An Agent-Based Approach to Classical-Marxian Value Theory and Labor Mobility

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Abstract

An agent-based model of a dynamic simple commodity economy inspired by the Classical/Marxian tradition is presented and discussed. The purpose of the model is to properly conceptualize and model the mobility of labor inherent in Marx's law of the tendency for the rate of surplus value to equalize across sectors of production regardless of skill heterogeneity in workers. The results of the model approximate the conclusions of the labor theory of value.

Keywords: Karl Marx; Long-Period Method; Rate of Surplus-Value; Mobility of Labor; Adam Smith; Agent-Based Model; Law of Value; Dynamic Computational Model.

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1 Introduction

The Classical Political Economy of Adam Smith, David Ricardo, and Karl Marx presents a vision of society as a self-organizing and open-ended system. The vision of the Classicals consists of an abstraction in which a sufficiently long period of time is considered, and the inputs of production (labor and capital) are fully mobile across different lines of production (Foley 2011, 15-20). This framework understands economies as being in a constant state of fluctuation while exhibiting some regular aggregate behaviors, but through the method of abstraction, the numerous fluctuations and disturbances are put aside in order to examine the recognizable aggregate behaviors that emerge as centers of gravitation for the ceaseless market fluctuations that characterize economies in reality. This overall approach to political economy of the Classicals is referred to as the long-period method (Garegnani 1970, 1976, 1984).

The long-period method's abstract assumption of perfectly mobile labor and capital best represents what Marx describes as the "pure" form of capitalism, or its "concept". Abstracting from any impedances or market frictions is necessary to reveal the inner laws and tendencies of capitalism which underly what is readily observable in reality (Marx 1981, 291).[1] The long-period method's use of mobile labor and capital leads Smith to the tendencies for the "whole of the advantages and disadvantages" of different employments of labor and capital ("stock" in Smith's terminology) to independently trend toward equalization (Smith 2000, Ch. 10). Marx's long-period method leads him to consider the tendencies for the rate of profit and rate of surplus-value to independently equalize across sectors (Cogliano 2011)(Marx 1981, 275, 297-298). The tendencies Smith and Marx present use different terminology, but are conceptually similar in their manifestation. Marx presents these tendencies as "economic laws" which emerge only as rough averages amidst turbulent and constant oscillations in the rates of profit and surplus-value and not as steady equilibria at which an economy always operates. Conceptualizing the central tendencies of an economy as

[1]Foley (2011) provides a succinct discussion of Marx's method of abstraction. The relevant passages in Marx's own work can be found in the introductions to the *Grundrisse* and volume one of *Capital*, as well as in the *Contribution to the Critique of Political Economy*. Many such passages are quoted at length in Cogliano (2011) and Foley (2011).

centers of gravitation, or attractors, is where the analogy of Classical Political Economy to modern complex systems theory, or complexity, can be made. The Classical vision "incorporates many insights of contemporary complex systems theory" (Foley 2003, 1) through its characterization of society as being in a persistent state of fluctuation yet exhibiting some recognizable aggregate regularities.

Complexity theory analyzes "highly organized but decentralized systems composed of very large numbers of individual components" (Foley 2003, 1). Complex systems are those which conform to the following criteria

> ...potential to configure their component parts in an astronomically large number of ways (they are *complex*), constant change in response to environmental stimulus and their own development (they are *adaptive*), a strong tendency to achieve recognizable, stable patterns in their configuration (they are *self-organizing*), and an avoidance of stable, self-reproducing states (they are *non-equilibrium systems*) (Foley 2003, 2).

The vision put forth by the Classical Political Economists fits the above definition of a complex system. Foley (2003) notes that the Classical vision of competition presents an example of how a decentralized system exhibits "a strong tendency to achieve recognizable, stable patterns" (Foley 2003, 2-3). The mobile labor and capital of the Classical long-period method facilitates such a stable pattern. If labor and capital are mobile across spheres of production and seek the highest possible returns to their respective contributions to the production process (effort in the case of labor and investment, or capital advanced in the case of capital), then any perturbation that causes the returns to either factor of production to be higher than in other sectors will cause an influx of that particular factor and a contraction in other sectors until the returns to both factors are once again independently equalized across sectors. These motions continue as the returns to labor and capital are constantly fluctuating across sectors, thus the equalization of the returns to labor and capital is not actually realized as a static equilibrium, it only reveals itself as a center of gravity for the oscillations in the returns to labor and capital over a long-period of time.[2] This description of competition applies to Smith, Ricardo, and Marx's vision of competition in general terms,

[2]It is important to note that in Marx's framework the movement of capital across spheres of production causes the expansion and contraction of sectors and labor is "flung"

but lacks their particular terminology and some of nuances that create differences in their frameworks.

The Classical conception of competition dovetails nicely with the complexity vision of the world. Examples of work that follows this line of thinking can be found in Cockshott, Cottrell, Michaelson, Wright, and Yakovenko (2009) and Wright (2008, 2011a,b).[3] The complementarity of the Classicals and the complexity vision is best summarized as follows:

> [The Classical vision] does not insist that each and every component of the economy achieve its own equilibrium as part of a larger master equilibrium of the system as a whole. In fact, it is precisely from the *disequilibrium* behavior of individual households and firms that the Classical vision of competition sees the orderliness of gravitation of market prices around natural prices as arising. In the language of complex systems theory, Classical gravitation is a self-organized outcome of the competitive economic system (Foley 2003, 4).

The rich analogy between the vision of the Classicals and modern complex systems theory lends itself to being modeled in an agent-based (ABM) or computational simulation. An ABM provides the kind of abstract, open-ended model with many agents/actors interacting with each other and behaving according to a set of pre-determined rules that is amenable to the Classical vision and allows one to view the aggregate regularities of gravitation described by the Classicals.

In its simplest form, an agent-based model is composed of an appropriate taxonomy of heterogeneous agents, a scale that fits the problem/phenomena under examination, and a set of rules that govern the actions and interactions of agents (LeBaron and Tesfatsion 2008). Together, the set(s) of agents and the behavioral rules constitute what can be called the 'micro-specification' of the model, and the scale of the model sets the scope for what types of 'macro' phenomena are being studied. Among the benefits of the agent-based approach to economics, perhaps the most important, is that

from one sector to another as capital migrates in search of the highest profit rate (Marx 1981, 298).

