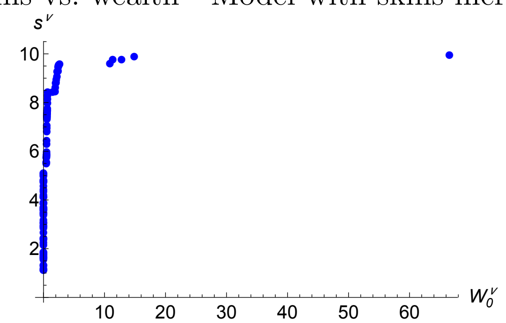
(a) Class status (b) CECP



13 Skills increasing in wealth

The simulations below introduce an alternative distribution of skills into the simulations of the paper to explore the robustness of our main results. The distribution of skills in each of the simulations in this section is such that s^ν are increasing in ω_0^ν. Specifically, at $t = 0$, s^ν are randomly drawn from the uniform distribution from 1 to 10, $s^\nu \in [1, 10]$, and then sorted so that they are increasing in agent's initial wealth ω_0^ν. This means that propertyless agents have skills ranging from the lowest in $\mathcal{N}$ to around the average, whereas the wealthiest agents at $t = 0$ are also the most skilled. The determination of Ω_0 in the simulations that follow is handled in the same manner as in the paper. A sample distribution of s^ν by ω_0^ν is provided in figure 132

Figure 132: Skills vs. wealth - Model with skills increasing in wealth



13.1 Model - skills increasing in wealth

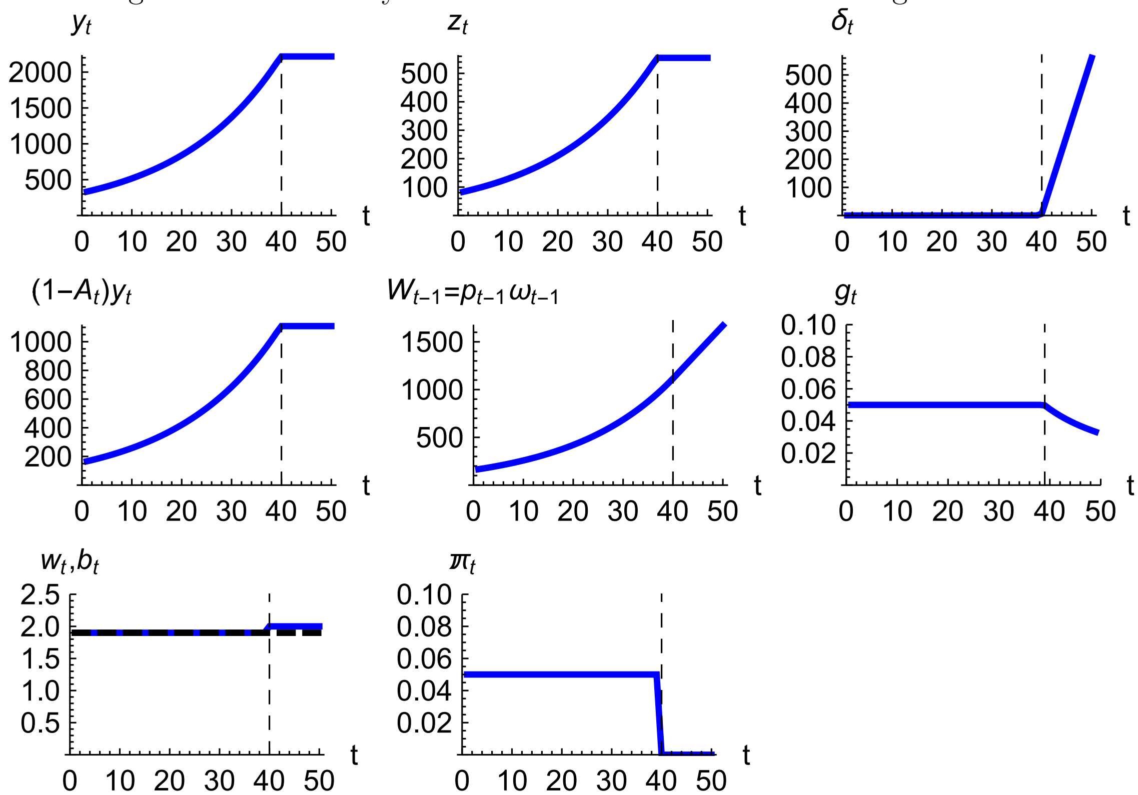
The simulation below implements the above described distribution of skills in the model. The initial parameters are $A = 0.5$, $L = 0.25$, $b = 1.9$, and $N = 100$. The simulation is initially capital constrained, starting far from the knife-edge.

Figure 133 shows the aggregate results. Consistent with the results of the model in the paper, there is steady growth in activity levels (x_t, y_t, δ_t), net output, and aggregate wealth while the simulation is capital constrained. The profit and accumulation rates are also stable until the simulation become labour constrained.

Figure 134 shows the dynamics of exploitation over the course of the simulation. As is clear in figure 134(a), there is a stable structure of exploitation while the simulation is capital constrained. Similarly, the distribution of exploitation intensity ε_t^ν is stable, with clear demarcations in agents' exploitation status until the simulation becomes labour constrained and ε_t^ν goes to 1 for all agents.

Figure 135 shows the dynamics of the distribution of wealth. As figure 135(a) shows, γ_t^W remains stable over the course of the simulation until the labour constraint

Figure 133: Summary results - Model with skills increasing in wealth

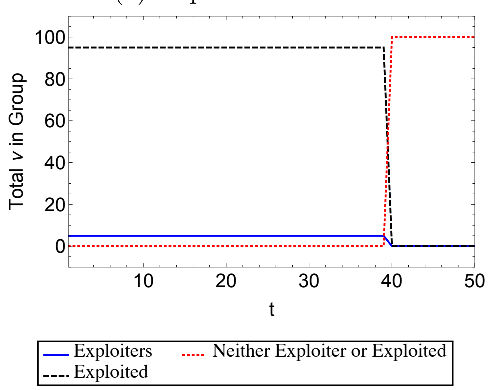


is binding, at which point it steadily declines. Figure 135(b) shows the distribution of wealth for select t.

Figure 136 shows the distribution of income over the simulation. Figure 136(a) shows a stable distribution of income shares over the course of the simulation, which is confirmed in the steady Gini coefficient of income in figure 136(b).

Figure 134: Exploitation - Model with skills increasing in wealth

(a) Exploitation status



(b) ε_t^ν

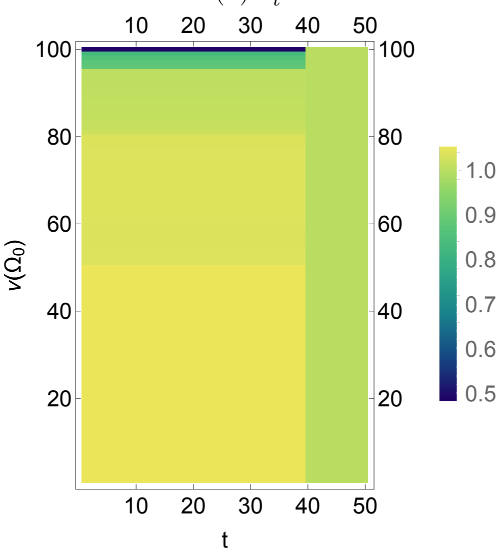
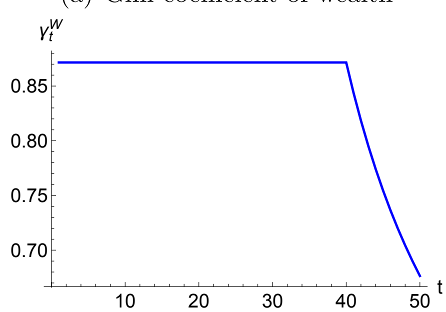


Figure 135: Distribution of wealth - Model with skills increasing in wealth

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

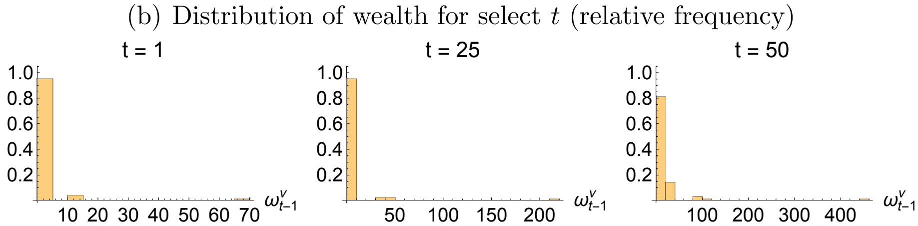
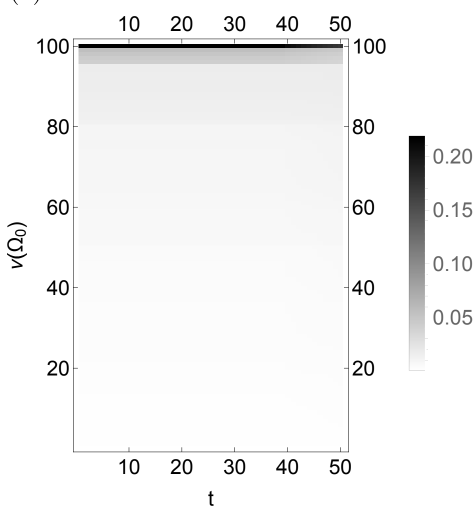
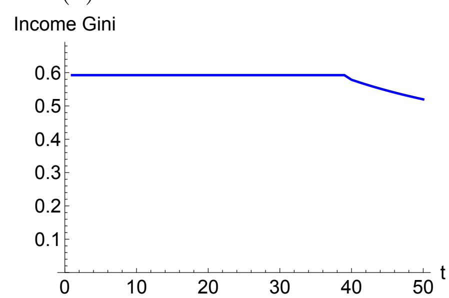


Figure 136: Distribution of income - Model with skills increasing in wealth

(a) Distribution of income shares over t



(b) Gini coefficient of income



13.2 Model with skills increasing in wealth and wealth taxes

The simulation below incorporates Piketty-type wealth taxes into the previous simulation of the model with skills increasing in initial wealth. The initial parameters are $A = 0.5$, $L = 0.25$, $b = 1.9$, and $N = 100$. The simulation is initially capital constrained, starting far from the knife-edge.

Figure 137 shows the aggregate results. Consistent with the results of the model in the paper, there is steady growth in activity levels (x_t, y_t, δ_t), net output, and aggregate wealth while the simulation is capital constrained. The profit and accumulation rates are also stable until the simulation become labour constrained.

Figure 137: Summary results - Model with skills increasing in wealth and wealth taxes

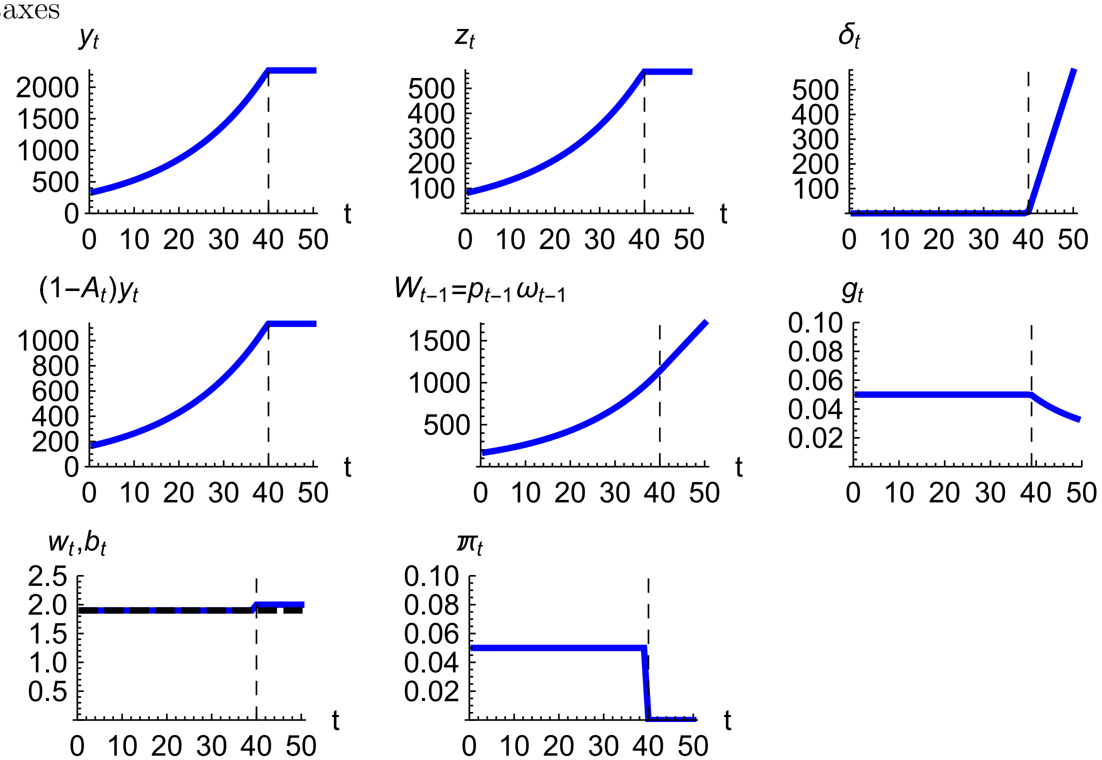
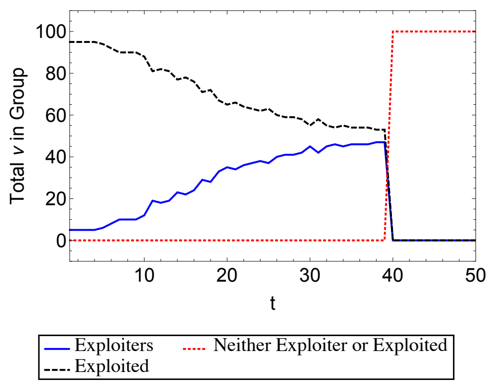


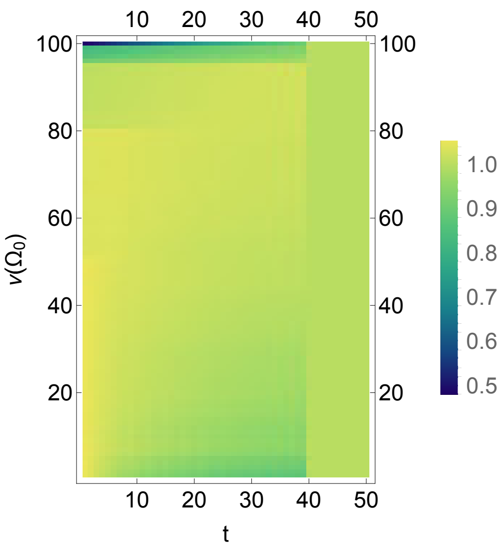
Figure 138 shows the dynamics of exploitation over the course of the simulation. As is clear in figure 138(a), wealth taxes quickly alter the number of exploited and exploiter agents while the simulation is capital constrained. Similarly, the distribution of exploitation intensity ε_t^{ν} shows that wealth taxes quickly alleviate exploitation intensity for propertyless agents at the bottom of the initial wealth distribution, shifting the burden of exploitation to agents with moderate-to-high skills and some wealth, until the simulation becomes labour constrained and ε_t^{ν} goes to 1 for all

Figure 138: Exploitation - Model with skills increasing in wealth and wealth taxes

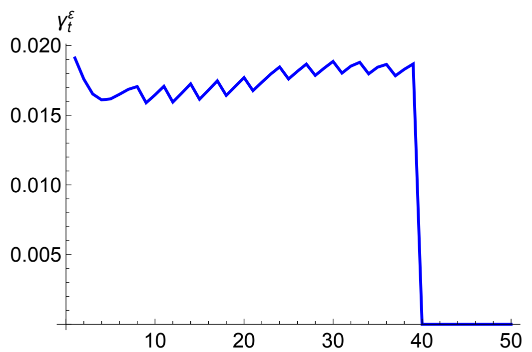
(a) Post-tax Exploitation status



(b) Post-tax ε_t^ν



(c) Gini coefficient of ε_t^ν



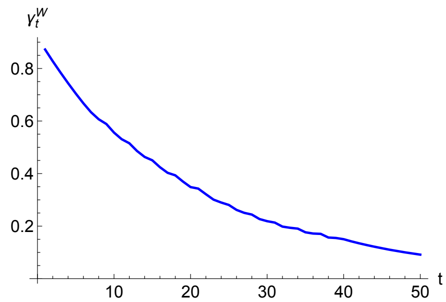
agents, as shown in figures 138(b). Figure 138(c) shows γ_t^ε, which increases over the course of the simulation until the simulation becomes labour constrained and exploitation ceases. The sawtooth pattern in γ_t^ν is due to groups of agents shifting their position in the tax scheme and experiencing different taxes rates over the course of the simulation.

Figure 139 shows the dynamics of the distribution of wealth. As figure 139(a) shows, γ_t^W quickly falls as wealth is redistributed through taxation. Figure 139(b) shows the distribution of post-tax wealth for select t.

Figure 140 shows the distribution of income over the simulation. Figures 140(a) and 28(c) show that the distribution of income is gradually compressed over the simulation, which is confirmed in the declining Gini coefficient in figure 140(b).

Figure 139: Distribution of wealth - Model with skills increasing in wealth and wealth taxes

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

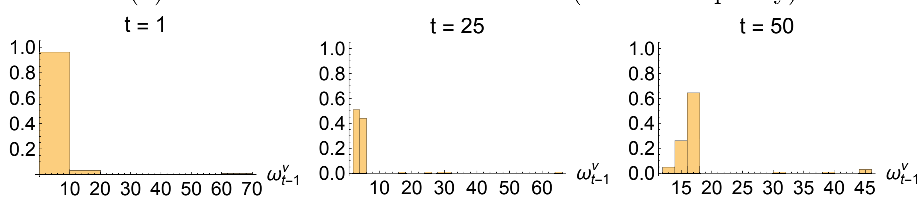
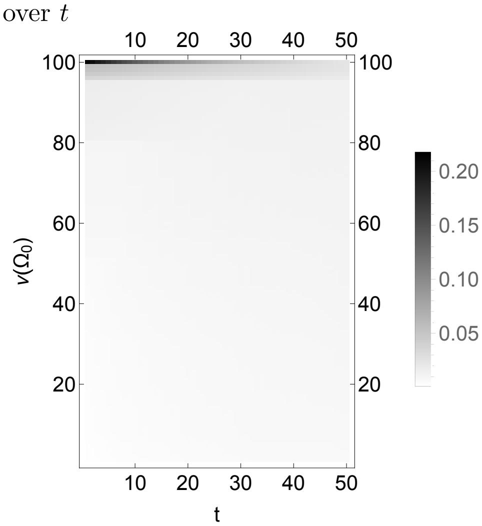
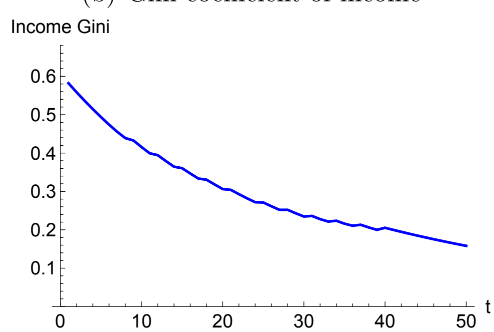


Figure 140: Distribution of income - Model with skills increasing in wealth and wealth taxes

(a) Post-tax Distribution of income shares over t



(b) Gini coefficient of income



13.3 Model with skills increasing in wealth and wealth equality

The simulation below applies Rule 1 to the simulation of the model with skills increasing in initial wealth. The initial parameters are $A = 0.5$, $L = 0.25$, $b = 1.9$, and $N = 100$. The simulation is initially capital constrained, starting far from the knife-edge.

Figure 141 shows the aggregate results. Consistent with the results of the model in the paper, there is steady growth in activity levels (x_t, y_t, δ_t), net output, and aggregate wealth while the simulation is capital constrained. The profit and accumulation rates are also stable until the simulation become labour constrained.

Figure 141: Summary results - Model with skills increasing in wealth and wealth taxes

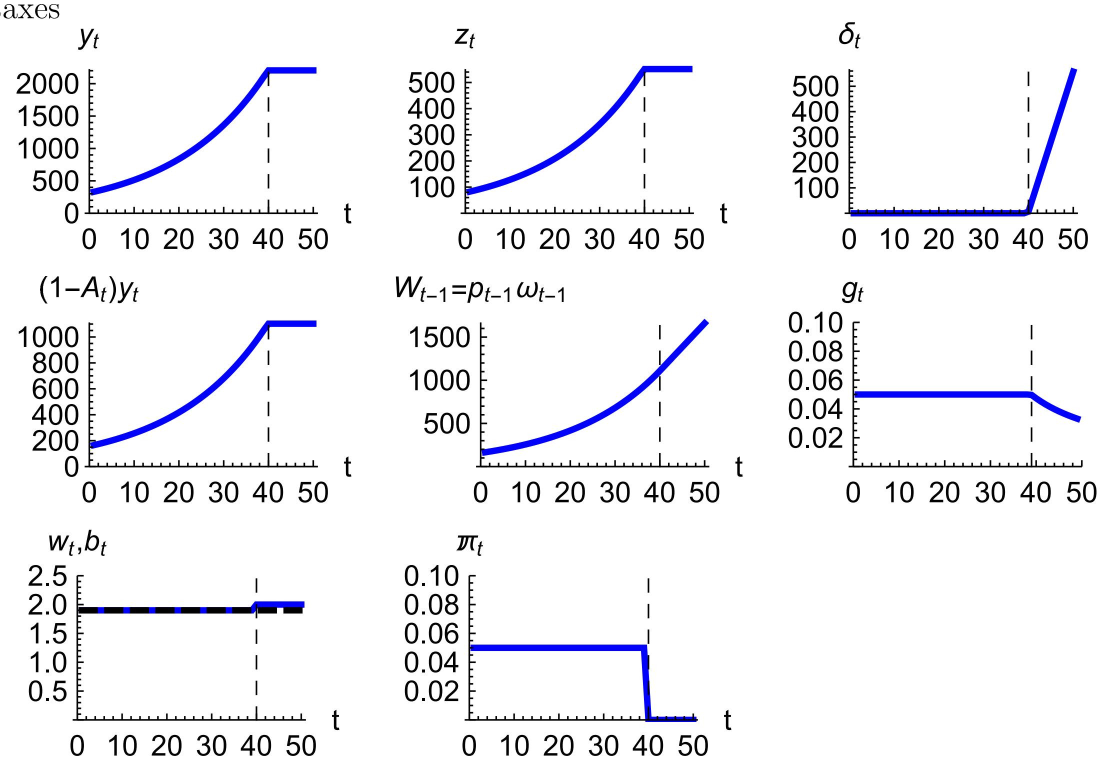


Figure 142 shows the dynamics of exploitation over the course of the simulation. As is clear in figure 142(a), wealth taxes using Rule 1 quickly alter the number of exploited and exploiter agents while the simulation is capital constrained. Similarly, the distribution exploitation intensity ε_t^ν shows that Rule 1 quickly alleviates exploitation intensity for propertyless agents at the bottom of the initial wealth distribution, shifting the burden of exploitation to agents with high skills, until the simulation becomes labour constrained and ε_t^ν goes to 1 for all agents, as shown in figures 142(b) and 29(b). Figure 142(c) shows γ_t^ε, which initially decreases and then rises to a stable value as wealth equality is achieved.

Figure 142: Exploitation - Model with skills increasing in wealth and wealth taxes

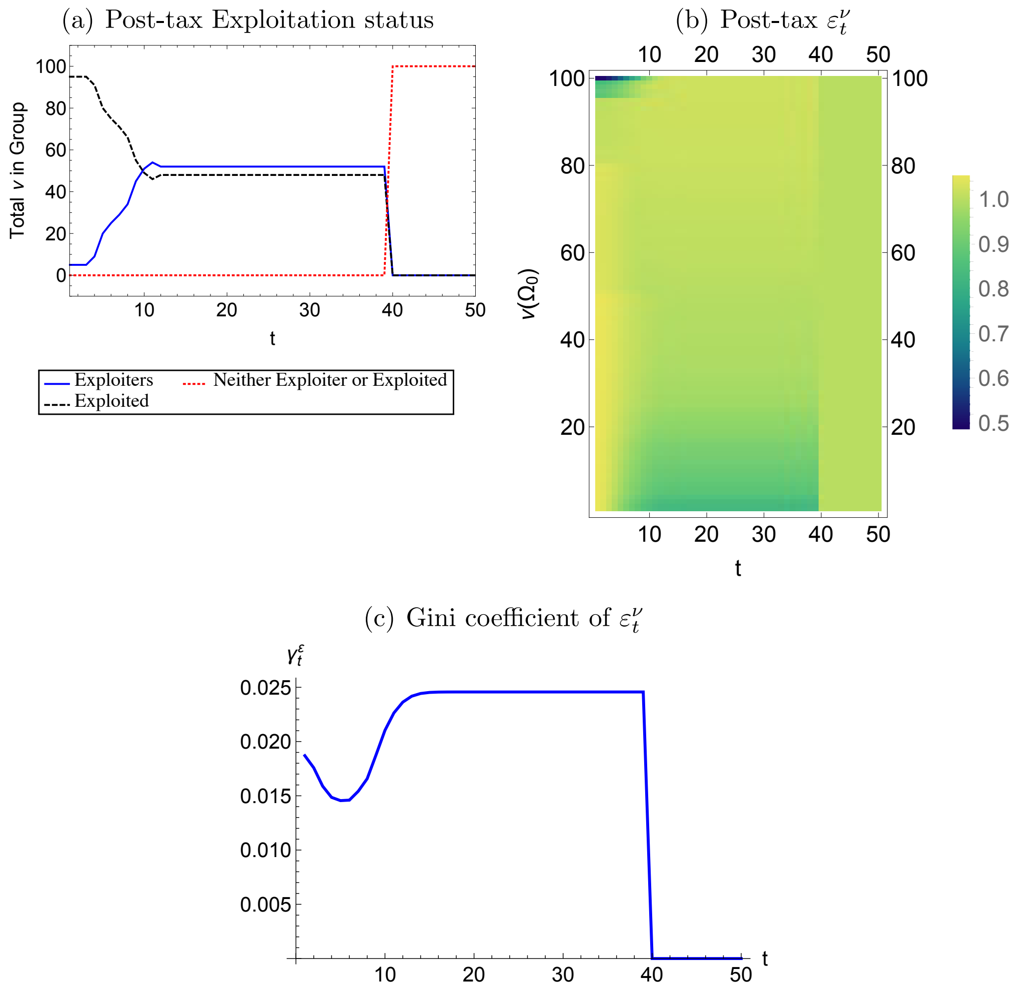
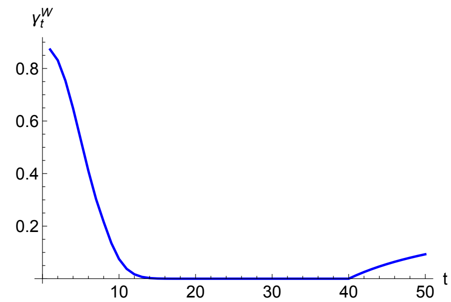


Figure 143 shows the dynamics of the distribution of wealth. As figure 143(a) shows, γ_t^W quickly falls to zero as wealth is redistributed. Figure 143(b) shows the distribution of wealth for select t.

Figure 144 shows the distribution of income over the simulation. Figure 144(a) show that the distribution of income is quickly compressed over the simulation, which is confirmed in the declining Gini coefficient of income in figure 144(b). Note that the realization of wealth equality does not entail income equality. The income inequality shown in figure 144(b), while small, is the result of the distribution of skills.

Figure 143: Distribution of wealth - Model with skills increasing in wealth and wealth taxes

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

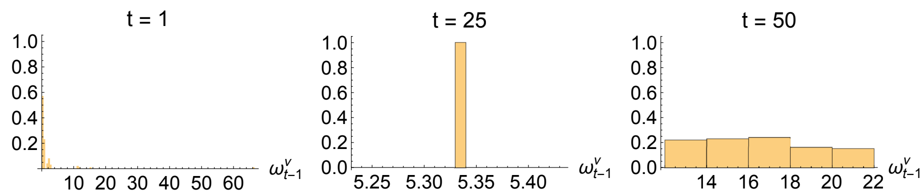
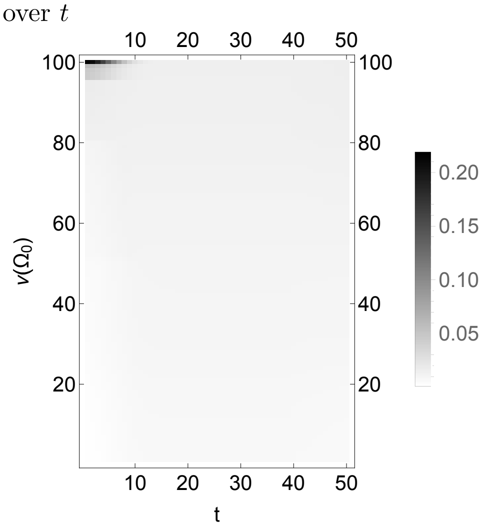
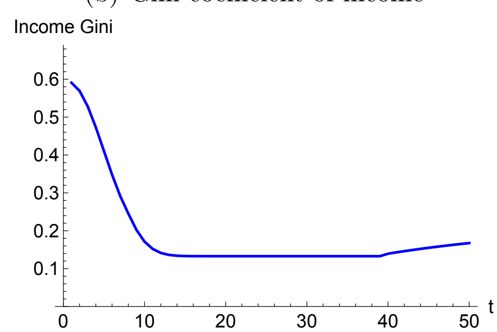


Figure 144: Distribution of income - Model with skills increasing in wealth and wealth taxes

(a) Post-tax Distribution of income shares over t



(b) Gini coefficient of income



13.4 Model with skills increasing in wealth and socialist allocation

The simulation below applies Rule 2 to the simulation of the model with skills increasing in initial wealth. The initial parameters are $A = 0.5$, $L = 0.25$, $b = 1.9$, and $N = 100$. The simulation is initially capital constrained, starting far from the knife-edge.

Figure 145 shows the aggregate results. Consistent with the results of the model in the paper, there is steady growth in activity levels (x_t, y_t, δ_t), net output, and aggregate wealth while the simulation is capital constrained. The profit and accumulation rates are also stable until the simulation become labour constrained.

Figure 145: Summary results - Model with skills increasing in wealth and socialist allocation

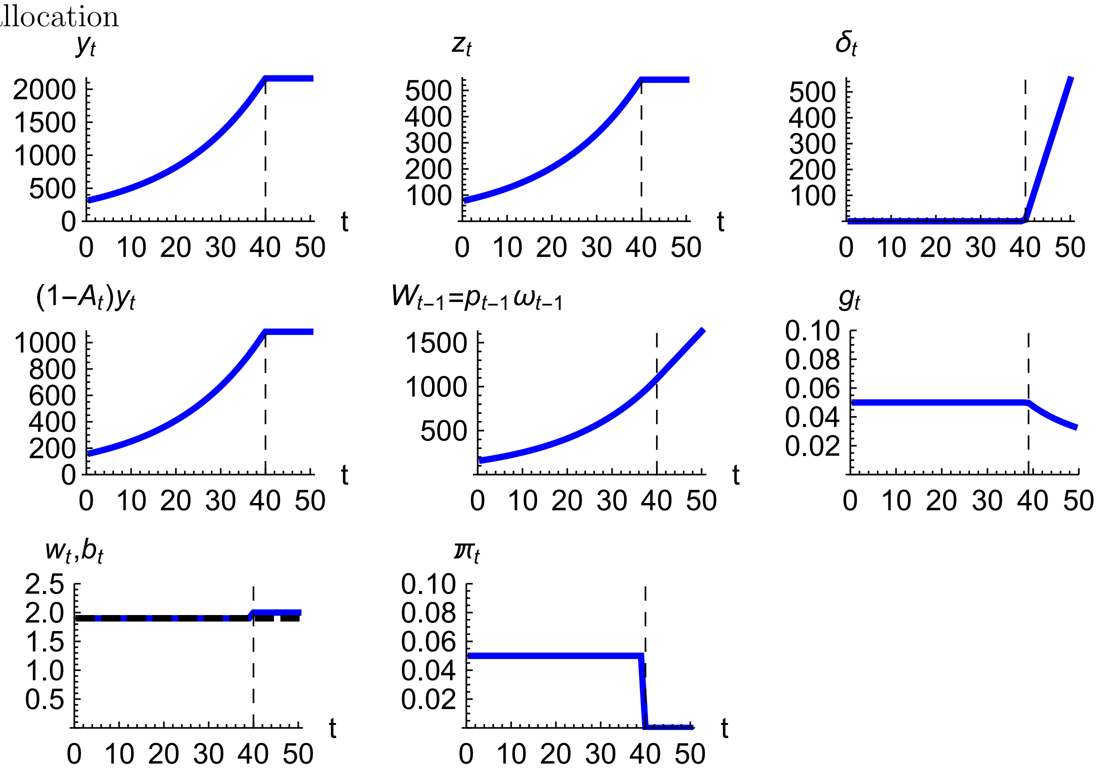


Figure 146 shows the dynamics of exploitation over the course of the simulation. As is clear in figure 146(a), wealth taxes using Rule 2 quickly alter the number of exploited and exploiter agents and quickly achieve the end of exploitation with the realization of a socialist allocation. Similarly, the distribution of exploitation intensity ε_t^ν shows that Rule 2 quickly eliminates exploitation with ε_t^ν going to 1 for all agents, as shown in figures 146(b) and 30(b). Figure 146(c) shows γ_t^ε, which quickly falls as

Rule 2 eliminates exploitation.