[3]Others pick up on this analogy between the Classicals and complex systems, but do not develop them as far. See Colander (2000) and Matthews (2000) for further mention of the relationship between the Classicals and Marx and complexity. Arthur (2006) and Miller and Page (2007) also locate thinking in terms of complex systems in the work of Adam Smith.

the models *generate* macroeconomic regularities from a microeconomic specification, and, given the recursive nature of agent-based modeling, the micro and macro "co-evolve" (Epstein 2006, 6). The microeconomic specification, or microstructure, sets the decision rules that groups of heterogeneous agents follow and how they interact with one another. At each stage of the simulation the agents carry out some actions according to these rules, and their actions are based on the limited information they have at their disposal—hence, they are boundedly rational. These actions carried out at each stage collectively influence the observed macroeconomic behavior of the system, or macrostructure. Conversely, the resulting macrostructure accounts for much, and often all, of the information and conditions upon which agents base their decisions in the next stage of the model.

Thus, the micro and macro feedback into one another and the macroeconomic regularities of the model unfold as emergent properties (Epstein 2006, 31-38). The agent-based approach is then concerned with developing "an account of" how the aggregate regularities are attained "by a decentralized system of heterogeneous autonomous agents", or how macroeconomic phenomena of interest can be *grown* (Epstein 2006, 8), and therein lies the explanatory power of agent-based models.[4] While ABMs can operate at a high level of abstraction and rest on assumptions that may not necessarily be realistic, they have an advantage over mathematical modeling techniques in that they can capture *real* dynamics operating in an economy (Axtell 2010, 36).

This paper explores an area of work recently taken up by Wright (2008, 2011a,b) that demonstrates the emergence of Marx's "law of value" in an agent-based framework. The basic framework of the agent-based approach

[4] Interestingly enough, Epstein (2006) notes that even perfect knowledge of individual decisions does not necessarily permit prediction of a macroeconomic structure: "...a crucial lesson of Schelling's segregation model, and of many subsequent Cellular Automation models, such as 'Life'—not to mention agent-based models themselves—is that *even perfect knowledge of individual decision rules does not always allow us to predict macroscopic structure* [original italics]" (Epstein 2006, 21). Additionally, this agent-based approach, which is inherently a complexity approach, yields the "deepest conceivable critique of rational choice theory" (Epstein 2006, 26), "One limitation [of rational choice] stems from the possibility that the agent's problem is in fact undecidable, so that no computational procedure exists which for all inputs will give her the needed answer in finite time. A second limitation is posed by computational complexity in that even if her problem is decidable, the computational cost of solving it may in many situations be so large as to overwhelm any possible gains from the optimal choice of action" (Albin 1998, 46).

to modeling the dynamics envisaged by the Classical Political Economists and Marx consists of an abstract model in which there are many agents who produce commodities and engage in decentralized exchange. These agents also decide how to allocate their labor time in production based on the prices that emerge through exchange. Thus, the quantities of different commodities available at each stage of the model are determined by the latest available prices, and the next prices to emerge from the market will be influenced by the available quantities of commodities. This formulation captures a type of dynamic price-quantity adjustment that causes the system to gravitate around its long-period equilibrium position.

As Wright (2011a,b) correctly notes, this particular class of models can be seen as an outgrowth of the "cross-dual" dynamics literature. The cross-dual dynamics literature explores models of dynamic price-quantity adjustment processes featuring feedbacks between prices and quantities.[5] This literature treats the price-quantity adjustment processes as systems of dynamic equations and introduces additional factors in the adjustment process described above. The most common addition is that firms (or capitalists) respond to profit rate fluctuations in making their output decision. Examples of such an approach can be found in Duménil and Lévy (1989, 1990, 1991), Flaschel and Semmler (1987), Flaschel (1990), and Semmler (1990). Additionally, a number of variations of the cross-dual setup exhibit convergence to, and stability of, the long-period equilibrium.

One benefit of an agent-based approach to Classical Political Economy and Marx's "law of value" over those of the "cross-dual" literature is the flexibility to explicitly incorporate decentralized market exchange in addition to production in the simulation procedures. A thorough study of the stability of systems of production and decentralized exchange has been undertaken by Fisher (1983). However, Fisher (1983)'s approach focuses on the general equilibrium model of Arrow and Debreu (1954) and the potential stability of an adjustment process toward a Walrasian equilibrium.[6] Agent-based approaches to understanding the stability of general equilibrium have been

[5]The term "cross-dual" is first used to describe models of dynamic price-quantity adjustment processes by Morishima (1976, 1977), and, as Semmler (1990) notes, Goodwin (1970) used the term "cross-field" to describe a similar dynamic adjustment process.

[6]The process studied by Fisher (1983) is known as the "Hahn process" originating in Hahn and Negishi (1962).

formulated by Gintis (2007, 2012) with certain caveats stemming from evolutionary game theory. The notion of equilibrium employed in this paper, and by the Classical Political Economists and Marx, is different.[7] The equilibrium is a center of gravity around which the system oscillates over long periods of time.

This paper aims to replicate Wright (2008)'s results of the emergence and stability of Marx's "law of value" in an agent-based computational model. However, adjustments are made to the behavior of the agents in his framework in order to better capture the dynamics of the mobility of laborers described by Cogliano (2011) and Adam Smith. The model presented also lays the foundation for future study of Marx's vision of a capitalist economy with the class division between labor and capital.[8]

2 Labor Mobility in Smith and Marx

The mobility of labor in the long-period method of the Classicals is of critical importance to our understanding of the dynamics of wages and the conditions of workers in capitalist society. Smith characterizes his long-period method as the condition in which "perfect liberty" exists. Smith's perfect liberty constitutes the conditions in which all of the producers in his "early and rude state of society", or labor and capital in a capitalist economy, are mobile across different lines of production and free to seek out what is most "advantageous" for themselves. Smith's long-period vision is clear in his discussion of the mobility of labor:

> The whole of the advantages and disadvantages of the different employments of labour and stock must, in the same neighbourhood, be either perfectly equal or continually tending to equality. If in the same neighbourhood, there was any employment evidently either more or less advantageous than the rest, so many people would crowd into it in the one case, and so many would desert it in the other, that its advantages

[7] See Garegnani (1976) for further discussion on the differences between the Walrasian and Classical notions of equilibrium.

[8] This is just one stage in a process that hopes to answer the call made by Yakovenko (2009) for more analysis of capitalist economies as systems which operate with two primary social classes. Yakovenko (2009) provides strong empirical evidence that supports treatment of capitalist economies as a two-class system as done by Marx (Yakovenko 2009, 20).

> would soon return to the level of other employments. This at least would be the case in a society where things were left to follow their natural course, where there was perfect liberty, and where every man was perfectly free both to chuse what occupation he though proper, and to change it as often as he thought proper. Every man's interest would prompt him to seek the advantageous, and to shun the disadvantageous employment. Pecuniary wages and profit, indeed, are every-where in Europe extremely different according to the different employments of labour and stock. But this difference arises partly from certain circumstances in the employments themselves, which, either really, or at least in the imaginations of men, make up for a small pecuniary gain in some, and counter-balance a great one in others; and partly from the policy of Europe, which no-where leaves things at perfect liberty (Smith 2000, 114).

The above passage makes clear Smith's long-period vision for the movements of the "whole of the advantages and disadvantages" of labor, and how the advantages and disadvantages constantly tend toward equality if the conditions of perfect liberty are present. However, he makes clear at the end of the passage that the conditions of perfect liberty amount to an abstraction, or theoretical tool, which allows one to uncover the motions of the advantages and disadvantages of labor, but there are all kinds of real-world frictions that mask and obscure the real tendency underlying observable reality.

Smith then continues and explains the five factors that account for differences in wages and finishes his discussion of the whole of the advantages and disadvantages of labor by noting that differences in wages (or frictions in labor markets) do not account for any change in the equalization process of the advantages and disadvantages of labor:

> The five circumstances above mentioned, though they occasion considerable inequalities in the wages of labour and profits of stock, occasion none in the whole of the advantages and disadvantages, real or imaginary, of the different employments of either. The nature of those circumstances is such, that they make up for a small pecuniary gain in some, and counter-balance a great one in others.
>
> In order, however, that this equality may take place in the whole of their advantages or disadvantages, three things are requisite even where there is the most perfect freedom. First the employments must be well known and long established in the neighbourhood; secondly, they must be in their ordinary, or what may be called their natural state; and, thirdly, they must be the sole or principal employments of those who occupy them (Smith 2000, 131-2).

Marx recognizes that the mobility of producers in Smith produces a tendency for wages and work conditions to equalize across sectors, while the mobility of capital induces the equalization of the rate of profit. Marx fully accepts Smith's mobility of labor that equalizes the "whole of the advantages and disadvantages" of labor across sectors of production and his description of the causes of wage differentials (Cogliano 2011). As much is evident in the following passage:

> As far as the many variation in the exploitation of labour between different spheres of production are concerned, Adam Smith has already shown fully enough how they cancel one another out through all kinds of compensations, either real or accepted by prejudice, and how therefore they need not be taken into account investigating the general conditions, as they are only apparent and evanescent (Marx 1981, 241).

However, he asserts that differences in wages are only "evanescent" and not relevant for an analysis which seeks to reveal the underlying regulative forces of a capitalist economy (Marx 1981, 241-242). Marx endorses Smith's analysis in regard to these points because he feels that it reveals the underlying tendency induced by the mobility of labor:

> If capitals that set in motion unequal quantities of living labour produce unequal amounts of surplus-value, this assumes that the level of exploitation of labour, or the rate of surplus-value, is the same, at least to a certain extent, or that the distinctions that exist here are balanced out by real or imaginary (conventional) grounds of compensation. This assumes competition among the workers, and an equalization that takes place by their constant migration between one sphere of production and another. We assume a general rate of surplus-value of this kind, as a tendency, like all economic laws, and as a theoretical simplification; but in any case this is in practice an actual presupposition of the capitalist mode of production, even if inhibited to a greater or lesser extent by practical frictions that produce more or less significant local differences, such as the settlement laws for agricultural labourers in England, for example. In theory, we assume that the laws of the capitalist mode of production develop in their pure form. In reality, this is only an approximation; but the approximation is all the more exact, the more the capitalist mode of production is developed and the less it is adulterated by survivals of earlier economic conditions with which it is amalgamated (Marx 1981, 275).

Hence, the equalization of the rate of surplus-value as outlined in the above passage should be treated as one of the central tendencies in Marx's theory of value. The purpose of this project is to build the foundation to correctly capture the mobility of labor inherent in Smith and Marx's vision so that it may be further built upon to fully represent the central tendencies and dynamics of Marx's theory of value and analysis of capitalism.

2.1 Two Laws of Exchange

The mobility of labor that Smith and Marx emphasize, and which is implicit in Ricardo, can be brought into proper focus by situating the labor theory of value in the long-period method as done by Foley and Duménil (2008a) and Foley (2011). This approach separates the labor theory of value into a two-layered abstraction that reveals the importance the mobility of labor and capital inherent in the long-period method. The first abstraction is the commodity law of exchange. The commodity law of exchange can be built by supposing there is a world in which there are multiple different lines of production (each with its own commodity), many producers who are mobile across lines of production, and producers possess their own means of production. The producers spend time working in the lines of production to make their tools and produce commodities and then exchange with producers in the other sectors to obtain other commodities for their consumption. If the producers in different sectors spend different amounts of time laboring to create their tools and commodities, then the rate at which the commodities exchange for one another across sectors will come to oscillate around centers of gravity at which the labor-times embodied in the commodities being exchanged are roughly equal. If the rates of exchange across sectors are not proportional to embodied labor-times, producers will migrate across sectors until the rate of exchange is once again proportional to embodied labor-times.