Figure 146: Exploitation - Model with skills increasing in wealth and socialist allocation

(a) Post-tax Exploitation status

(b) Post-tax ε_t^ν

(c) Gini coefficient of ε_t^ν

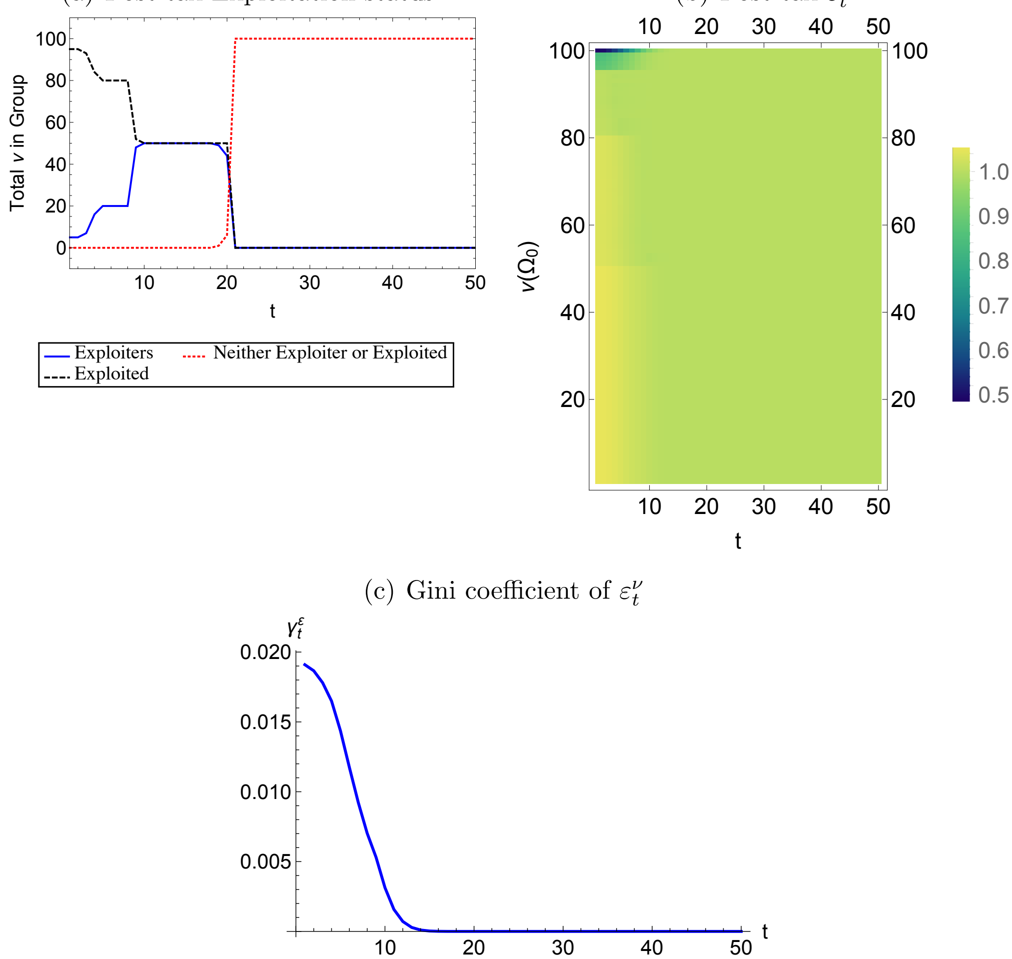
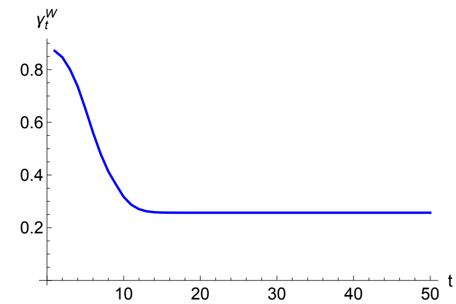


Figure 147 shows the dynamics of the distribution of wealth. As figure 147(a) shows, γ_t^W quickly falls to a stable value as the socialist allocation is achieved. As expected, the socialist allocation does not entail wealth equality, rather the socialist allocation entails a distribution of wealth consistent with each agents' labour capacity l^ν. Thus, some wealth inequality is a necessity of the socialist allocation. Figure 147(b) shows the distribution of post-tax wealth for select t.

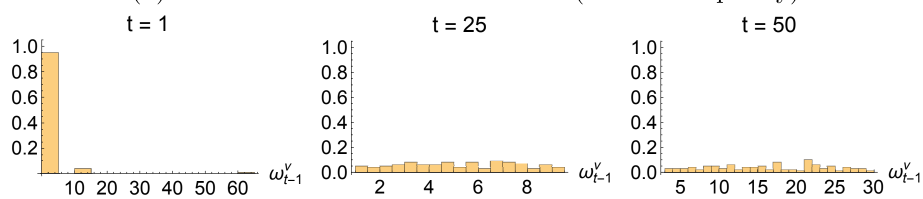
Figure 148 shows the distribution of income over the simulation. Figure 148(a) show that the distribution of income is quickly compressed over the simulation, which is confirmed in the declining Gini coefficient of income in figure 148(b). Note that

Figure 147: Distribution of wealth - Model with skills increasing in wealth and socialist allocation

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

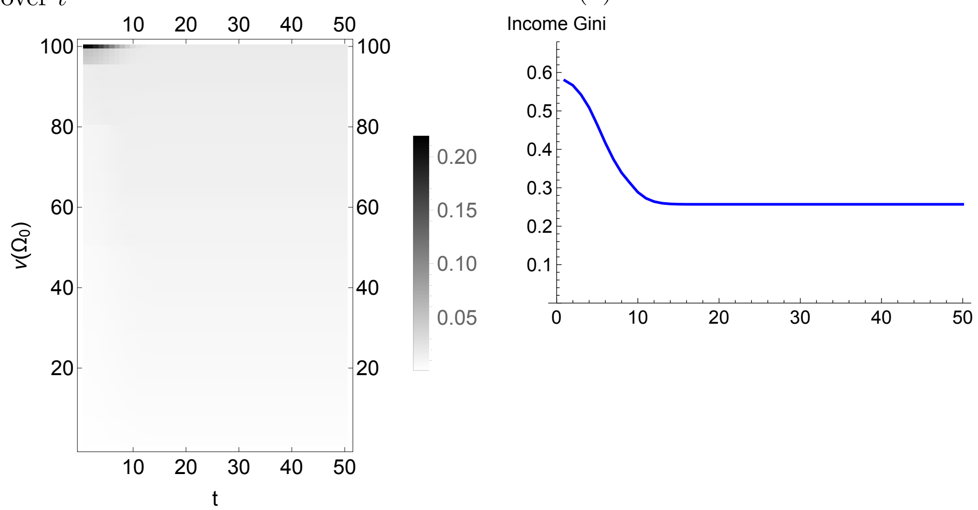


the realization of the socialist allocation does not entail income equality. The income inequality shown in figure 148(b) during the socialist phase of the simulation is the result of the distribution of skills (since wealth is distributed in proportion to skills) and is higher than that of the bourgeois equality allocation with a uniform wealth distribution.

Figure 148: Distribution of income - Model with skills increasing in wealth and socialist allocation

(a) Post-tax Distribution of income shares over t

(b) Gini coefficient of income



13.5 Class in the model with skills increasing in wealth

The figures below present the class dynamics of the simulations with skills increasing in Ω_0.

Figure 149: Class

(a) Class status (b) CECP

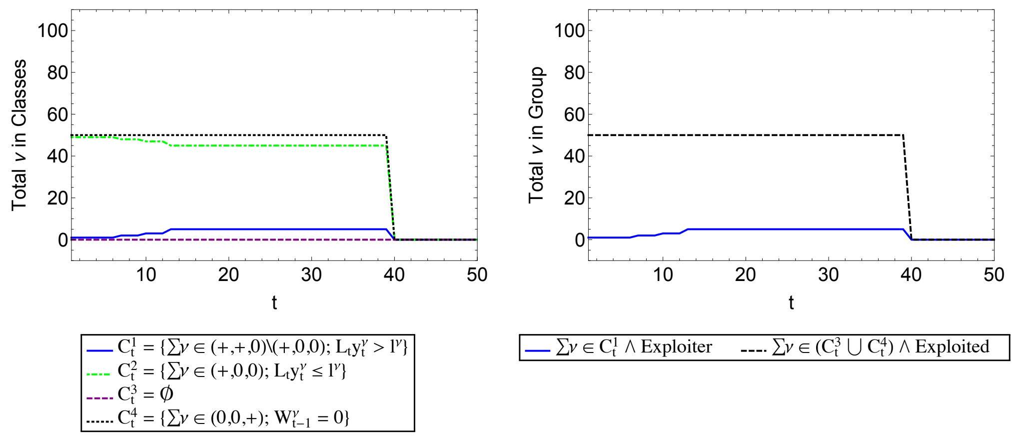


Figure 150: Class - Model with wealth taxes

(a) Class status (b) CECP

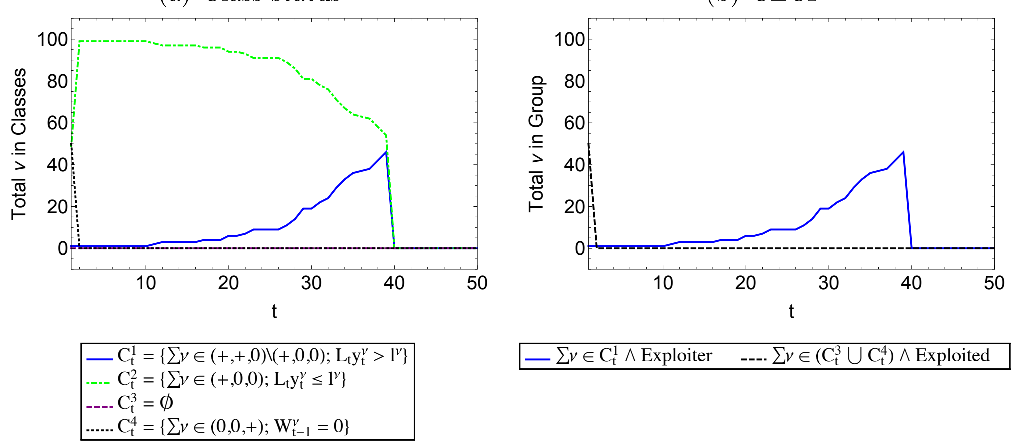


Figure 151: Class - Model with wealth equality

(a) Class status (b) CECP

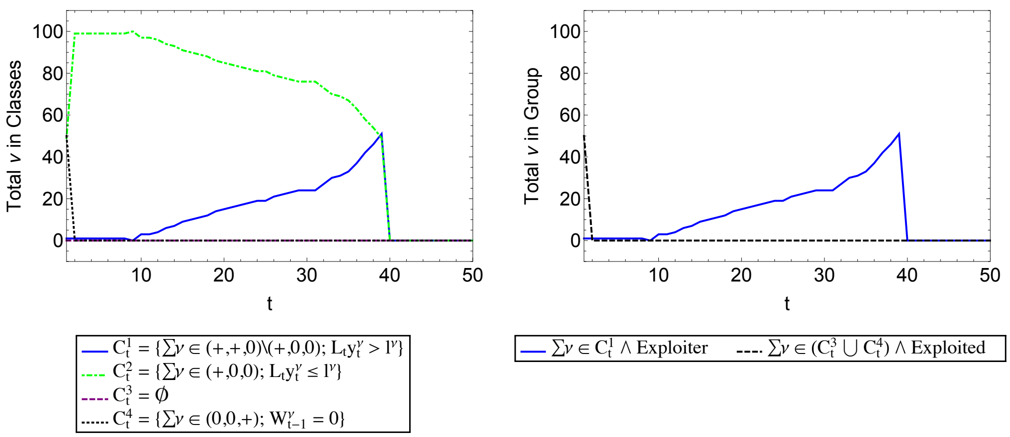
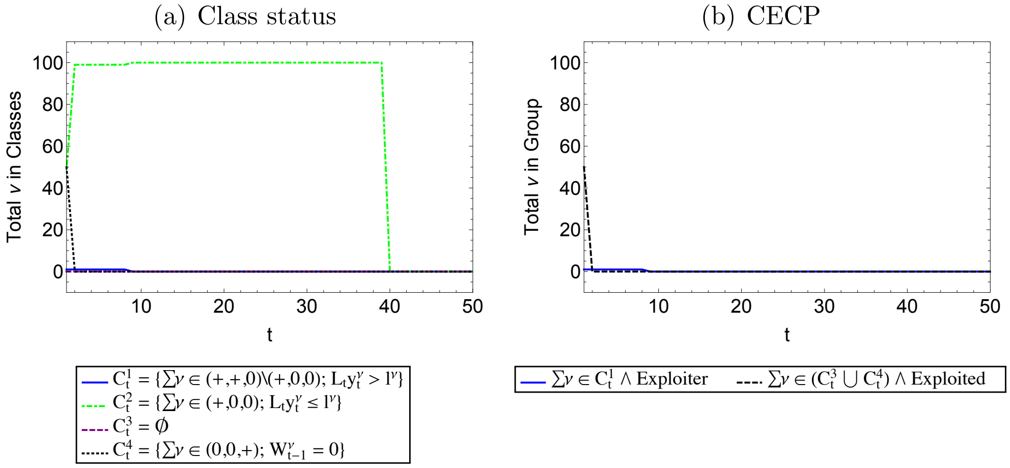


Figure 152: Class - Model with socialist allocation

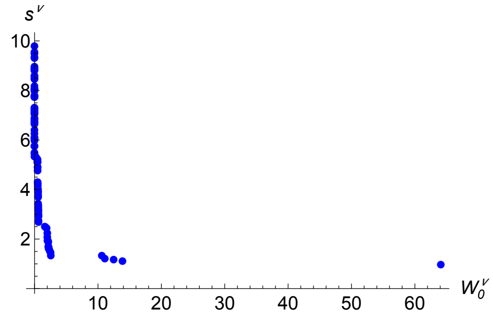
(a) Class status (b) CECP



14 Skills decreasing in wealth

The simulations below present results for the simulations run with the distribution of skills as a decreasing function of agents' initial wealth. The s^ν are drawn from a uniform distribution in the range of 1 to 10 and assigned to agents so that s^ν are decreasing in Ω_0, an example of skills in relation to Ω_0 is shown in figure 153.

Figure 153: Skills vs. wealth - Model with skilled-labour



14.1 Model with s^ν decreasing in wealth

The following presents the model of the paper with s^ν decreasing in agents' wealth ω_{t-1}^ν. The simulation begins with the parameters: $N = 100$, $A = 0.5$, $L = 0.25$, and $b = 1.9$. Figure 154 reports the summary results for the model. The simulation shows steady growth of activity levels (y_t, z_t), net output $(1 - A)y_t$, and output per capita $\frac{(1-A)y_t}{N}$ until the simulation becomes labour constrained. The growth rate of aggregate endowments and the profit rate are also steady as long as the simulation is capital constrained.

Figure 155 reports the dynamics of exploitation over the course of the simulation. Similar to the benchmark model, the structure of exploitation and classes is relatively stable as long as the simulation is capital constrained, but as soon as the simulation is labour constrained exploitation and classes disappear. Figures 155(b) displays the distribution of ε_t^ν over t. In figure 155(b) there is a clear pattern of exploitation up until the point at which the economy becomes labour constrained. Agents who are exploited experience $\varepsilon_t^\nu > 1$ consistently until labour is abundant, and agents who are exploiters experience $\varepsilon_t^\nu < 1$ for all t while the economy is capital constrained.

Figure 156(a) shows that the Gini coefficient of wealth is stable until the economy is labour constrained, at which point wealth inequality begins to steadily decline as all agents accumulate. Figure 156(b) shows the distribution of wealth for select t.

Figure 157 shows the distribution of net income over the course of the simulation.

Figure 154: Summary results - Model with skilled-labour

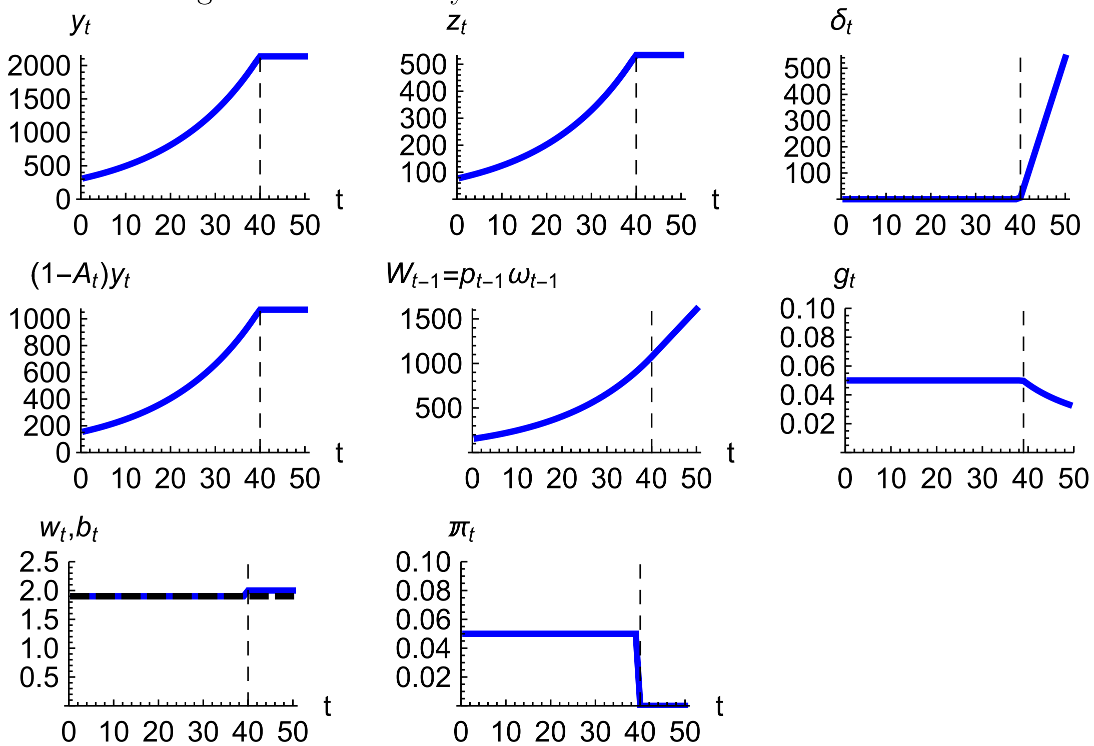


Figure 155: Exploitation - Model with skilled-labour

(a) Exploitation status

(b) ε_t^ν

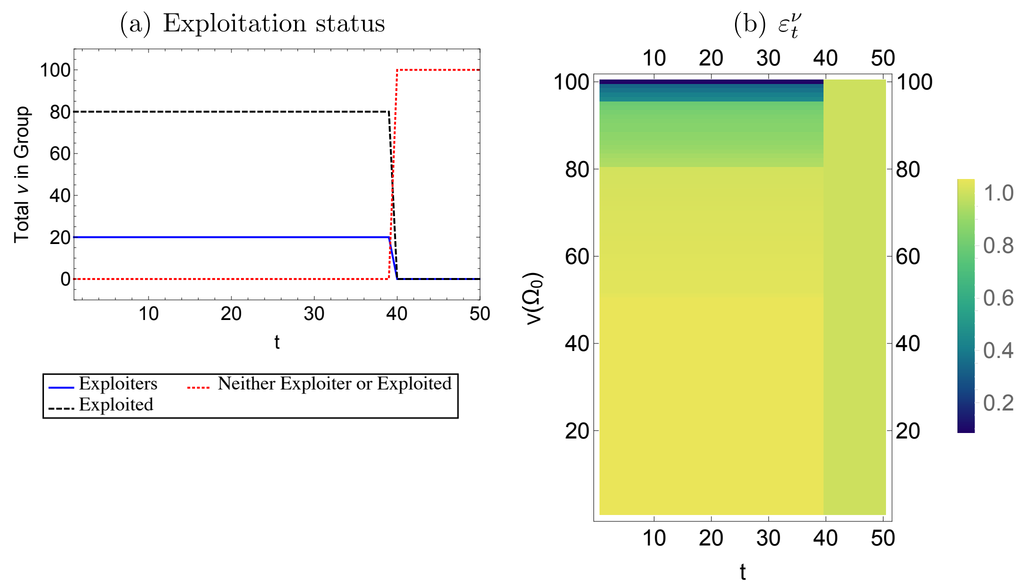
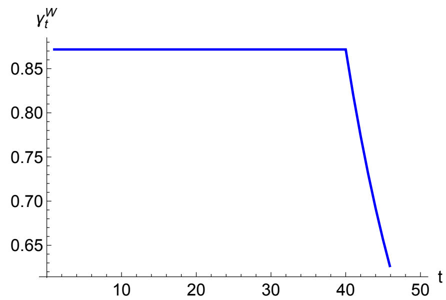


Figure 156: Distribution of wealth - Model with skilled-labour

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

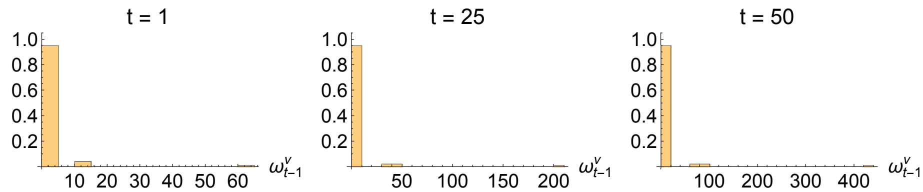
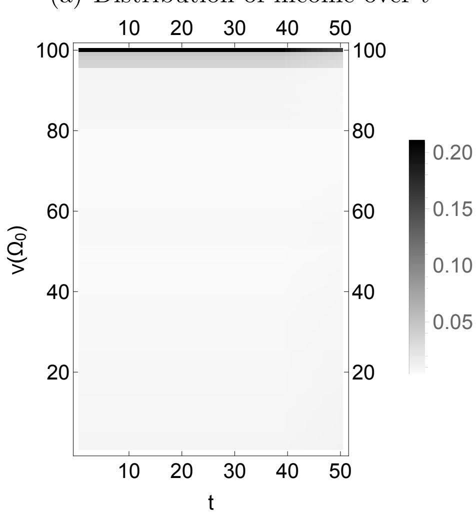
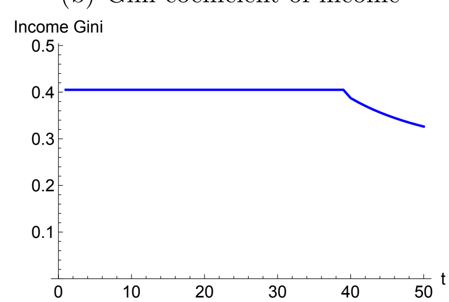


Figure 157: Distribution of income - Model with skilled-labour

(a) Distribution of income over t



(b) Gini coefficient of income



14.2 Model with s^ν decreasing in wealth and wealth taxes

The following presents the model with heterogeneous labour and wealth taxes, where s^ν is decreasing in agents' wealth ω^ν_{t-1}. The simulation begins with the same benchmark parameters as earlier simulations of the model. The relationship between s^ν and wealth is the same as the previous simulation. The addition of wealth taxes has no qualitative impact on the aggregate results of the model.

Figure 158: Summary results - Model with skilled-labour and wealth taxes

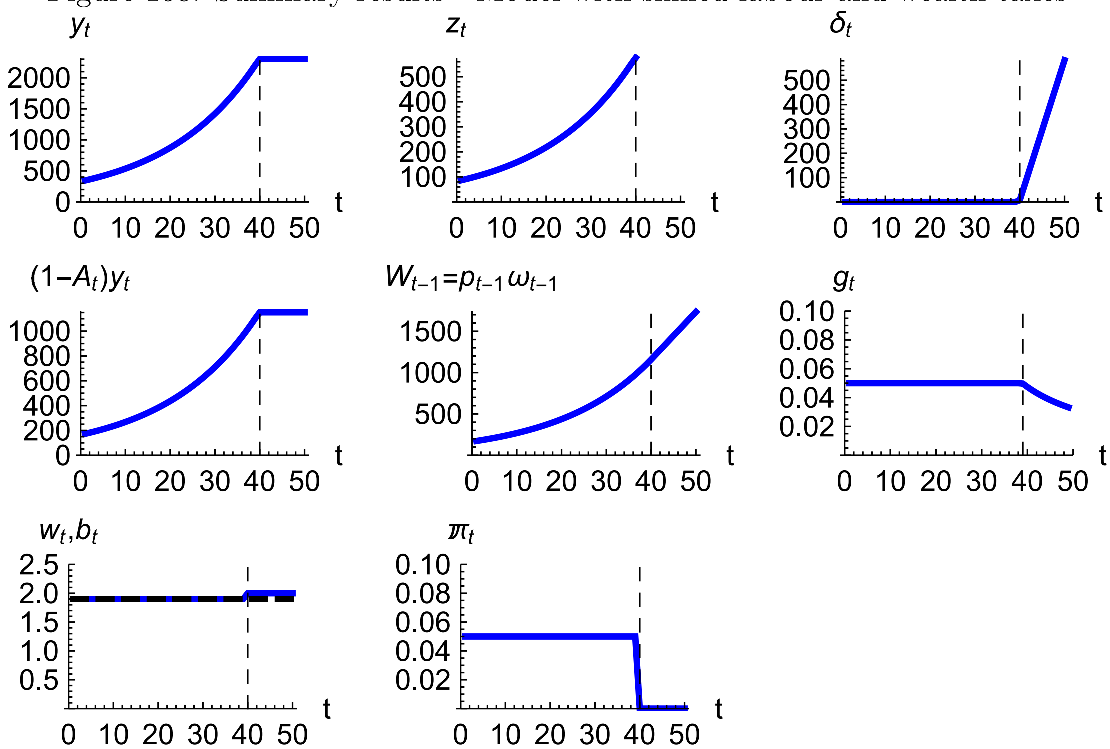


Figure 159 reports the dynamics of exploitation over the course of the simulation. Figure 159(b) displays the distribution of ε^ν_t over t.

Figure 160(a) shows that the Gini coefficient of wealth. Figure 160(b) shows the distribution of wealth for select t.

Figure 161 shows the distribution of income over the course of the simulation.

Figure 159: Exploitation - Model with skilled-labour and wealth taxes

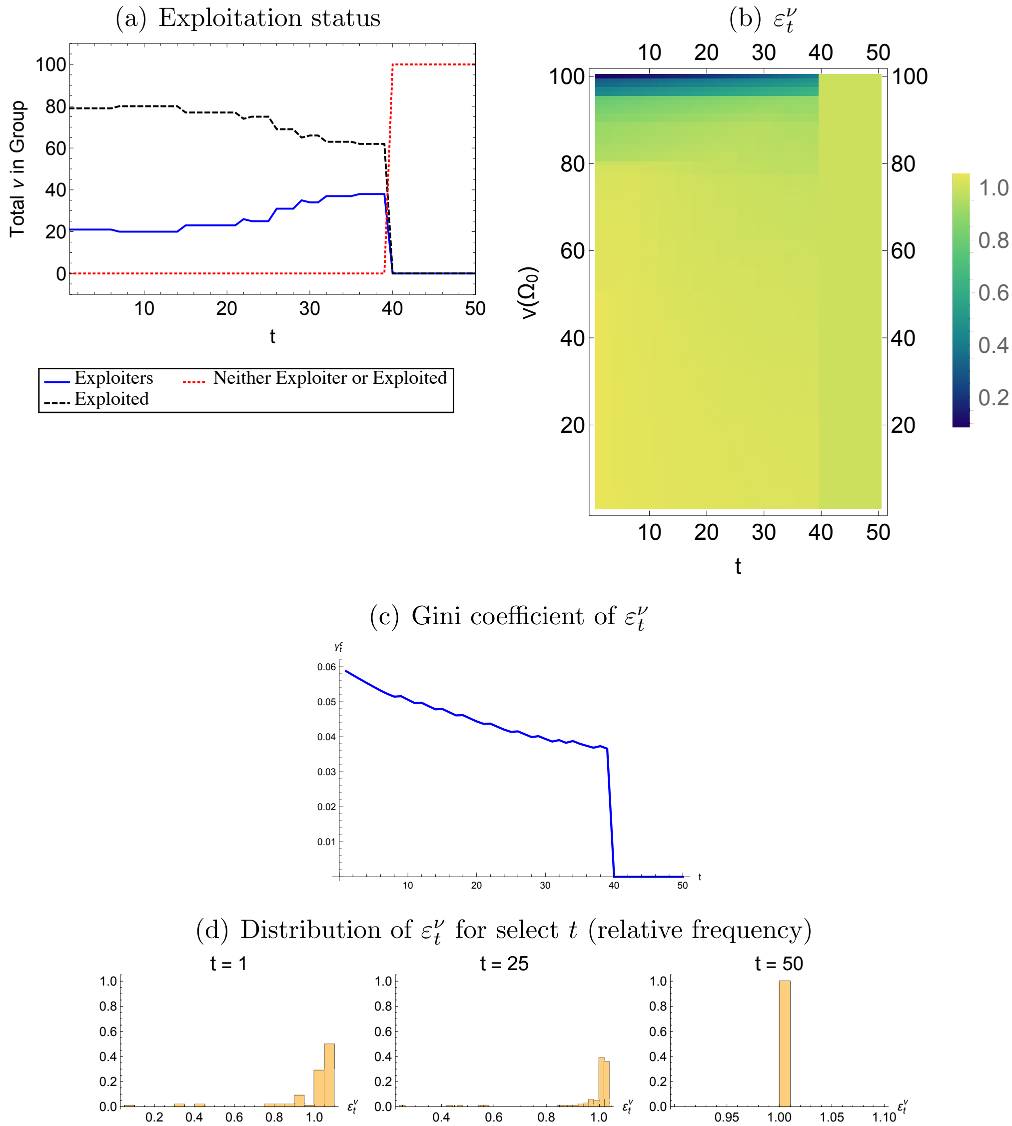
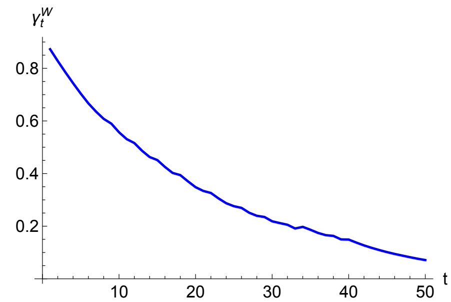


Figure 160: Distribution of wealth - Model with skilled-labour and wealth taxes

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

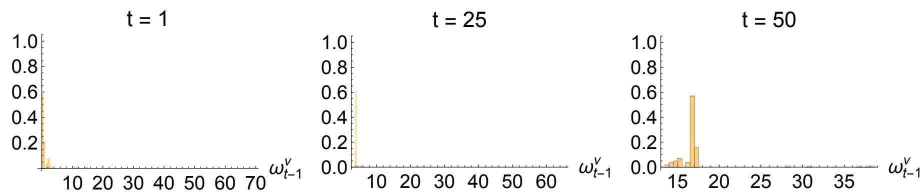
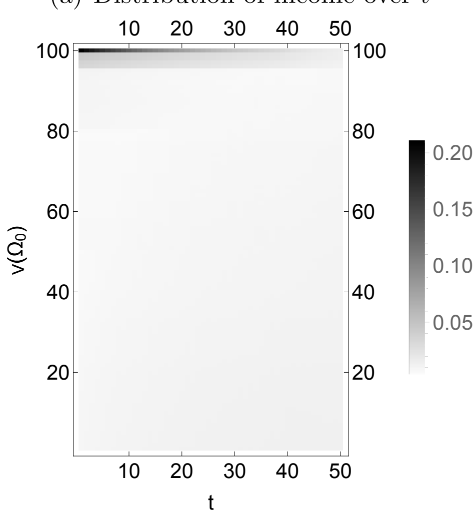
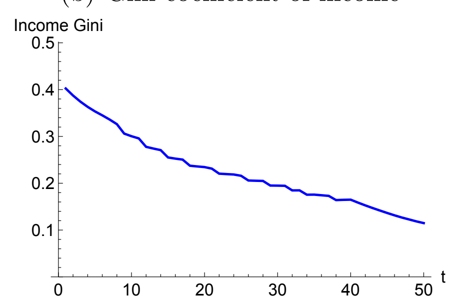


Figure 161: Distribution of income - Model with skilled-labour and wealth taxes

(a) Distribution of income over t



(b) Gini coefficient of income



14.3 Model with wealth equality

The following modifies the model with wealth taxes to include endogenous tax rates designed to eliminate wealth inequality using Rule 1. This is done to examine the properties of exploitation in the scenario where skills are decreasing in agents' wealth. Figure 163 reports the dynamics of exploitation over the course of the simulation. Figure 163(b) displays the distribution of ε_t^{ν} over t. Figure 164(a) shows the Gini coefficient of wealth. Figure 164(b) shows the distribution of wealth for select t. Figure 165 shows the distribution of income over the course of the simulation.

Figure 162: Summary results - Model with skilled-labour and wealth equality

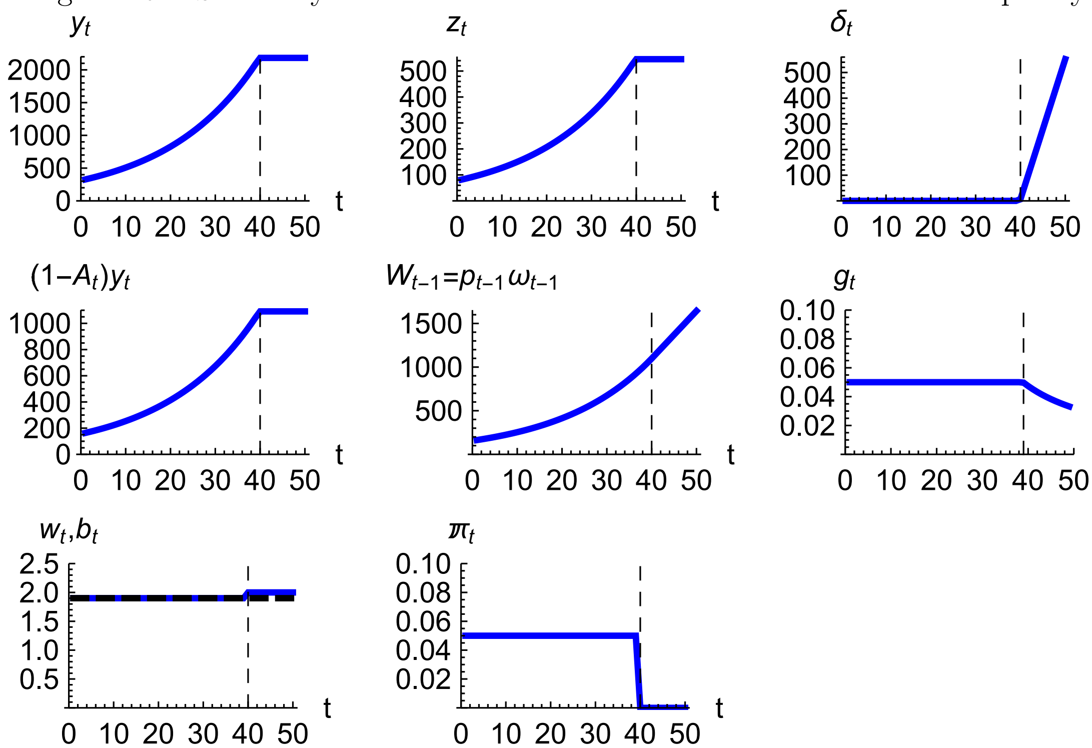


Figure 163: Exploitation - Model with skilled-labour and wealth equality

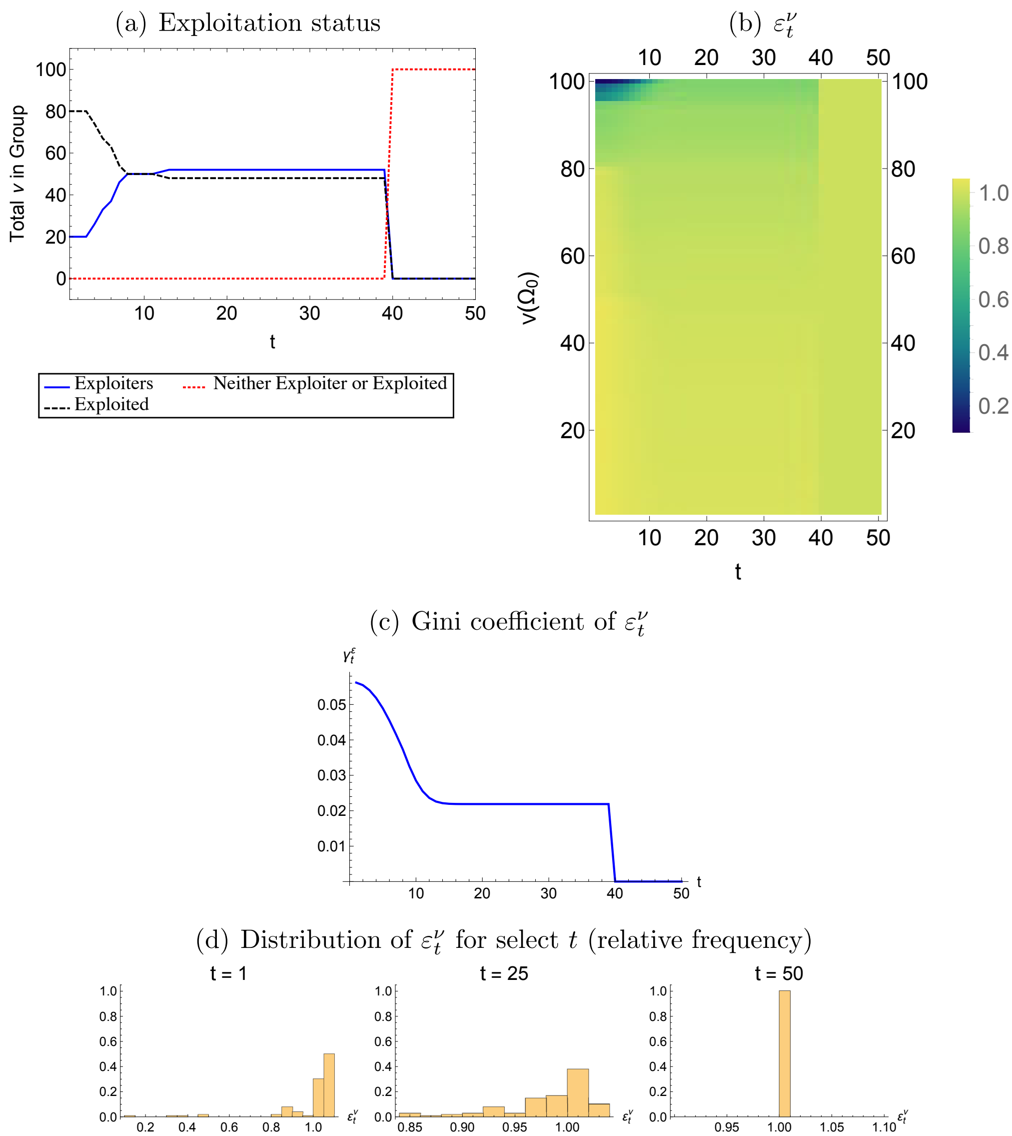
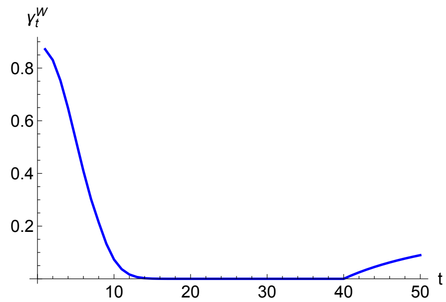