The constant movement of producers into and out of lines of production continually expands and contracts the supply in those sectors and causes the system to continually oscillate around its long-period equilibrium position. The long-period position is where the distribution of producers across lines of production is such that the rate of exchange of commodities across sectors is exactly proportional to embodied labor-times. The mobility of producers also has the effect of equalizing the "reproductive condition" (Foley 2011, 16), or the returns to individual effort, of producers across lines of production. The abstraction of the commodity law of exchange closely resembles Smith's "early and rude state of society", which is accepted by Marx, and acts as the abstraction in which commodities exchange at their values. The mobility of producers in the commodity law of exchange is the source from which Marx later develops the turbulently equalizing rate of surplus-value across spheres of production, and this mobility is of particular concern for this paper.

The commodity law of exchange can be expanded to better resemble the dynamics of a capitalist economy by introducing ownership of means of production (constant capital) which are bought, sold, and consumed as part of the production process, as well as the class division between labor and capital. This expanded abstraction is called the "capitalist law of exchange" (Foley and Duménil 2008a; Foley 2011). In the capitalist law of exchange capital is mobile across lines of production and this mobility induces the turbulent equalization of the profit rate. The capitalist law of exchange is seen as incorporating the commodity law of exchange into its depiction of a capitalist economy (Foley 2011, 21-22). Thus, the tendency for the rate of profit to equalize exists simultaneously with the tendency for the rate of surplus-value, or reproductive condition, to equalize across sectors (Cogliano 2011). The capitalist law of exchange captures the whole of the labor theory of value through a multi-layered abstraction in which the commodity law of exchange operates at the highest level of abstraction.[9] As a first step to building Marx's theory of value in an agent-based framework, the model discussed in the following sections simulates the commodity law of exchange and the mobility of producers—which evolves into the mobility of labor under the capitalist law of exchange—causing the tendency for the reproductive condition to equalize across sectors of production.

3 The Model

The commodity law of exchange as described in the previous section is precisely the type of abstraction which lends itself to the agent-based, or computational, framework. The turbulent motions and gravitation described in

[9] A similar emphasis on the layering of abstraction in Marx's theory of value can be found in Rubin (1990).

the commodity law are the type of properties which give it features of a complex system. Thus, an agent-based model can be developed to better understand the mobility of labor and the turbulently equalizing reproductive condition in the commodity law of exchange. Modeling the commodity law of exchange, or something similar, in a computational framework has already been pursued by Wright (2008, 2011b). The model developed in this paper draws heavily on Wright (2008) and aims to replicate Wright's results while making adjustments to a handful of aspects of the model.

The following model consists of N producers who are engaged in two sectors of production. Each sector i produces a unique commodity x_i,[10] and producers have free access to means of production. At this abstract level of direct commodity producers in a world with only two commodities, it makes sense to abstract from money and its associated dynamics. Hence, market exchange in the economy takes place by barter. The model is run for a number of time steps T, and has rules to control the dynamics of the producers' actions and interactions with one another.

3.1 Production

The two commodities produced in the model are made entirely by the producers and do not require other commodities as inputs of production. At the initialization stage producers in both lines of production are created and randomly placed throughout the world. All producers produce one unit of commodity x_i every l_i time steps (Wright 2008, 370). This yields a production vector $\mathbf{l} = (1/l_1, 1/l_2)$ with $l_1, l_2 > 0$. l_1 and l_2 are respectively labeled as `Value-Commodity1` and `Value-Commodity2` in the simulation code. l_1 and l_2 are also greater than zero but less than or equal to one so that the rate of production is rapid. Production occurs at the same rate for all producers within a sector, but can differ across sectors, hence, labor is homogeneous, but different sectors have different labor-time requirements. Thus, $1/l_i$ also provides the quantity of commodity x_i produced by a single producer in sector i during each step of the model. Once a commodity is produced it is

[10] This is the first deviation from Wright (2008). Wright's model has L commodities labeled $1 \ldots L$ and, thus, has L possible sectors of production in which producers are engaged (Wright 2008, 370). The current model begins with only two commodities in order to simplify the approach.

held in the endowment of the producer.[11]

3.2 The Market

The subjective pricing by producers and market exchange differ from the procedures outlined in Wright (2008). Wright employs a subjective pricing mechanism that generates subjective evaluations as quantities of money randomly drawn from the producers' total holding of money. The current model employs a Cobb-Douglas utility function to allow subjective pricing to be based on producers' stocks of commodities rather than a random quantity of a producer's money holdings. This method of building decentralized exchange is directly adapted from Albin and Foley (1992), and is chosen so that the prices manifesting in the market reflect the available stocks of commodities across producers in the economy. The specific form of the utility function used in the simulations is $u = x_1^{\alpha} x_2^{\beta}$. The simulation is run for the case in which $\alpha = \beta$.

3.2.1 Subjective Pricing

Producers j in sector i subjectively determine the initial offer price $p_{i,j}$ of their commodities before they enter the market to exchange with other producers. The initial price evaluation of commodity x_i is taken to be the marginal rate of substitution between the two commodities:

$$p_{i,j} = \frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = \frac{\alpha x_2}{\beta x_1}$$

with $\alpha = \beta$:

$$\frac{x_2}{x_1} \tag{1}$$

These subjective prices are determined independently by every producer in each time step of the simulation, and are always a reflection of their stocks of commodities. The use of a utility function and marginal rates of substitution is done to capture some willingness to trade of the producers and does not "imply any utilitarian theory of motivation" (Foley 2010, 117 fn. 3), or undo the model's foundation on the labor theory of value.

[11] The endowment of commodities 1 and 2 are labeled as `Commodity1` and `Commodity2` in the simulation itself.