Figure 164: Distribution of wealth - Model with skilled-labour and wealth equality

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

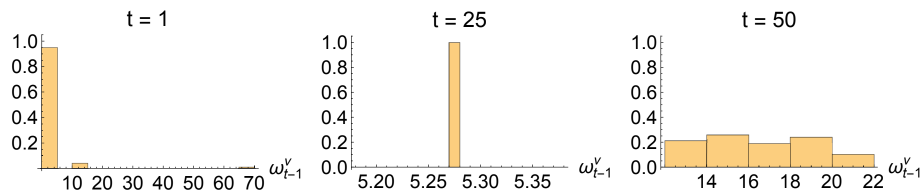
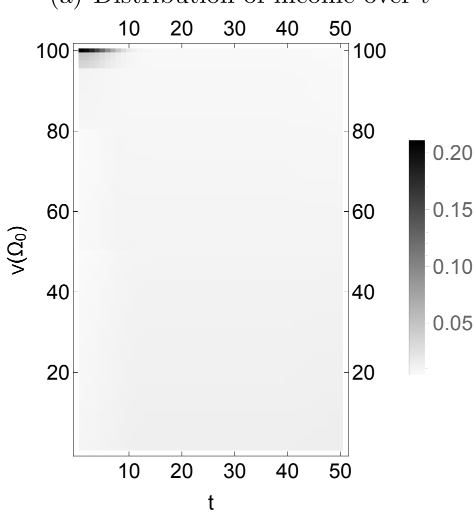
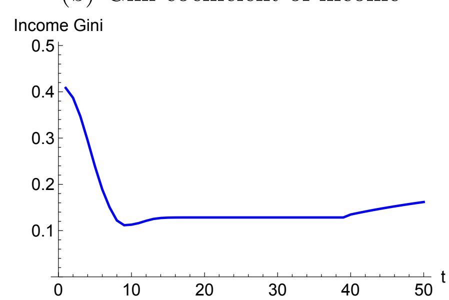


Figure 165: Distribution of income - Model with skilled-labour and wealth equality

(a) Distribution of income over t



(b) Gini coefficient of income



14.4 Model with socialist allocation

The following modifies the model with wealth taxes to include endogenous tax rates designed to achieve a socialist allocation using Rule 2. This is done to examine the properties of exploitation in the scenario where skills are decreasing in agents' wealth. The application of Rule 2 has no qualitative impact on the aggregate results. Figure 167 reports the dynamics of exploitation over the course of the simulation. Figure 167(b) displays the distribution of ε_t^{ν} over t. Figure 168(a) shows the Gini coefficient of wealth. Figure 168(b) shows the distribution of wealth for select t. Figure 169 shows the distribution of income over the course of the simulation.

Figure 166: Summary results - Model with skilled-labour and socialist allocation

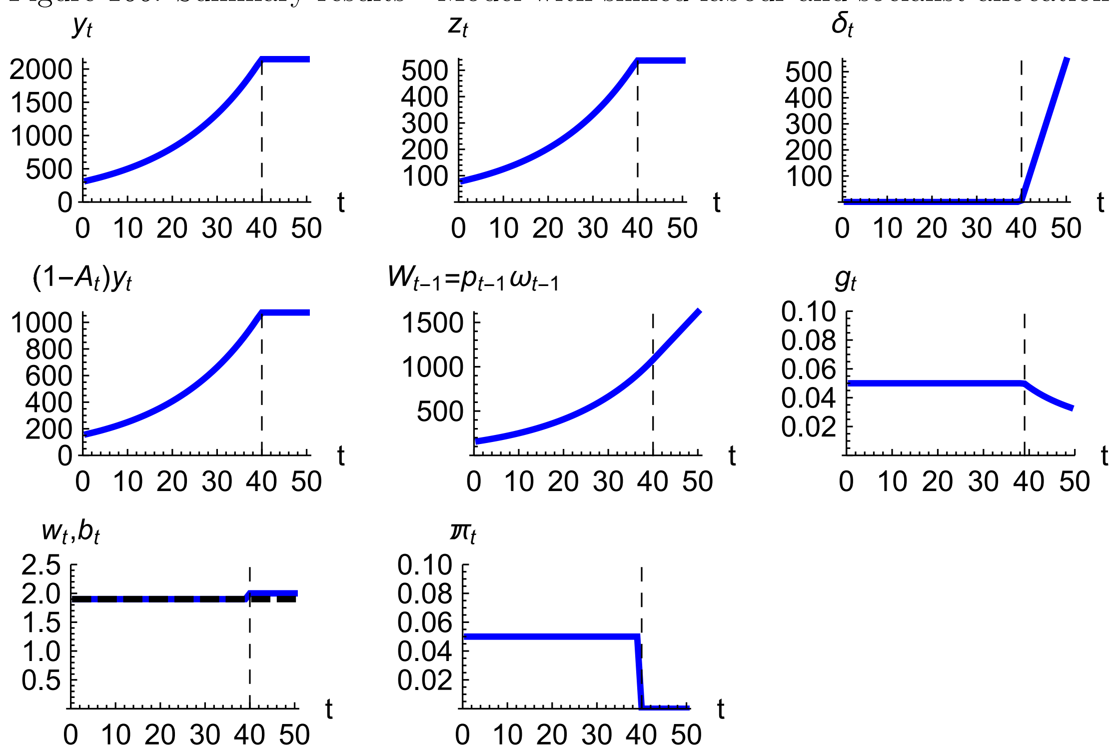
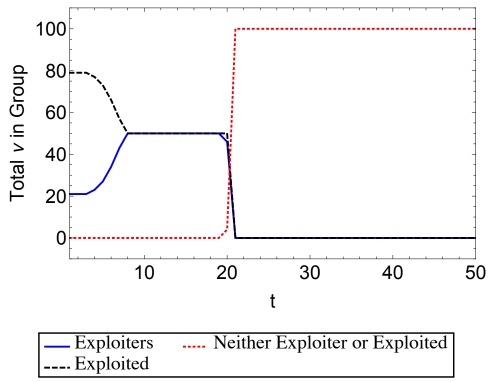
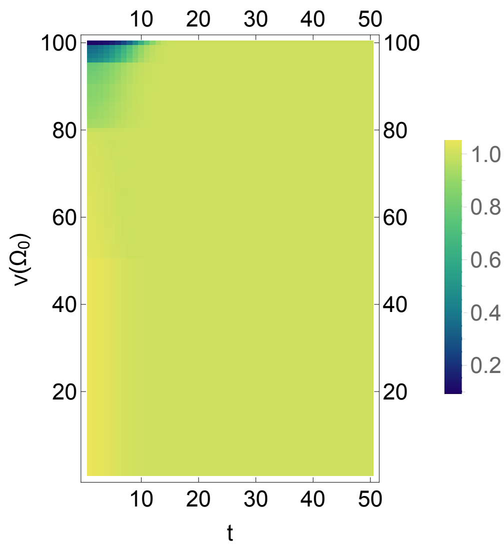


Figure 167: Exploitation - Model with skilled-labour and socialist allocation

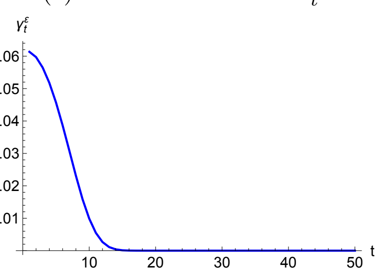
(a) Exploitation status



(b) ε_t^ν



(c) Gini coefficient of ε_t^ν



(d) Distribution of ε_t^ν for select t (relative frequency)

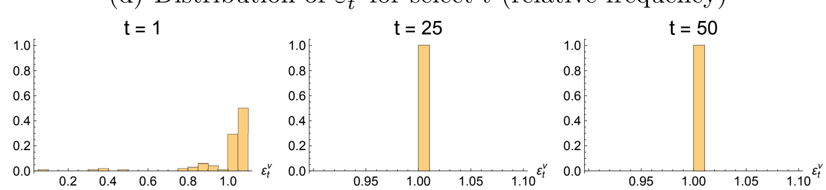
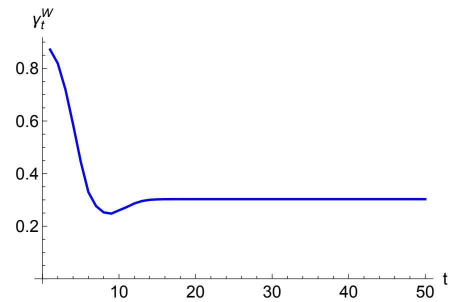


Figure 168: Distribution of wealth - Model with skilled-labour and socialist allocation

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

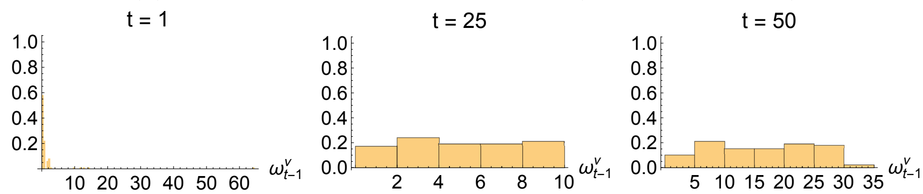
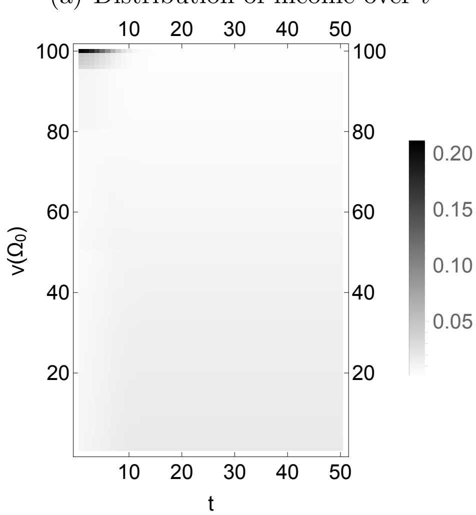
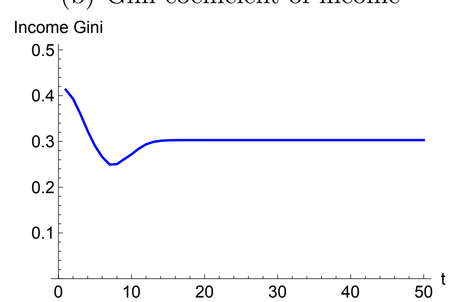


Figure 169: Distribution of income - Model with skilled-labour and socialist allocation

(a) Distribution of income over t



(b) Gini coefficient of income



14.5 Class in the model with s^{ν} decreasing in wealth

The figures below show the dynamics of classes and the CECP for the simulations with skills decreasing in Ω_0.

Figure 170: Class

(a) Class status (b) CECP

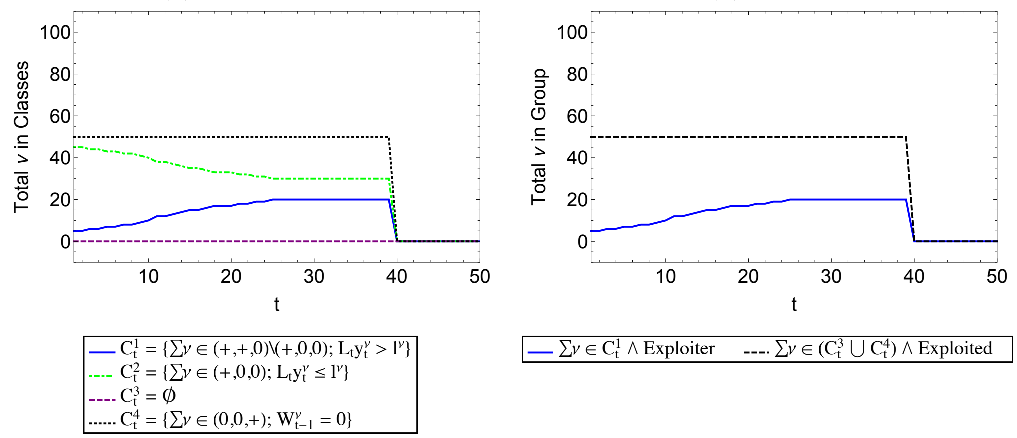


Figure 171: Class - Model with wealth taxes

(a) Class status (b) CECP

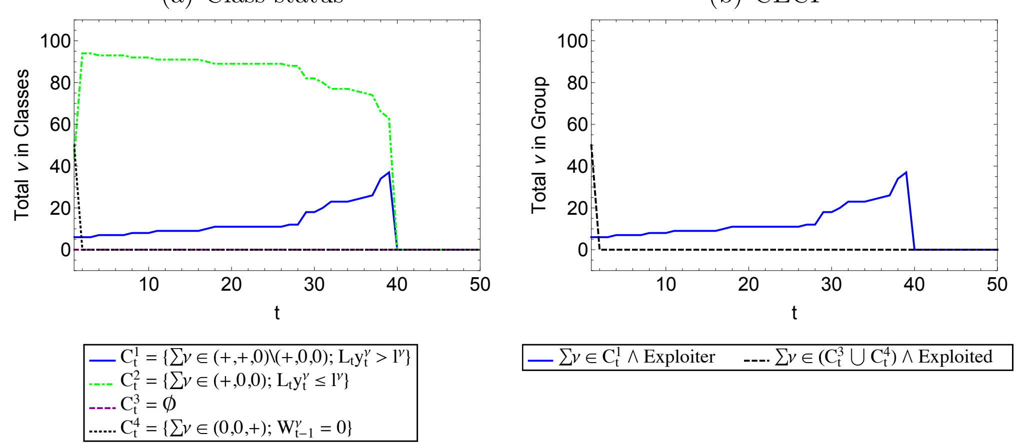
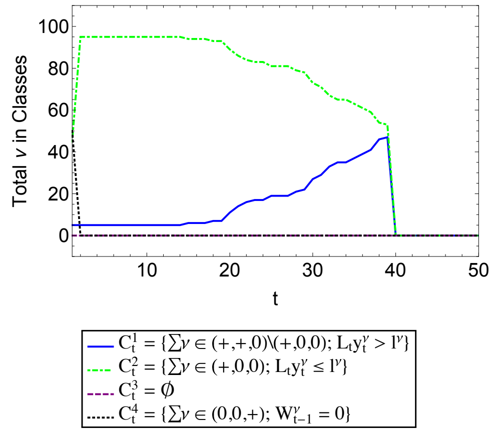


Figure 172: Class - Model with wealth equality

(a) Class status



(b) CECP

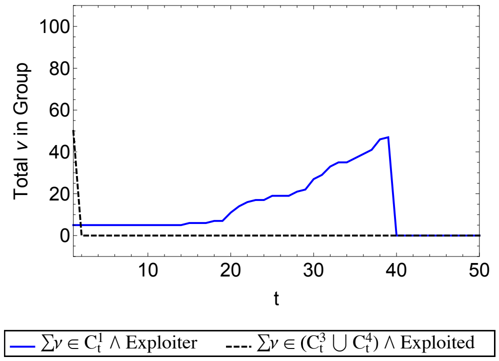
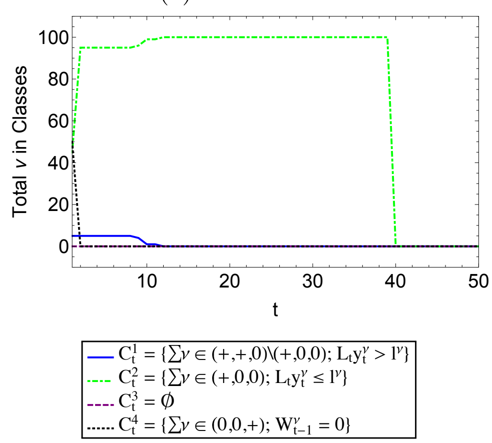
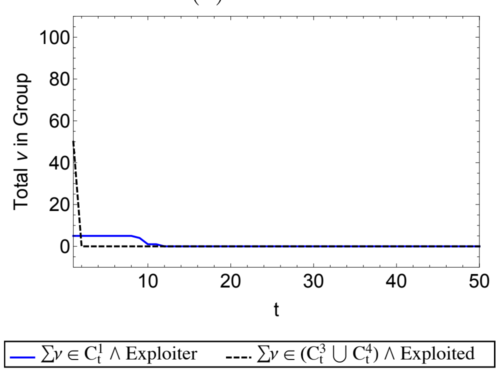


Figure 173: Class - Model with socialist allocation

(a) Class status



(b) CECP

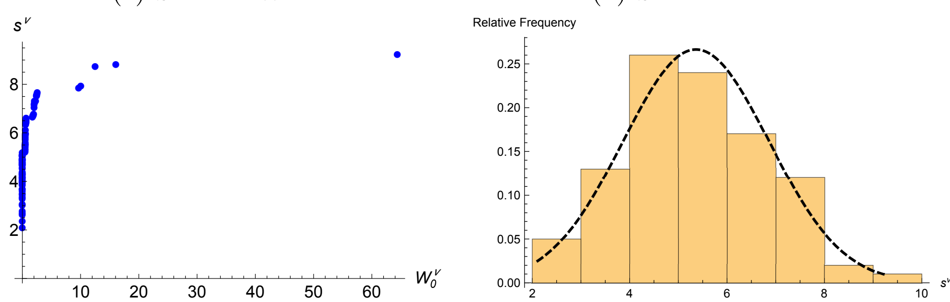


15 Normally distributed skills

The following simulations include skill factors that are normally distributed and ordered according to the initial distribution of wealth. Thus, the poorest agents have the lowest skill factors, the wealthiest have the highest, and there is a robust middle of agents who may or may not begin the simulation with small amounts of wealth. An example of the distribution of skills and the distribution of skills in relation to wealth at $t = 0$ for a typical run of the simulations that follow is shown below in figure 174.