3.2.2 Exchange

Both commodities x_1 and x_2 are exchanged in quantities determined by the parameter `Trade-Step`.[12] Provided there is a buyer b of commodity x_i, and a seller s as well, each producer brings their respective offer price, $p_{i,b}$ and $p_{i,s}$, to the market. An exchange price is then selected as the geometric mean of the offer prices $p_{i,b}$ and $p_{i,s}$.[13] Then the buyer and seller exchange their respective commodities with each other—each receives a quantity of the commodity which they do not produce themselves, and each producer gives up some quantity of the commodity they produce. This exchange generates an exchange price ρ_z, where z represents the exchange taking place between a buyer and seller at the current moment.[14] The exchange price generated in this barter exchange scheme is a relative price of the two commodities with x_2 taken as the numéraire. During each exchange between producers the amount of x_2 traded is given by `Trade-Step` and the amount of x_1 changing hands is given by $x_1 = \texttt{Trade-Step}/\rho_z$. Multiple exchange prices manifest at each time step of the simulation because there are numerous independent exchanges taking place between buyers and sellers through a subjective pricing and bargaining process. Additionally, these exchange prices are not necessarily equilibrium prices. This exchange procedure is repeated until the average offer prices in both sectors are close.[15] Thus, the producers trade until they are in the neighborhood of equilibrium prices. This is done to ensure that the market operates efficiently so that the insights of the simulation

[12] Use of a fixed trade step in exchange is adapted from Albin and Foley (1992)'s use of a fixed trade step that splits the gains from trade between two agents. The current implementation does not split the gains from trade in the same way, but sets the quantity of the numéraire commodity exchanged. The current simulation is built with the size of the trades (denoted as `Trade-Step` in the simulation code) as a parameter which can be varied, but only simulation runs with `Trade-Step` $= 0.20$ are examined.

[13] The use of the geometric mean is drawn from Albin and Foley (1992): "The choice of the geometric average is suggested by the form of the utility function and tends to protect agents who are outliers in endowment proportions from trading at very unfavorable prices" (Albin and Foley 1992, 30 fn. 3). Since the current model employs the same utility function adopting this convention is sensible. The calculation thus appears in the simulation code as $\sqrt{p_{i,b} \cdot p_{i,s}}$.

[14] Each producer's vector of exchange prices is labeled as `exchange-price` in the simulation code. This vector stores the ten most recent exchange prices for each producer.

[15] The order of which set of producers, those in sector 1 or 2, who act as the initial set of buyers and sellers is randomized over the iterations of the exchange procedure. Sometimes the producers in sector 1 will be the initial set of buyers, and sometimes it will be the producers in sector 2.

focus on the labor theory of value, and not on any peculiarities of market exchange. This type of exchange process and the equilibrium it attains are termed "catallactic" by Foley (2010), and analyses of exchange procedures with a similar flavor can be found in Axtell (2005) and Smale (1976).

3.3 Consumption

Producers consume both commodity types during each step of the simulation. During each time step the producers consume a certain proportion of their endowment of commodities. The portion of consumption is denoted by `prop-consume` in the simulation code and is applied evenly across both commodities held by the producers. This type of consumption is also adapted from Albin and Foley (1992), and the simulation is run for `prop-consume` $= 0.5$.

3.4 Reproduction

The current model produces cases where production is equal to consumption on average over the producers due to the agents' consumption of a fixed fraction of their endowment during each time step. The approach of proportional consumption holds the benefit that the producers can consume during every time step of the simulation. This is one of the bigger differences between the current model and Wright (2008)'s use of pre-determined rates of consumption that behave like a "subsistence bundle." In the current model, what could be called the "subsistence bundle" is co-determined through the proportional consumption and the overall allocation of commodities resulting from market exchange, and behaves more as an emergent property of the interactions of the agents over time.

3.5 The Division of Labor

Each producer specializes in one of two sectors, thus dedicating themselves to producing one commodity at a time. However, in order to faithfully represent the commodity law of exchange the producers must be able to move across sectors. The mobility of producers across sectors in reaction to changes in their own reproductive condition or the reproductive condition in the other sector is the key dynamic for the commodity law of exchange to reach a state where it oscillates around its long-period equilibrium. Wright (2008) establishes a sector-switching rule which works through finding an inter-temporal consumption error for all the agents. If the consumption error of an agent increases from one time step to the next then the agent switches sectors with some probability. As stated by Wright, "Dissatisfied actors switch to new sectors in search of sufficient income to meet their consumption requirements" (Wright 2008, 374). Wright's rule works through the agents laboring in one sector over enough time steps to produce, trade, and consume at least one commodity (the number of time steps is just the number of time steps it takes to consume one unit of a commodity). The sector-switching rule employed for the current model is different.

At the end of each time step the producers compare their individual reproductive condition to the average reproductive condition in the other sector.[16] The reproductive condition of producer j in sector 1 is

$$R_{1,j} = \frac{1}{\tau} \sum_{t=1}^{\tau} \rho_{1,j,t} \tag{2}$$

and the reproductive condition of producer j in sector 2 is

$$R_{2,j} = \frac{1}{\tau} \sum_{t=1}^{\tau} \rho_{2,j,t} \tag{3}$$

Equations (2) and (3) take a moving average of producer exchange prices over some number of iterations τ of the model.[17] Stated differently, an individual producer's reproductive condition is the moving average of their individual exchange prices received in the market.

The average reproductive condition $\overline{R}_i$ for sector i is the moving average of the average exchange prices $\overline{P}_{i,t}$ of producers in that sector. Calculation of the average reproductive condition follows a similar procedure as for the individual producers' reproductive condition.[18] If the $\overline{R}_i$ for the other sector

[16] The sector-switching rule does not take effect until after the first time step has passed. In the code for the simulation the sector-switching rule appears at the beginning of the program, but is not active during the first time step. Logically speaking, the sector-switching rule then occurs at the end of the production, exchange, and consumption steps because it works with information from the previous time step that has not yet been updated and modified by the production, exchange, and consumption steps.

[17] The simulation is run for the case in which $\tau = 10$.

[18] The average reproductive condition $\overline{R}_i$ that is actually used to calculate the conditions

is greater than an individual producer's $R_{i,j}$ the producer will probabilistically move to the other sector in search of better returns to their individual effort in production.[19] For the case of two sectors $(1, 2)$, if, for instance, $R_{1,j}$ of an individual producer in sector 1 is less than $\overline{R}_2$, the producer in sector 1 will migrate to sector 2 with some probability.[20] Movement of producers of this kind faithfully represents the mobility of producers in the commodity law of exchange, and as presented in Cogliano (2011). By having the producers migrate across sectors according to the conditions under which they reproduce themselves, the proper foundation is laid to develop Marx's equalized rate of surplus-value when the commodity law ABM is expanded to model the capitalist law of exchange and Marx's full theory of value.