Figure 174: Skill distribution - Model with normally distributed skilled-labour

(a) Skill vs. Wealth (b) Skill distribution



15.1 Model with heterogeneous labour and normally distributed skills

The following simulation modifies the model to include skill factors that are normally distributed and ordered according to the initial distribution of wealth. The simulation uses the parameters: $N = 100$, $A = 0.5$, $L = 0.25$, and $b = 1.9$. Figure 175 reports the summary results for the model. Figure 176 reports the dynamics of exploitation over the course of the simulation. Figure 177(a) shows γ_t^W and figure 177(b) shows the distribution of wealth for select t. Figure 178 shows the distribution of income over the course of the simulation.

Figure 175: Summary results - Model with skilled-labour

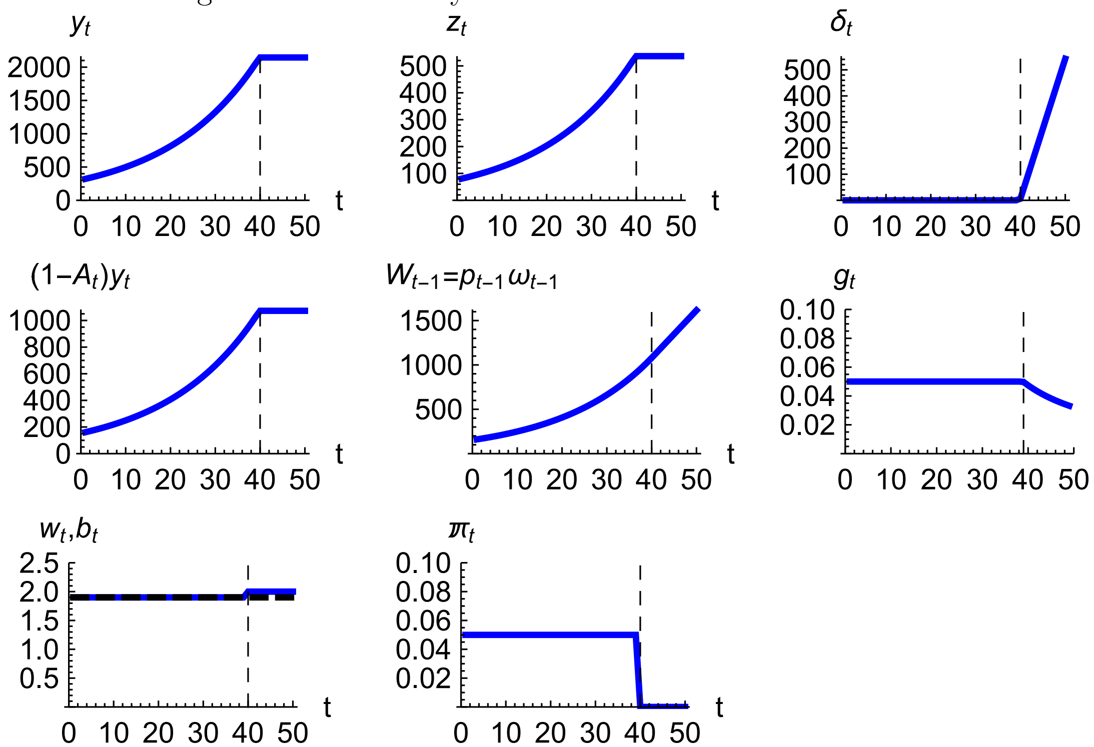


Figure 176: Exploitation - Model with skilled-labour

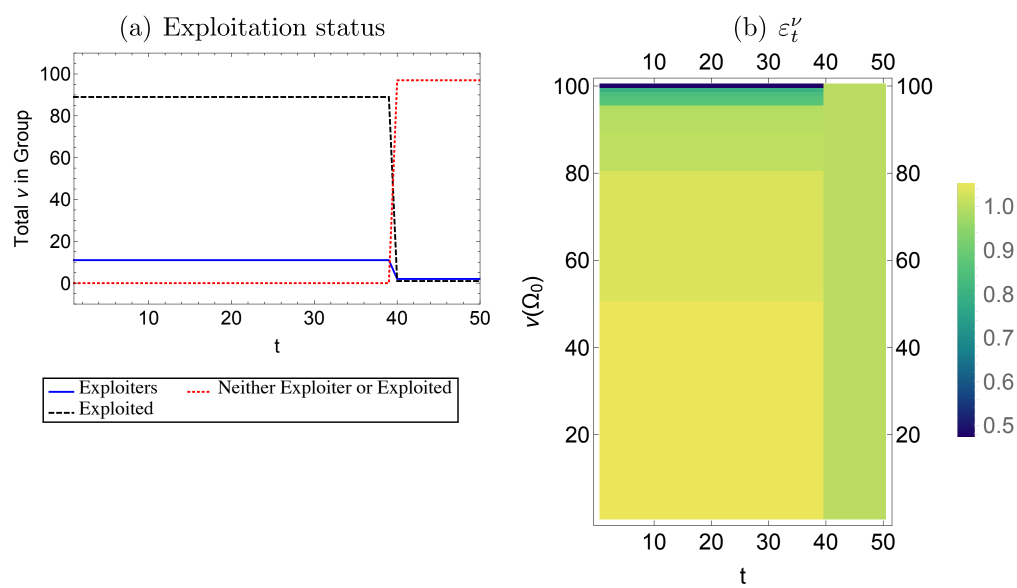
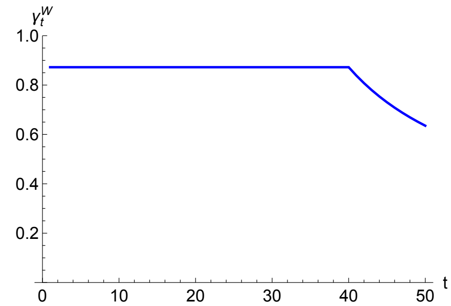


Figure 177: Distribution of wealth - Model with skilled-labour

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

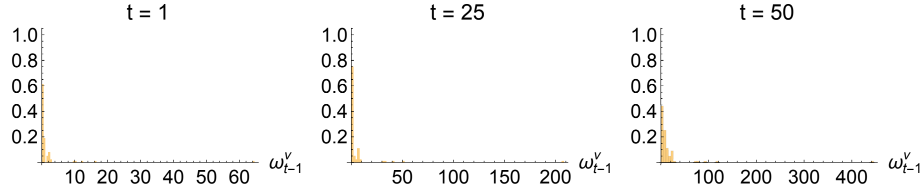
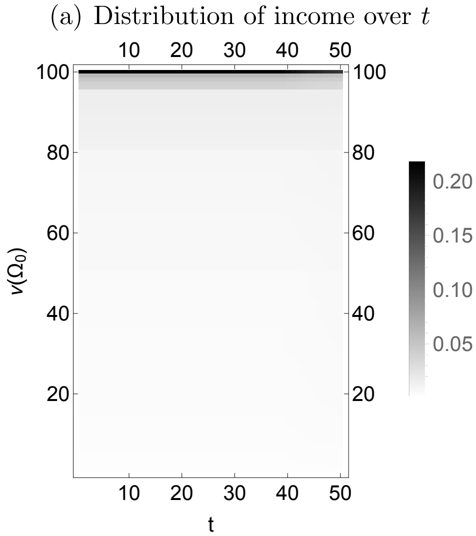
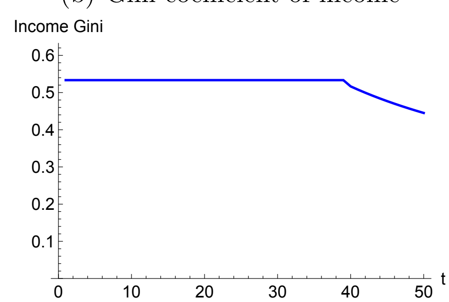


Figure 178: Distribution of income - Model with skilled-labour

(a) Distribution of income over t



(b) Gini coefficient of income



15.2 Model with heterogeneous labour, normally distributed skills, and wealth taxes

The following simulation modifies the model to include skill factors that are normally distributed and ordered according to the initial distribution of wealth, and Piketty-type wealth taxes. The simulation uses the parameters: $N = 100$, $A = 0.5$, $L = 0.25$, and $b = 1.9$. Figure 179 reports the summary results for the model. Figure 180 reports the dynamics of exploitation over the course of the simulation. Figure 181(a) shows γ_t^W and figure 181(b) shows the distribution of wealth for select t. Figure 182 shows the distribution of income over the course of the simulation.

Figure 179: Summary results - Model with wealth taxes

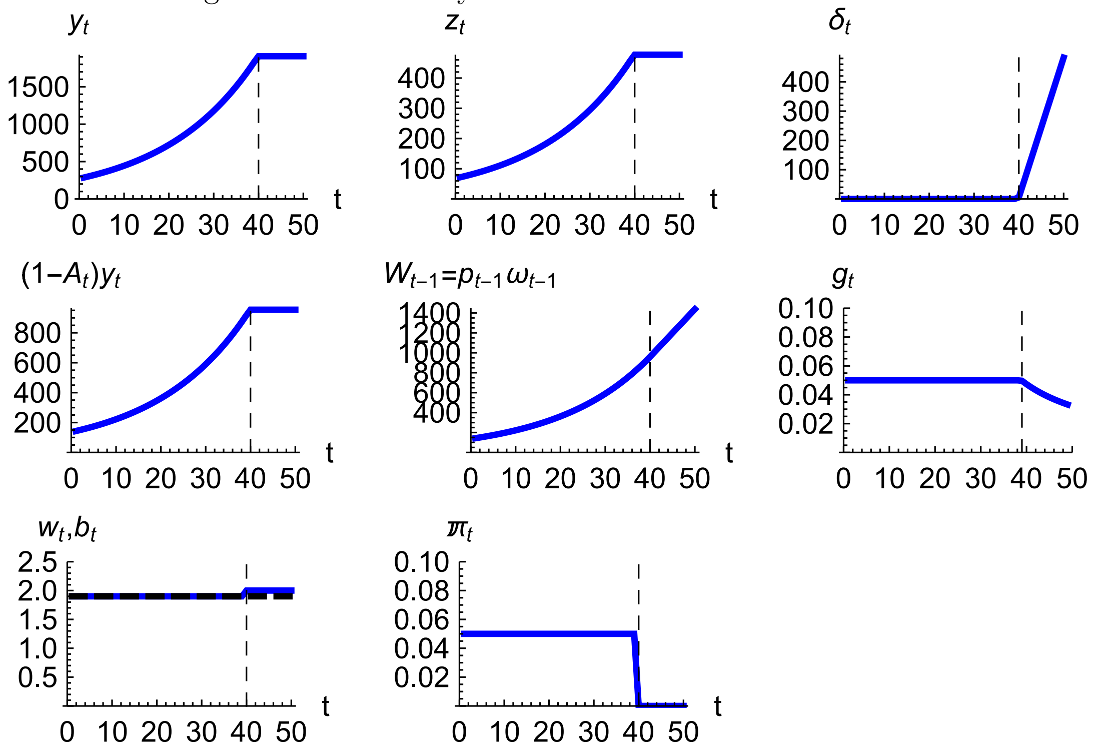


Figure 180: Exploitation - Model with wealth taxes

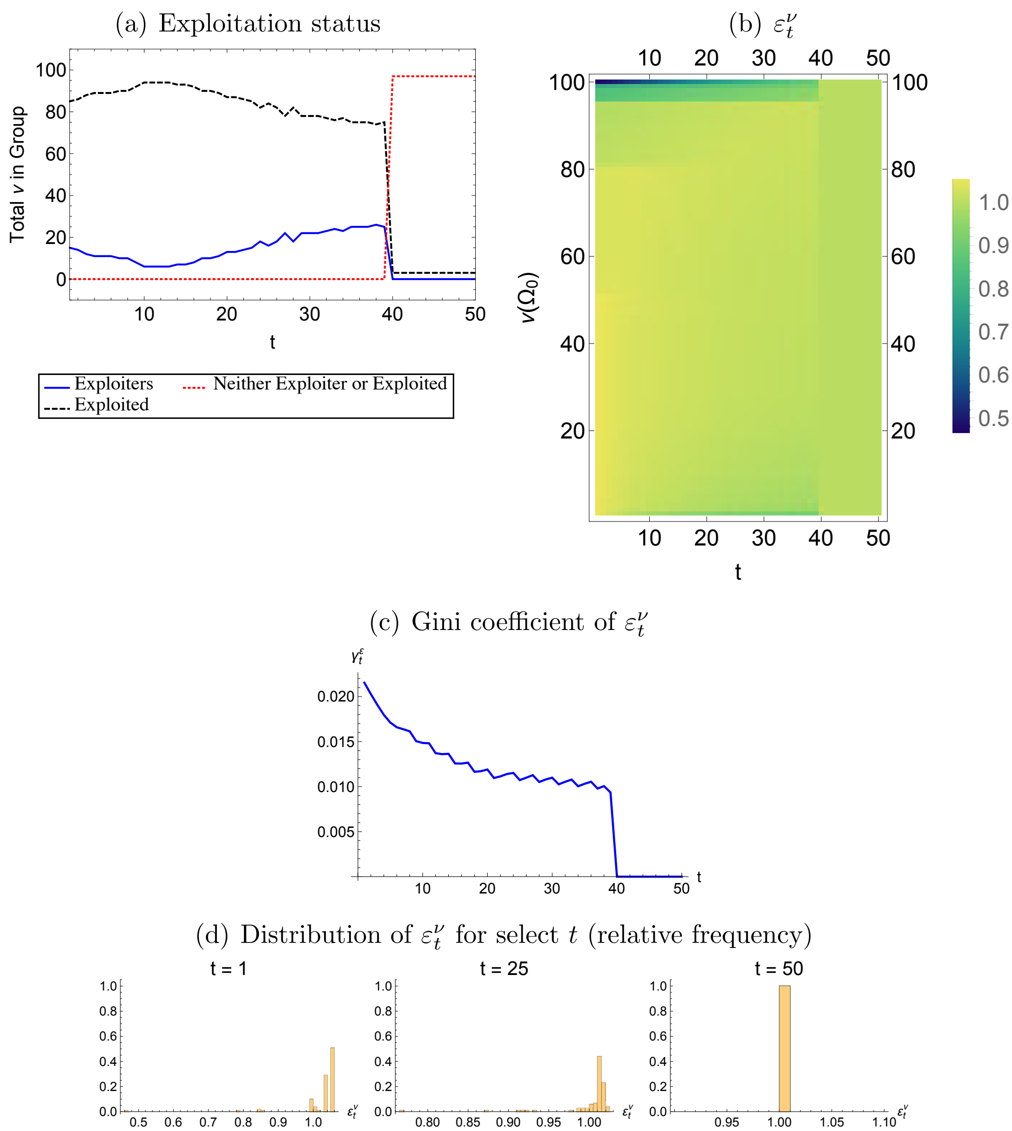
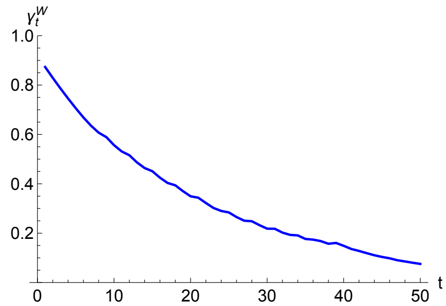


Figure 181: Distribution of wealth - Model with wealth taxes

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

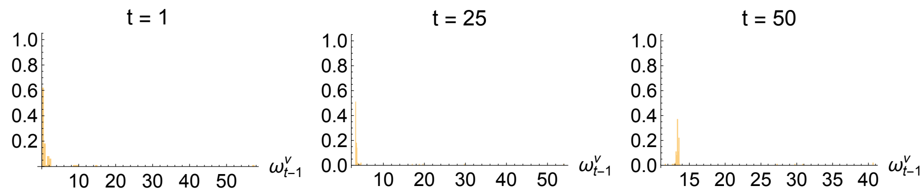
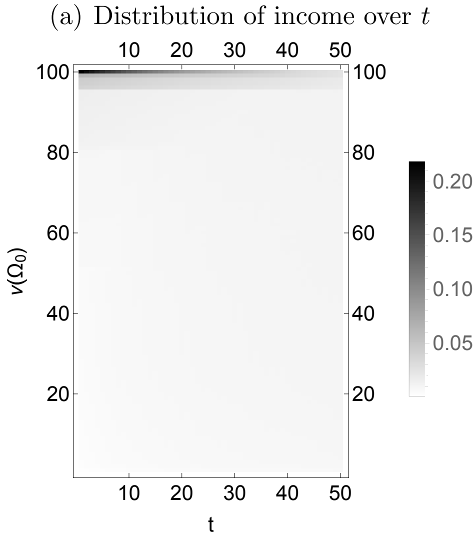
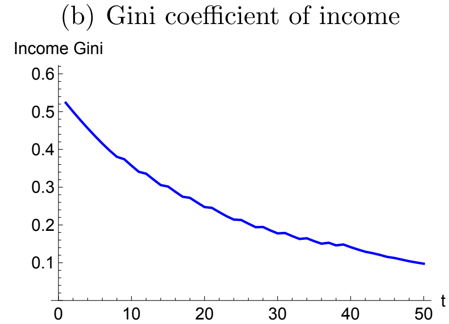


Figure 182: Distribution of income - Model with wealth taxes

(a) Distribution of income over t



(b) Gini coefficient of income



15.3 Model with normally distributed skills and wealth equality

The following simulation modifies the tax routine from the previous simulation to include endogenous tax rates that eliminate wealth inequality using Rule 1. The simulation uses the parameters: $N = 100$, $A = 0.5$, $L = 0.25$, and $b = 1.9$. Figure 183 reports the summary results for the model. Figure 184 reports the dynamics of exploitation over the course of the simulation. Figure 185(a) shows γ_t^W and figure 185(b) shows the distribution of wealth for select t. Figure 186 shows the distribution of income over the course of the simulation.

Figure 183: Summary results - Model with wealth equality

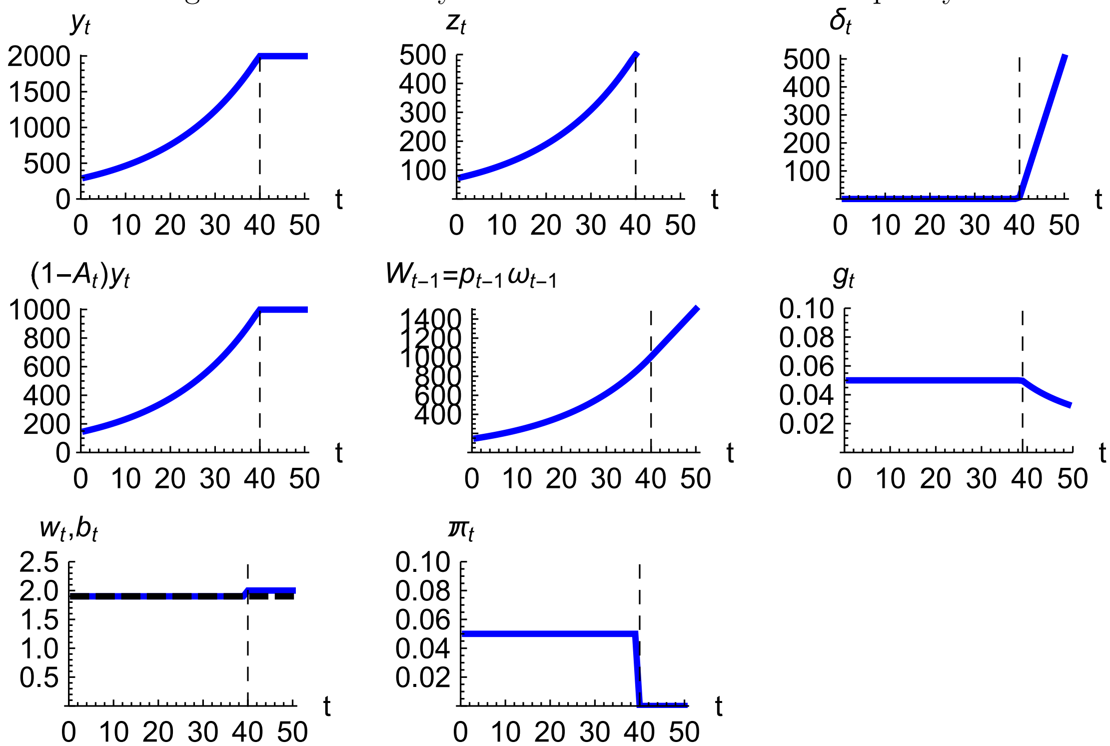


Figure 184: Exploitation - Model with wealth equality

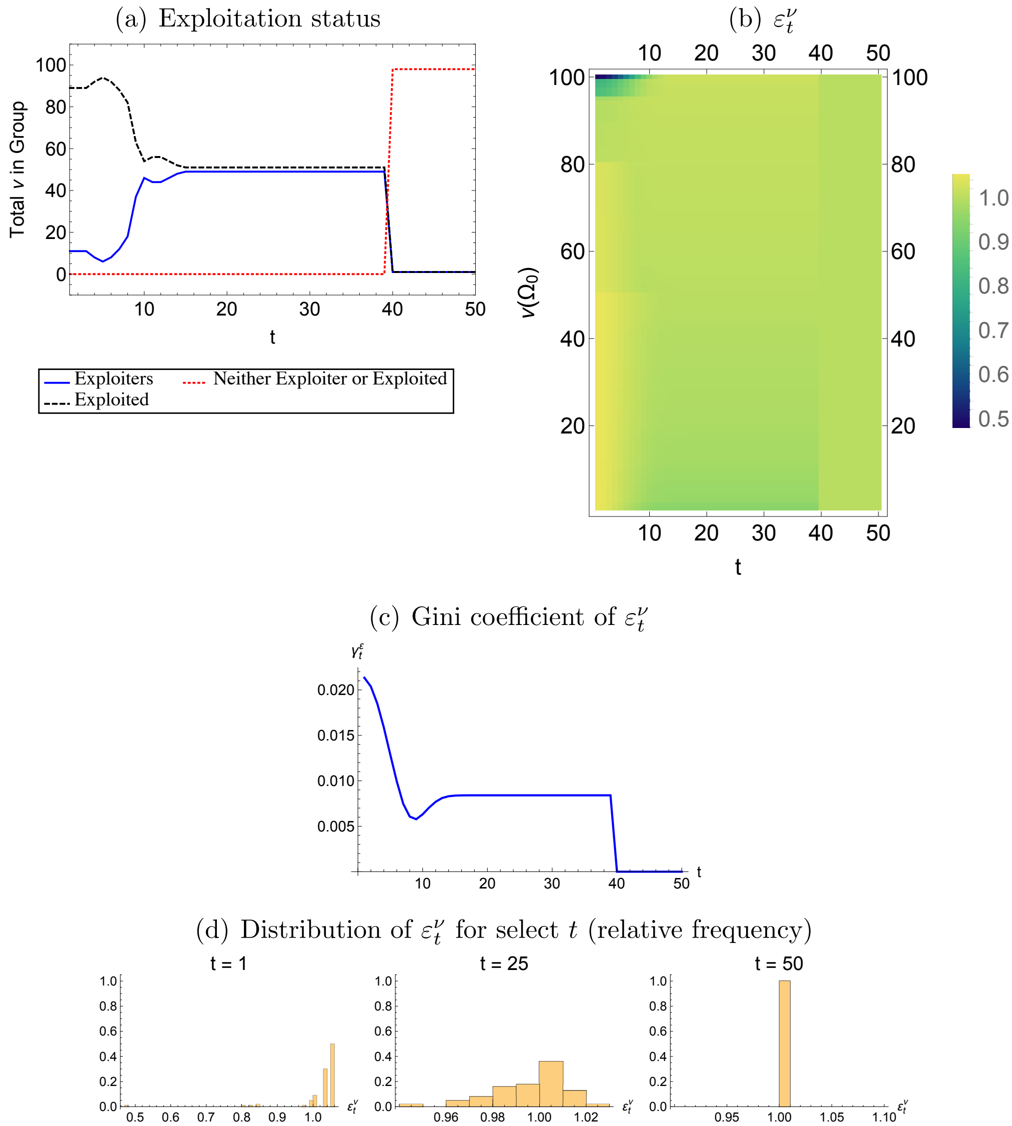
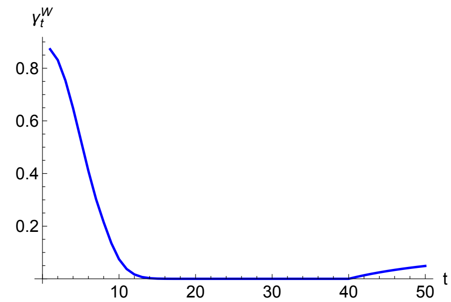


Figure 185: Distribution of wealth - Model with wealth equality

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

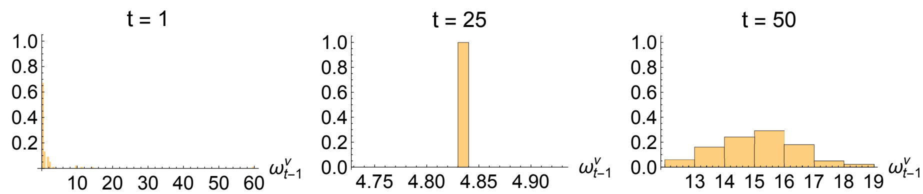
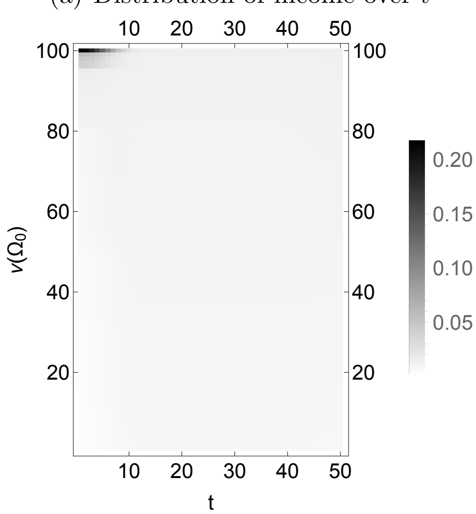
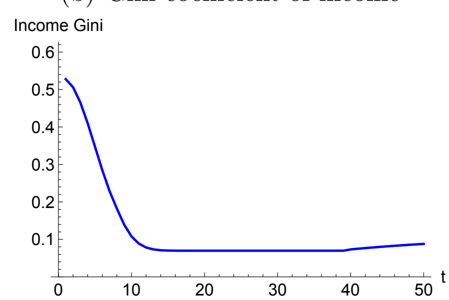


Figure 186: Distribution of income - Model with wealth equality

(a) Distribution of income over t



(b) Gini coefficient of income



15.4 Model with normally distributed skills and socialist allocation

The following simulation modifies the tax routine from the previous simulation to include endogenous tax rates that that achieve a socialist allocation using Rule 2. The simulation uses the parameters: $N = 100$, $A = 0.5$, $L = 0.25$, and $b = 1.9$. Figure 187 reports the summary results for the model. Figure 188 reports the dynamics of exploitation over the course of the simulation. Figure 189(a) shows γ_t^W and figure 189(b) shows the distribution of wealth for select t. Figure 190 shows the distribution of income over the course of the simulation.

Figure 187: Summary results - Model with socialist allocation

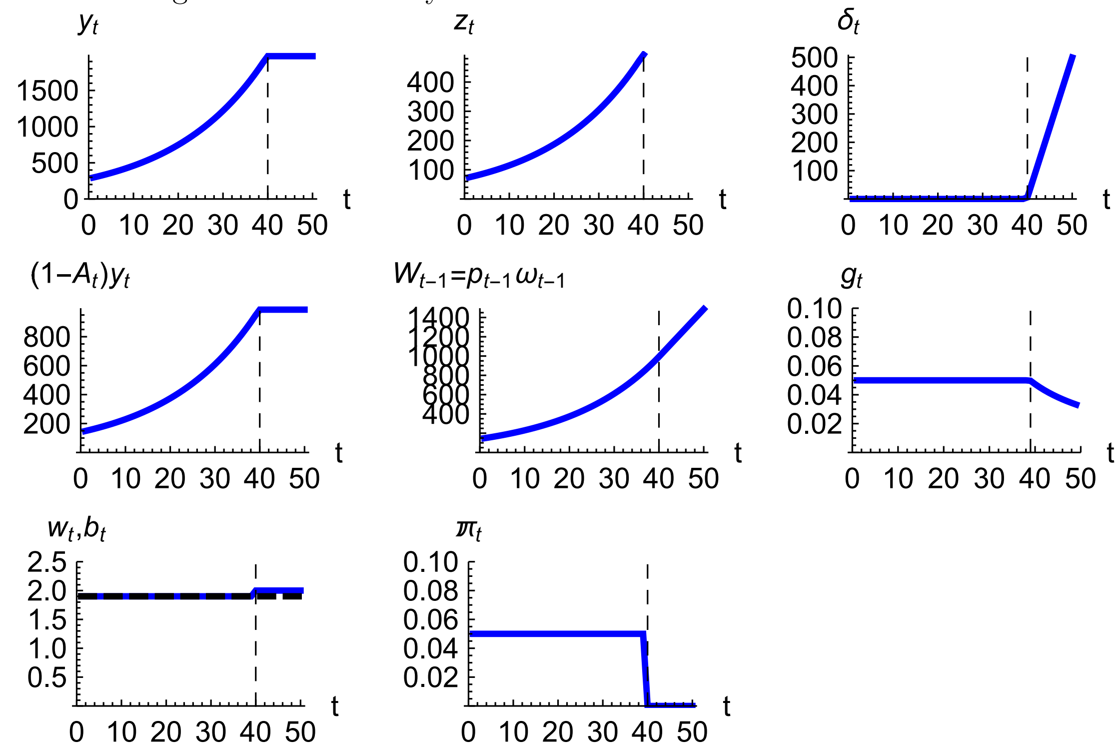
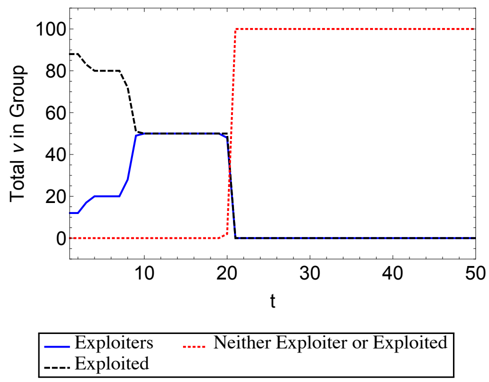
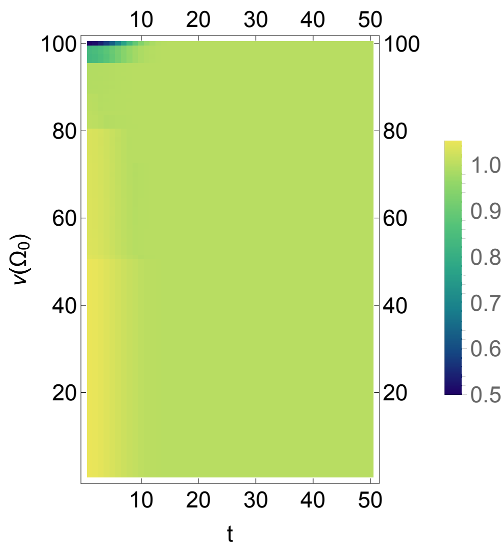


Figure 188: Exploitation - Model with socialist allocation

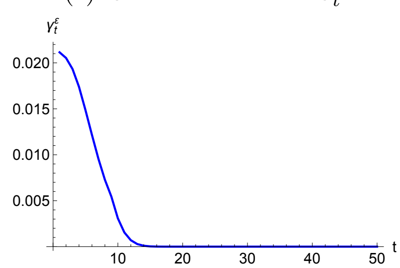
(a) Exploitation status



(b) ε_t^{ν}



(c) Gini coefficient of ε_t^{ν}



(d) Distribution of ε_t^{ν} for select t (relative frequency)

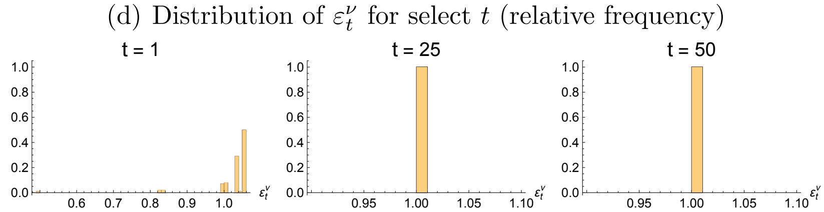
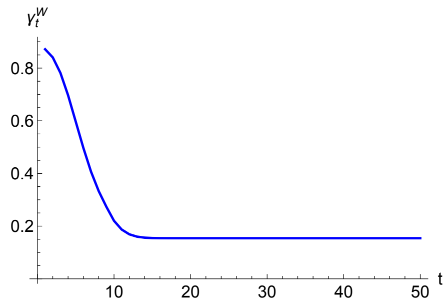


Figure 189: Distribution of wealth - Model with socialist allocation

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

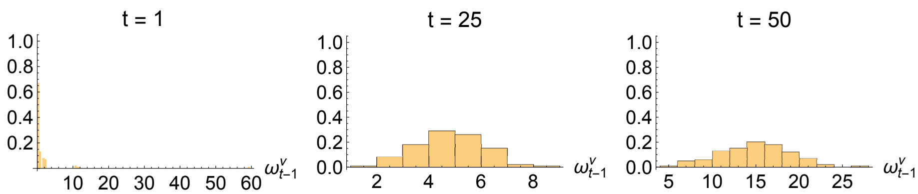
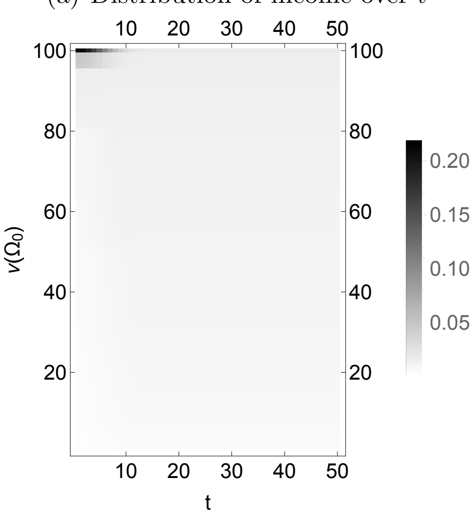
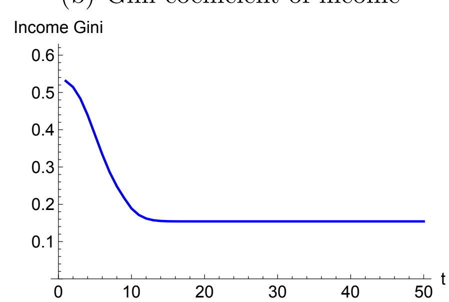


Figure 190: Distribution of income - Model with socialist allocation

(a) Distribution of income over t



(b) Gini coefficient of income

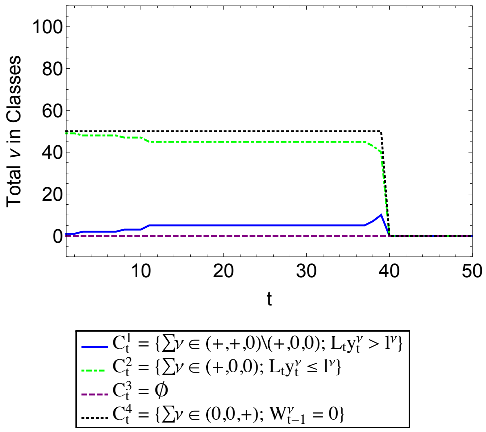


15.5 Class in the simulations with normally distributed skills

The figures below present the dynamics of classes and the CECP for the simulations with normally distributed skills.