3.6 Simulation and Parameters

Wright (2008)'s model of the emergence of Marx's law of value in a simple commodity economy has five parameters: (1) "the number of actors;" (2) "the number of commodities;" (3) the quantity of money in the econ-

for which producers migrate across sectors is calculated during each time step of the model according to a moving average so as to incorporate some "history" into the model. The moving average is calculated as $\frac{1}{\tau}\sum_{t=1}^{\tau}\overline{P}_{i,t}$, where τ is the ten previous time steps of the model and $\overline{P}_i$ is the average price of commodity i during one time step of the model as given by $\frac{1}{n_i}\sum_{j=1}^{n_i}\rho_j$ with n_i being the number of producers in sector i and ρ_j being the exchange prices of the n_i producers. This is done to slow down the migration process and stabilize the model.

[19] In addition to switching sectors based on differentials in the reproductive condition, producers can switch if they have not taken part in the market exchange, and their offer prices are sufficiently high.

[20] The probability of a producer migrating from one sector to another is determined through a logistic function of the difference between the individual producer's reproductive condition and the average reproductive condition of the other sector. For producer j in sector 1, the function is as follows: $\psi = \frac{1}{\left(1+\exp\left\{\gamma\left[(\epsilon(\overline{R}_2 - R_{1,j}) + ((1-\epsilon)\frac{x_2}{x_1}))\right]\right\}\right)}$; the respective function for a producer in sector 2 can be written similarly. ϵ is used as a dummy variable to signal whether the producer will be evaluating their reproductive condition based on the moving average of their exchange prices, or based on their offer price if they have not taken part in exchange. γ is a parameter used to adjust the shape of the logistic function as needed. ψ is taken as the probability that an individual producer will switch sectors and is compared to a number drawn randomly from a normal distribution with $\mu = 0$ and $\sigma = 0.75$. If ψ is less than the randomly drawn number, the producer switches to the other sector. One possible area of improvement to the sector switching algorithm would be a more carefully tuned distribution from which producers draw a random number to compare their probability of migrating. This could reduce some of the randomness in movements across sectors, and yield more information regarding the conditions that lead to sector switches.

omy; (4) an upper bound on the maximum possible consumption period in order to "constrain the random construction of production and consumption vectors;" (5) "a switching parameter C that is the constant multiple of the maximum consumption period required" by the sector-switching rule (Wright 2008, 374). The order of execution of Wright (2008)'s model is as follows: "Increment the global time step" $\rightarrow$ invoke the production rule for each actor $\rightarrow$ invoke the consumption rule for each actor $\rightarrow$ invoke the market clearing rule $\rightarrow$ invoke the sector-switching rule for each actor $\rightarrow$ repeat (Wright 2008, 374).

The model of the commodity law of exchange described in this paper has similar parameters (although money is notably absent), but the order of execution is slightly different. The parameters are as follows:

1. The number of agents.
2. The number of sectors.
3. The labor requirements of the commodities.
4. The size of the exchange that producers make with one another.
5. Damping parameters in the sector switching algorithm.
6. The α and β parameters of producers.

The order of execution is as follows: (i) producers in different sectors produce; (ii) producers consume both commodities; (iii) producers trade; (iv) producers switch sectors if they perceive their reproductive condition as deviating too far from the average reproductive condition of the other sector; (v) repeat until the number of time steps reaches the `end-of-world` condition in the simulation. The simulation is built using the NetLogo language and software package (Wilensky 1999).

4 Results

The current setup of the model quickly achieves the long-period equilibrium described by the commodity law of exchange for cases in which $l_1 = l_2$ and $l_1 \neq l_2$ when the simulation is started in disequilibrium. Some results for a case in which the simulation finds the desired equilibrium of the commodity law of exchange are shown in Figures 1, 2, 3, and Table 1. In this case $l_1 = l_2 = 0.20$. Figures 4, 5, 6, and Table 2 show results for a case in which

$l_1 \neq l_2$ in which the model finds the desired equilibrium as well. For this case $l_1 = 0.30$ and $l_2 = 0.10$.

Figure 1 displays the evolution of relative price and the allocation of producers across sectors beginning from the initialization stage of the model for the case in which $l_1 = l_2 = 0.2$. Figure 2 shows how the relative prices in the simulation come to oscillate around relative value. Figure 3 compares the distribution of producers across sectors 1 and 2. Figures 1, 2, and 3 fit the theoretical conclusions of the commodity law of exchange. These results are also summarized in Table 1, which provides summary statistics of the key variables in the model, and includes information on the average relative prices achieved during each time step of the model ("Relative Price") and the moving average of relative prices across all agents ("MA Relative Price"). As one can see, the mean and median values of the relative prices are close to 1, and the mean and median values of the allocation of producers across sectors are close the desired value of 150.

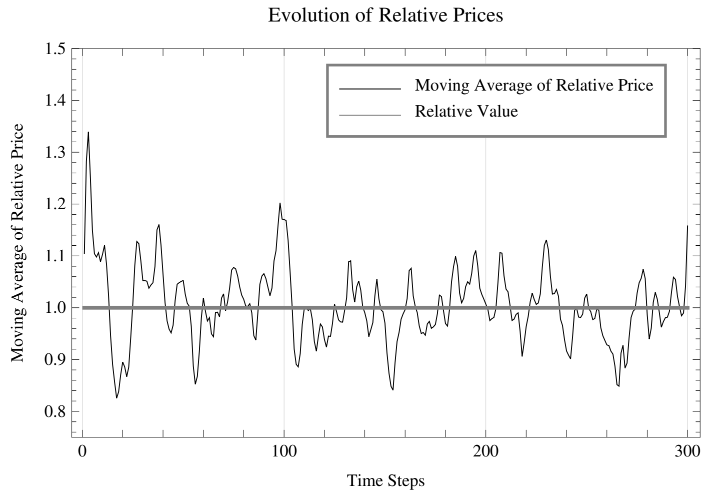
Table 1: Summary Statistics for $N = 300$, $l_1 = 0.20$, and $l_2 = 0.20$.

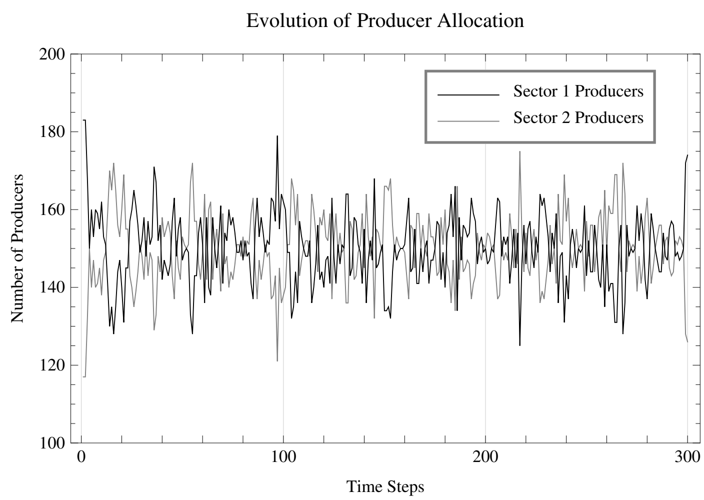
Variable	T	Mean	Median	Min	Max	Std. Dev.
MA Relative Price	1000	0.9996	0.9945	0.8124	1.339	0.0643
Relative Price	1000	1.0073	1.0038	0.8025	1.484	0.0783
Relative Value	1000	1	1	1	1	0
Sector 1	1000	150.383	150	117	177	8.854
Sector 2	1000	149.617	150	123	183	8.854

Figures 4, 5, 6, and Table 2 present results for a simulation with $l_1 = 0.30$ and $l_2 = 0.10$ that finds the desired equilibrium for relative prices. Figure 4 displays the evolution of prices and the allocation of producers from the initialization of the model and 300 iterations. Figure 5 also displays the distribution of relative prices around relative value, and Figure 6 displays the allocation of producers across sectors. The summary statistics presented in Table 2 reinforce these results. The mean of relative prices and the moving average of relative prices are close to the desired number given by relative value, and the distribution of producers across sectors fits the parameters given by the producers' utility function.

Different simulations with different parameters achieve stability and the results described by the commodity law of exchange with relative prices os-

Figure 1: Evolution of Relative Price and Allocation of Producers for: $N = 300$, $l_1 = 0.20$, and $l_2 = 0.20$ for 300 time steps.





cillating around relative values. It is easy to see when running the simulation that the relative prices are governed by relative values. This behavior can be seen by running the simulation and adjusting the values of the commodities (`Value-Commodity1`, `Value-Commodity2`) to change the relative values, and as relative values change, the trend of relative prices follows the movement of relative values. Hence, relative values are acting as an attractor for relative prices.

Figure 2: Stable Distribution of Relative Prices around Relative Value for: $N = 300$, $l_1 = 0.20$, and $l_2 = 0.20$ for 1000 time steps. Relative value is denoted by the thick vertical line visible at the top of the graph.

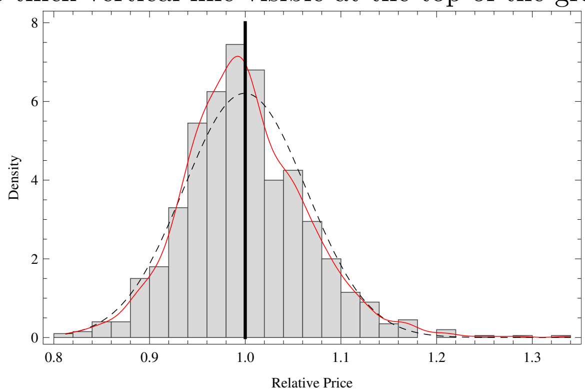


Figure 3: Distribution of Producers Across Sectors for: $N = 300$, $l_1 = 0.20$, and $l_2 = 0.20$ for 1000 time steps.

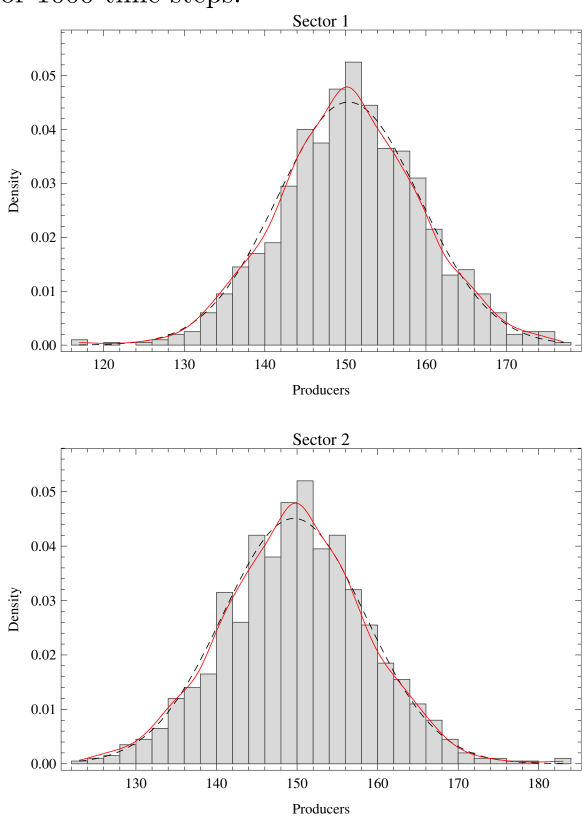
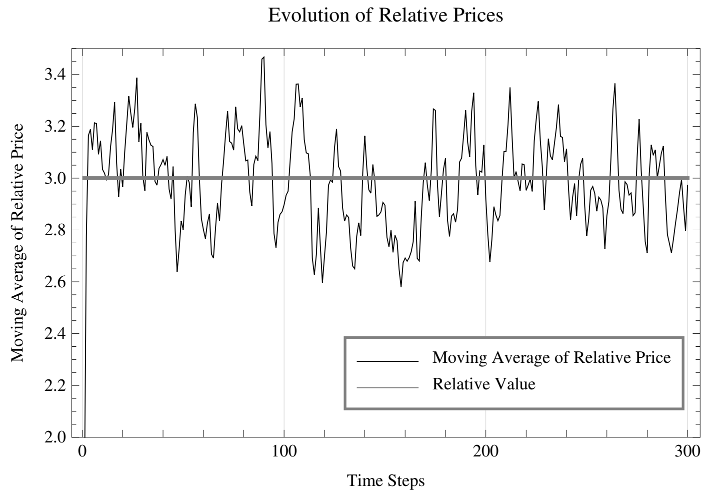
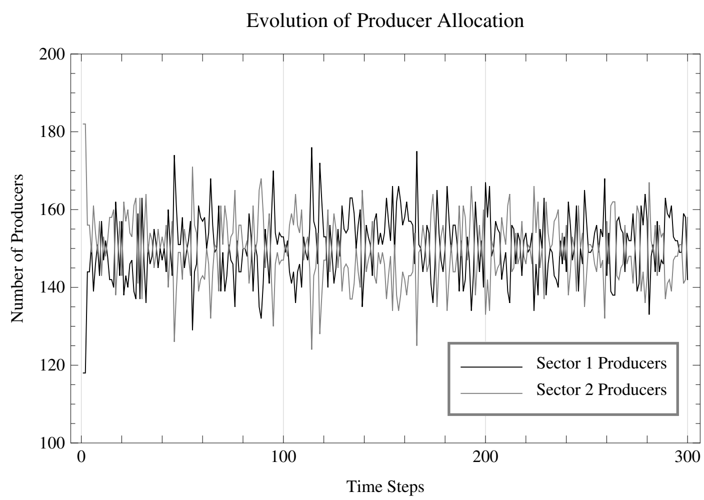