Figure 191: Class

(a) Class status



(b) CECP

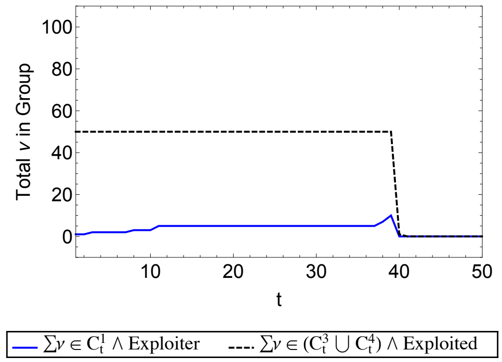
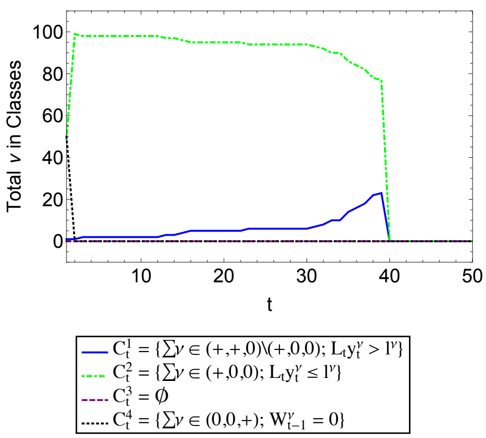


Figure 192: Class - Model with wealth taxes

(a) Class status



(b) CECP

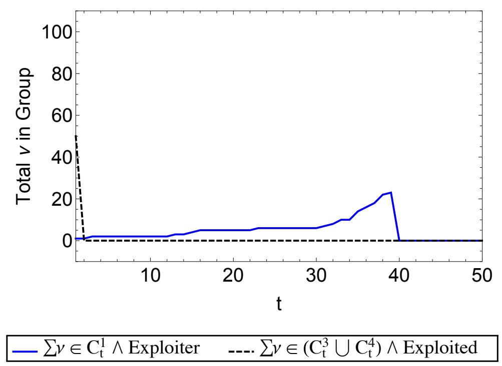


Figure 193: Class - Model with wealth equality

(a) Class status (b) CECP

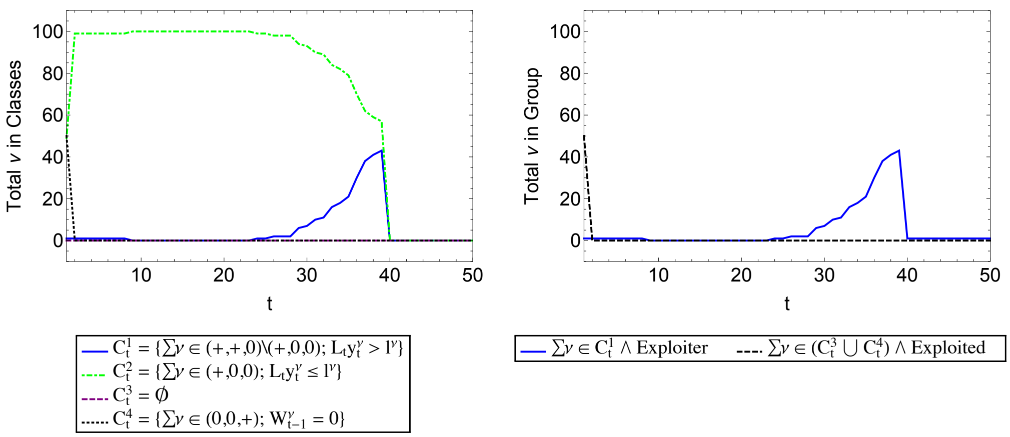
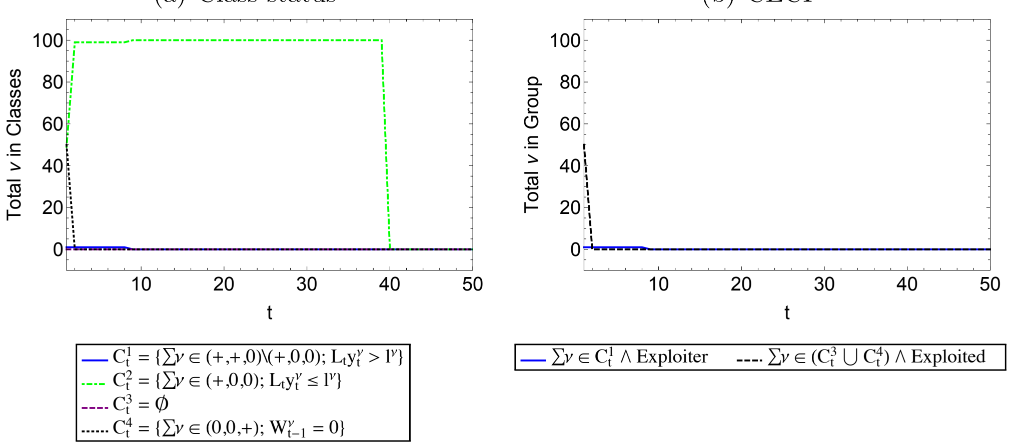


Figure 194: Class - Model with socialist allocation

(a) Class status (b) CECP



16 Alternative view on heterogeneous labour

The following section incorporates heterogeneous labour through providing agents with differing amounts of *labour time*. Results for simulations applying this extension to the versions of these models with different types of taxes are not shown since the results are not qualitatively different from the results presented in section 11 here, i.e. where skills are decreasing in wealth at $t = 0$.

The typical classical-Marxian assumption of homogeneous labour is present with the uniform labour endowments $l^\nu = 1$ in previous models. However, in the models that follow agents' labour endowments at $t = 0$ are constructed such that each $\nu \in \mathcal{N}_t$ has $l^\nu \in \{1, 2, 3, 4\}$. l^ν are assigned by partitioning all $\nu \in \mathcal{N}_t$ into quartiles and assigning values 4, 3, 2, and 1 to each quartile respectively. This is done so that the first two quartiles possess greater labour endowments than the upper two quartiles. The lower two quartiles are those ν who begin the simulation with little to no wealth, while the upper two quartiles consist of those agents who begin the simulation with moderate to considerable wealth, relative to the other $\nu \in \mathcal{N}_0$. Assigning different values to l^ν effectively makes it so that different agents have *different amounts of time* available to spend working. This is just one possible way to account for heterogeneous labour.

The initial distribution of wealth Ω_0 is handled in the same way as in all previous simulations.

16.1 Model with heterogeneous labour time

The following presents the economy with heterogeneous labour time. The simulation uses the parameters: $N = 100$, $A = 0.5$, $L = 0.25$, and $b = 1.9$. Figure 195 reports the summary results. Figure 196 reports the dynamics of exploitation over the course of the simulation. Similar to earlier versions of the model, the structure of exploitation is relatively stable as long as the simulation is capital constrained, but as soon as the simulation is labour constrained exploitation and classes disappear. Figure 197(a) shows that the Gini coefficient of wealth is stable until the economy is labour constrained, at which point wealth inequality begins to steadily decline as all agents accumulate. Figure 197(b) shows the distribution of wealth for select t. Figure 198 shows the distribution of income across agents.

Figure 195: Summary results - Model with heterogeneous labour time

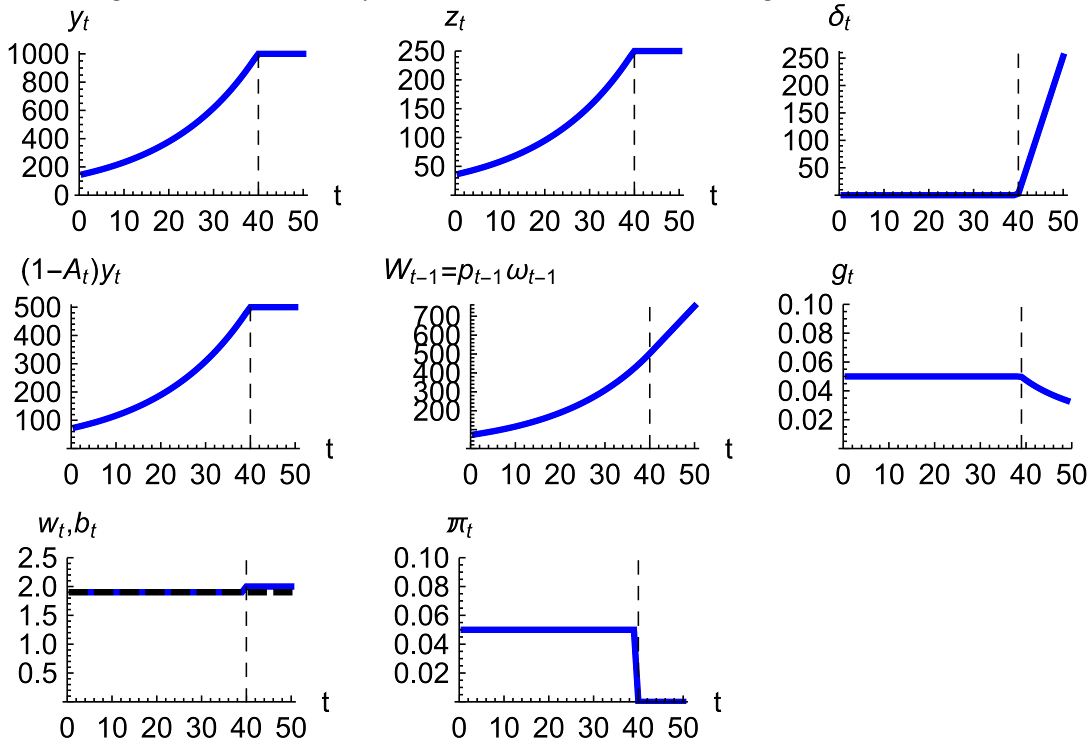
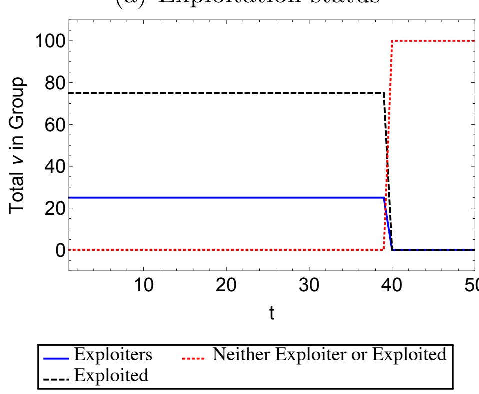


Figure 196: Exploitation - Model with heterogeneous labour time

(a) Exploitation status



(b) Exploitation intensity

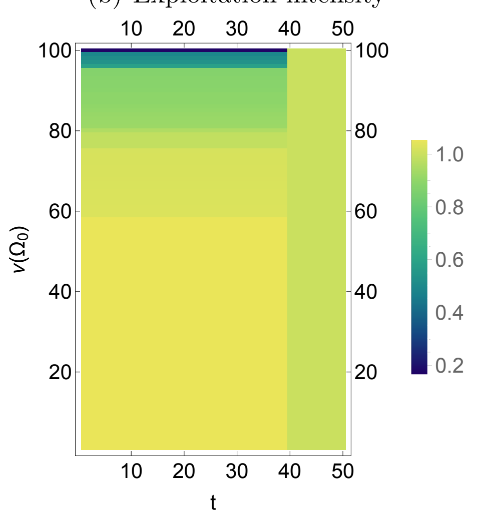
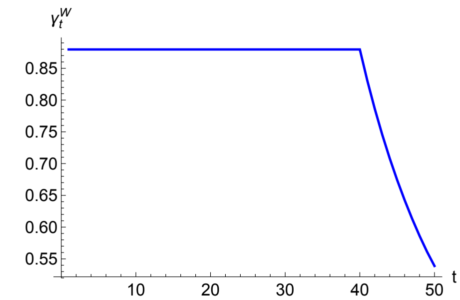


Figure 197: Distribution of wealth - Model with heterogeneous labour time

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

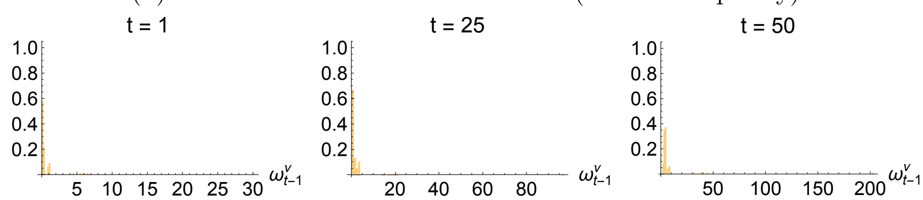
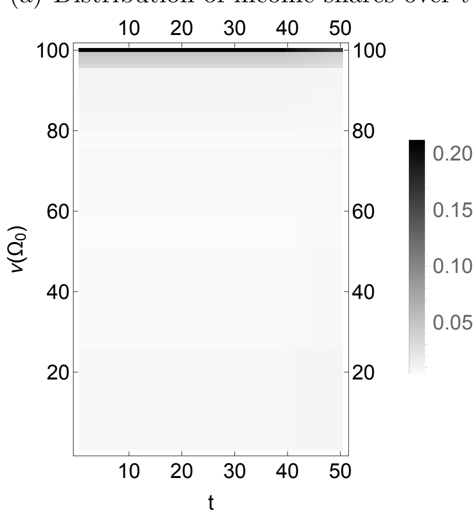
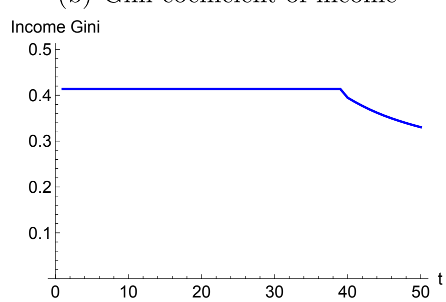


Figure 198: Distribution of Income - Model with heterogeneous labour time

(a) Distribution of income shares over t



(b) Gini coefficient of income



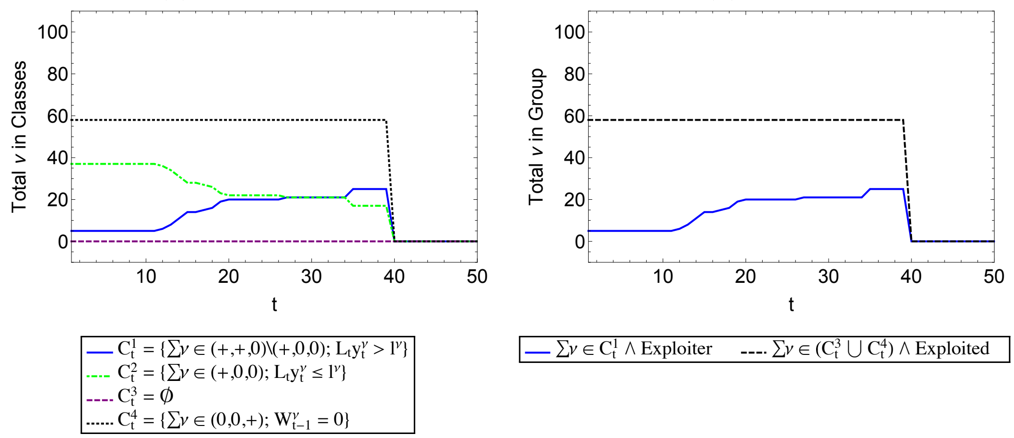
16.2 Class in the model with heterogenous labour time

The figures below show the dynamics of classes and the CECP for the simulations with heterogenous labour time.

Figure 199: Class - Model with heterogenous labour time

(a) Class status

(b) CECP



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[2] Bowles, S., Gintis, H., Osborne, M., 2001. The Determinants of Earnings: A Behavioral Approach. *Journal of Economic Literature* 39, 1137-1176.

[3] Roemer, J.E., 1982. *A General Theory of Exploitation and Class*. Harvard University Press, Cambridge, MA.