Figure 4: Evolution of Relative Price and Allocation of Producers for: $N = 300$, $l_1 = 0.30$, and $l_2 = 0.10$ for 300 time steps.





5 Conclusions and Further Work

Two ways in which this model can be expanded are the addition of more sectors and development to include the capitalist law of exchange and Marx's complete theory of value. The two-commodity setup of the model begs the question: what happens in a more general setting with three or more sectors and commodities? It has been shown that models of exchange with three or more commodities can result in cycling that does not find an equilibrium po-

Figure 5: Stable Distribution of Relative Prices compared to Relative Value for: $N = 300$, $l_1 = 0.30$, and $l_2 = 0.10$ for 1000 time steps. Relative value is denoted by the thick vertical line visible at the top of the graph.

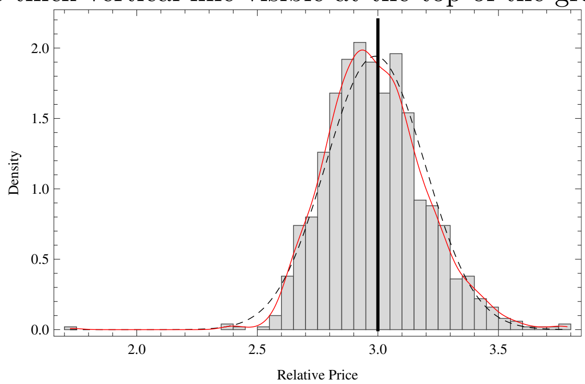


Figure 6: Distribution of Producers Across Sectors for: $N = 300$, $l_1 = 0.30$, and $l_2 = 0.10$ for 1000 time steps.

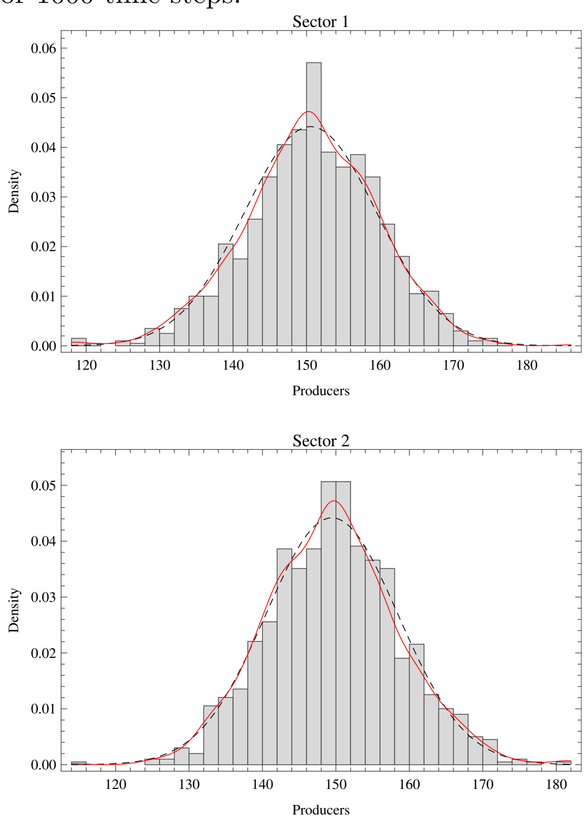


Table 2: Summary Statistics for $N = 300$, $l_1 = 0.30$, and $l_2 = 0.10$.

Variable	T	Mean	Median	Min	Max	Std. Dev.
MA Relative Price	1000	2.9895	2.9756	1.7247	3.7845	0.2056
Relative Price	1000	3.00078	2.9787	2.2222	3.8882	0.2388
Relative Value	1000	3	3	3	3	0
Sector 1	1000	150.544	151	118	186	9.036
Sector 2	1000	149.456	149	114	182	9.036

sition (Scarf 1960, 1973). This model could benefit from analyzing whether or not the addition of more commodities produces the cycling found in Scarf (1960)'s examples, and whether or not such cycling matters for the notion of equilibrium employed in this model. Expansion of the model to include the capitalist law of exchange and the independent equalization of the sectoral rates of surplus-value and profit rates will also yield improvements in the insights that can be garnered regarding macroeconomic phenomena like business cycles and unemployment, and microeconomic issues concerning market structure, industrial organization, and labor mobility.

However, the results lend credence to the overall approach to value theory and political economy presented in the paper. The results show that an agent-based computational model of production and exchange, in which agents engage in barter exchange and make decisions regarding the allocation of their labor time in production, approximates the long-period equilibrium of the labor theory of value envisaged by the Classical Political Economists and Marx.

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